Non-Abelian topology: how to model it, and how to search for its effects?

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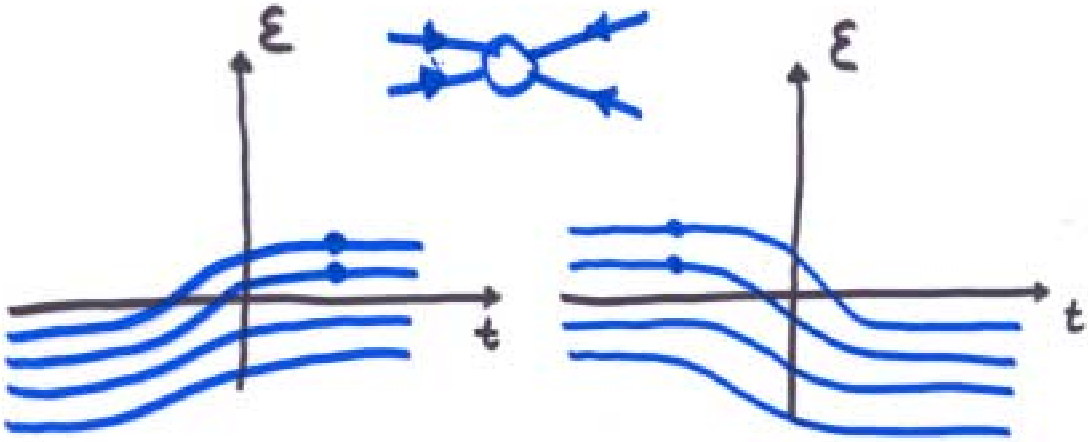
- P-odd bubbles are characterized by non-zero topological charge:

$$\nu[G] = \frac{g^2}{32\pi^2} \int d^4x \, Tr\left(G_{\mu\nu} \tilde{G}^{\mu\nu}\right)$$

and cause chirality violation:

$$\Delta Q_5 = 2 N_f \, \nu[G] = N_R - N_L$$

Slide 2



LEFT RIGHT

FERMIONS
IN THE EXTERNAL TOPOLOGICAL
FIELDS

- Problem: do not know how to model non-Abelian classical gauge fields
- Is it possible to find a simpler model which would incorporate the same (similar) physics?

Slide 3

What is the most important feature of topological solutions?

- the link between the spatial and internal symmetries

sphere $S_3 \longleftrightarrow SU(2)$

3 angles $\qquad 2^2 - 1 = 3$ parameters

- "spin from isospin"

$$\vec{J} = \vec{L} + \vec{T}$$

total momentum ($\vec{J}$); orbital momentum ($\vec{L}$); $\vec{T}$ — SU(2) generator

Slide 4

e.g., instanton,

$$A_\mu = -i\,\eta_{\mu\nu a}\, x_\nu\, \tau_a \frac{1}{r^2+\rho^2}$$

$$\eta_{0ia} = -\eta_{i0a} = \delta_{ia}$$

$$\eta_{ija} = \varepsilon_{ija}$$

$$F_{\mu\nu} = 2i\,\eta_{\mu\nu a}\,\tau_a \frac{\rho^2}{(r^2+\rho^2)^2}$$

different colors ⟷ different directions

... but how to model?

- lesson: in SU(2), no difference between quarks and antiquarks
  meson = $q\bar{q}$ baryon = qq
  $2 \otimes 2$

Slide 5

Consider "magnetic monopole"

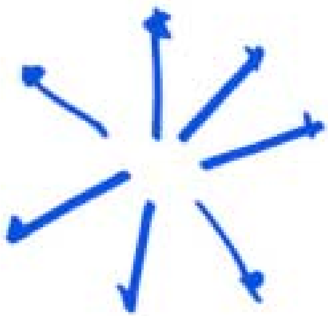
't Hooft
Polyakov, '74

again, $SU(2) \leftrightarrow S_3$

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2}(D_\mu \varphi)^a (D_\mu \varphi)^a - \frac{\lambda}{4}(\varphi^a \varphi^a - v^2)^2$$

solution

$$A_i^a = \frac{1}{gr} \underbrace{\varepsilon^{aij}}_{\text{like 't Hooft symbol}} \hat{n}_j$$



magnetic field

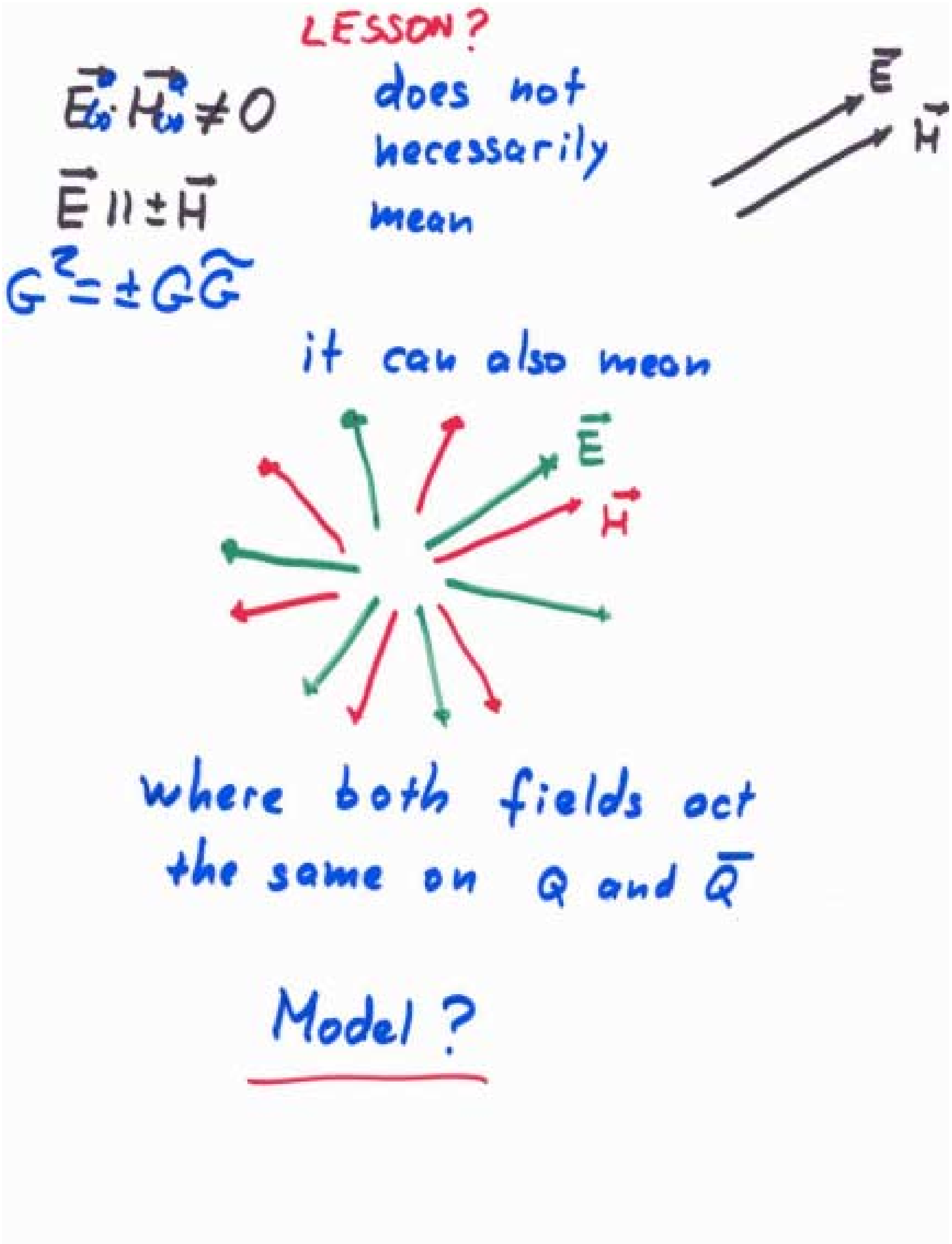
$$H_i^a = \frac{1}{gr^2} \hat{n}_i \hat{n}_a$$

$\downarrow$

$$H_i = \frac{1}{gr^2} n_i$$

$\frac{1}{g} \equiv g_M$ "magnetic charge"

Slide 6


LESSON?
$\vec{E}^a_{c\omega} \cdot \vec{H}^a_{c\omega} \neq 0$
$\vec{E} \parallel \pm \vec{H}$
$G^2 = \pm G\tilde{G}$
does not necessarily mean
$\vec{E}$
$\vec{H}$
it can also mean
$\vec{E}$
$\vec{H}$
where both fields act the same on Q and $\bar{Q}$
Model?