Parity Violation

in

High Energy Nuclear Collisions

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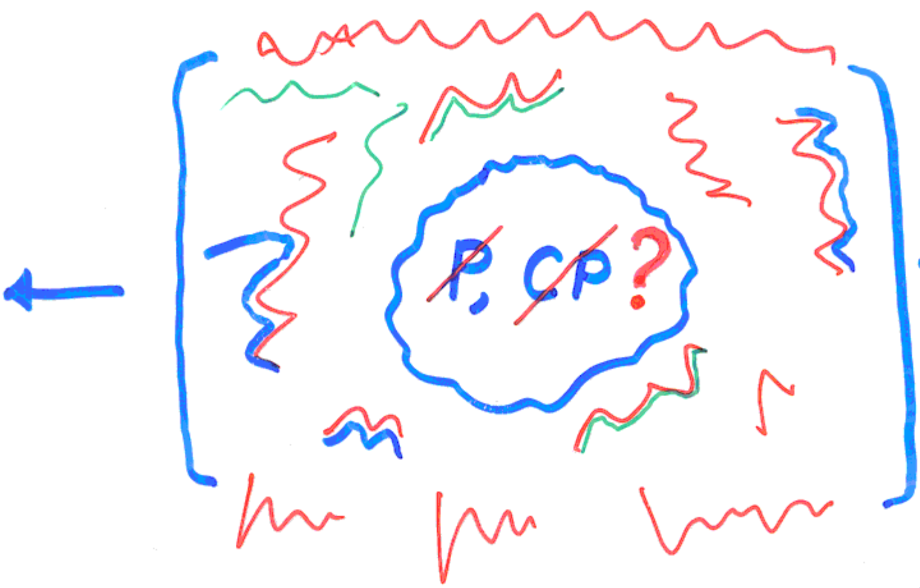


1) D. K., R. Pisarski, M. Tytgat Phys. Rev. Lett. 81(9

2) D. K., R. Pisarski Phys. Rev. D '99

SYMMETRIES ARE THE MOST FUNDAMENTAL PROPERTIES OF THE WORLD

YET, IN QCD, EVEN P, CP, T INVARIANCES REMAIN AN OPEN PROBLEM

WHY? P, CP VIOLATION IN STRONG INTERACTIONS WAS NEVER OBSERVED...

IN PHENOMENOLOGY,

- THE PROBLEM IS CAUSED BY ONE SINGLE PARTICLE – THE $\eta'(958)$

WHAT IS SO SPECIAL ABOUT THE η'?

CHIRAL $U_L(3) \times U_R(3)$ SYMMETRY OF QCD IS SPONTANEOUSLY BROKEN

$$\Downarrow$$

$$3^2 = 9 \text{ GOLDSTONE BOSONS}$$

$$3\pi\text{'s},\ 4K\text{'s},\ \eta,\ \eta'\ ?$$

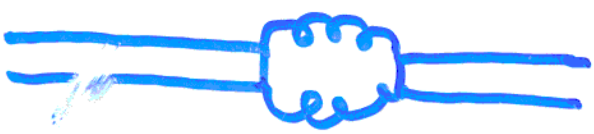
$$3+4+1+1$$

BUT: η' IS VERY MASSIVE, $M_{\eta'} > M_{\text{PROTON}}$

WHY? CONSIDER THE FIELD

$$\eta_0 = \frac{1}{\sqrt{3}} |\bar{u}u + \bar{d}d + \bar{s}s\rangle$$

SINCE IT IS FLAVOR SINGLET, IT CAN ANNIHILATE INTO GLUONS:

η_0 

THIS INTRODUCES GLUONIC PIECE IN THE DIVERGENCE OF THE FLAVOR SINGLET AXIAL CURRENT:

$$\partial^\mu J^0_{5\mu} = 2i \sum_f m_f \bar{q}_f \gamma_5 q_f + \underline{2N_f \frac{g^2}{16\pi^2} Tr(G\tilde{G})}$$

THIS PIECE DOES NOT VANISH IN THE CHIRAL LIMIT: "AXIAL ANOMALY"

J. Bell, R. Jackiw, S. Adler '69

- NOT A PROBLEM YET, SINCE THE "ANOMALOUS" PIECE IS A FULL DIVERGENCE:

$$2N_f \frac{g^2}{16\pi^2} Tr(G\tilde{G}) = \partial^\mu K_\mu ,$$

$$K_\mu = 2N_f \frac{g^2}{16\pi^2} \epsilon_{\mu\nu\lambda\rho} Tr(G^{\nu\lambda} A^\rho)$$

– GAUGE DEPENDENT GLUONIC CURRENT...

RE-DEFINE

$$J_{5\mu} \equiv J^{0}_{5\mu} - K_{\mu} ;$$

NOW, THIS CURRENT IS CONSERVED:

$$\partial^{\mu} J_{5\mu} \xrightarrow[m_f \to 0]{} 0$$

AND THE CORRESPONDING CHARGE MUST BE CONSERVED AS WELL:

$$Q_5 = \int d^3x \, J_{50} \qquad \frac{dQ_5}{dt} = 0 \; ?$$

LET US CHECK THIS:

$$\int_{-\infty}^{\infty} dt \, \frac{dQ_5}{dt} = 2N_f \, \nu[G],$$

$$\nu[G] = \frac{g^2}{32\pi^2} \int d^4x \, \mathrm{Tr}\left(G_{\mu\nu} \tilde{G}^{\mu\nu}\right)$$

- IN QCD, THERE EXIST CLASSICAL SOLUTIONS WITH $\nu \neq 0$!

FOR THE ONE-INSTANTON CONFIGURATION, FOR EXAMPLE,

$$\nu[G_{inst}] = 1$$

$\Rightarrow$ Q_5 IS NOT CONSERVED:

FROM $t = -\infty$ TO $t = +\infty$ IT CHANGES BY

$$\Delta Q_5 = 2N_f \, \nu[G]$$

NON-PERTURBATIVE TOPOLOGICAL SOLUTIONS EXPLICITLY BREAK $U_A(1)$

$$\Downarrow$$

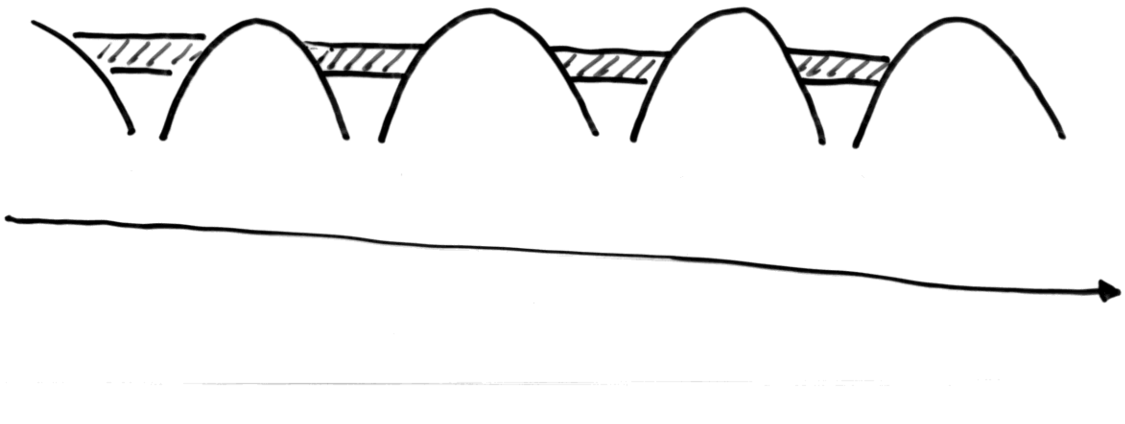
THERE SHOULD BE NO Goldstone, η' CAN BE MASSIVE

- IN THE PRESENCE OF VACUUM SOLUTIONS WITH DIFFERENT TOPOLOGICAL NUMBERS, ν, THE VACUUM WAVE FUNCTION TAKES THE FORM

$$|\theta\rangle = \sum_{\nu} e^{i\theta\nu} |\nu\rangle$$

ANALOGOUS TO BLOCH WAVE IN A CRYSTAL:

$$|\vec{k}\rangle = \sum_{\vec{x}} e^{i\vec{k}\vec{x}} |\vec{x}\rangle$$



LET US COMPUTE AN EXPECTATION VALUE OF AN OBSERVABLE:

$$\langle O \rangle = \sum_{\nu} e^{i\theta\nu} \int d[\psi, \bar{\psi}]\, d[A]\, e^{i\int \mathcal{L}(\psi,A)\, d^4x} \left[O(\psi,A)\right]_{\nu}$$

WHERE $\nu = \frac{g^2}{32\pi^2} \int d^4x \, \mathrm{Tr}(G\hat{G})$

THIS PRESCRIPTION IS EQUIVALENT TO ADDING A NEW TERM TO THE QCD LAGRANGIAN:

$$\mathcal{L}_{QCD} \longrightarrow \mathcal{L}'_{QCD} = \mathcal{L}_{QCD} + \underline{\theta \frac{g^2}{32\pi^2} \mathrm{Tr}(G\tilde{G})}$$

- THIS TERM IS P, CP ODD! ($G\hat{G} \sim \vec{E}\cdot\vec{H}$)

... WE TRADED $U_A(1)$ PROBLEM FOR AN EVEN MORE SERIOUS ONE — STRONG CP PROBLEM...

Example:

an effective Lagrangian including $U_A(1)$ term
(non-linear σ-model)

G. Veneziano,
P. di Vecchia;
E. Witten

$$\mathcal{L} = \frac{F_\pi^2}{2}\Big\{ \underbrace{\mathrm{Tr}\,\partial_\mu U \partial_\mu U^{-1}}_{U(3)\times U(3)\ \text{invariant}} + \underbrace{(\mathrm{Tr}\,MU + \mathrm{Tr}\,MU^+)}_{\text{under } SU(3)\times SU(3) \text{ transforms as quark mass term}} -$$

$$- \underbrace{\frac{a}{N}\left(-i \ln \det U - \theta\right)^2}_{\text{preserves } SU(3)\times SU(3),\ \text{reflects } U_A(1) \text{ anomaly}} \Big\}$$

$$U = \exp\left(i \frac{\Phi}{F_\pi}\right)$$

$$a \sim \int d^4x \left\langle T\{ G_{\mu\nu}\tilde{G}^{\mu\nu}(x), G_{\mu\nu}\tilde{G}^{\mu\nu}(0)\} \right\rangle_{YM}$$

The angle θ is severely constrained

by D_n, $\eta \to \pi\pi$ $\quad |\theta| < 10^{-9}$

We will assume $\theta = 0$

Effective potential (vacuum energy)

$$V_{eff}(U) = F_\pi^2\left(-\frac{1}{2}\,\mathrm{Tr}\,MU - \frac{1}{2}\,\mathrm{Tr}\,MU^+ + \frac{a}{2N}\left(-i\ln\det U\right)^2\right)$$

assume $m_u = m_d$

$$M = \mathrm{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2) \equiv \mathrm{diag}(\mu^2, \mu^2, \mu_s^2)$$

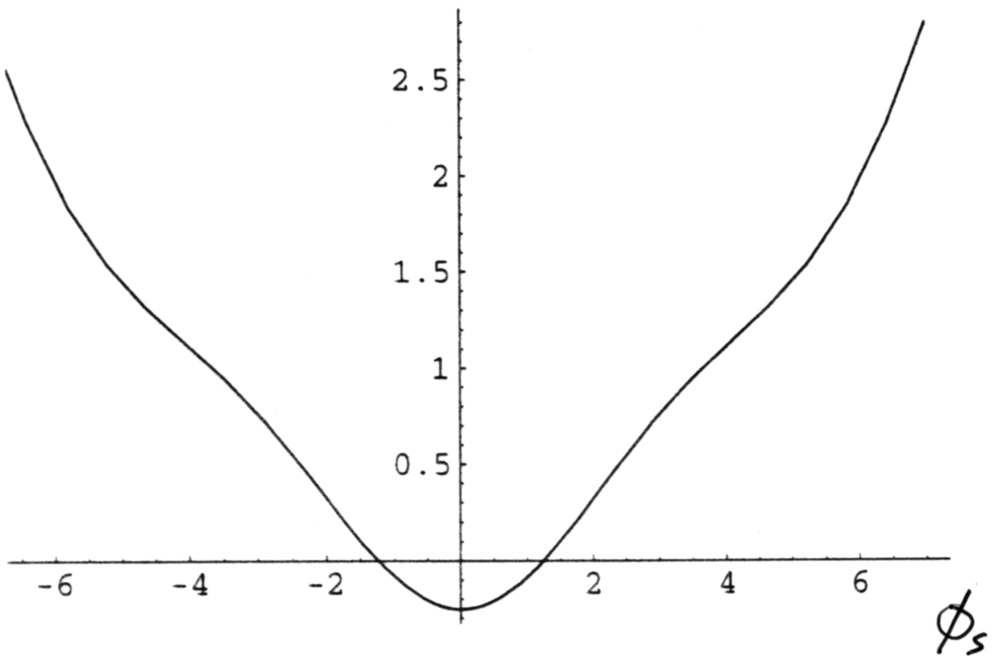
$$U = \begin{pmatrix} e^{i\phi_1} & & 0 \\ & e^{i\phi_2} & \\ 0 & & e^{i\phi_3} \end{pmatrix}$$

In terms of ϕ's, the effective potential

$$V_{eff}(\phi_i) = F_\pi^2\left[-\sum_i \mu_i^2\cos\phi_i + \frac{a}{2N}\left(\sum_i \phi_i - \theta\right)^2\right]$$

How does it look like?

$\rightarrow$ Fig



2.5
2
1.5
1
0.5
-6
-4
-2
2
4
6
ϕ_s

At $\theta=0$, (we do not consider $\theta \neq 0$, Dashen phenomena, etc)

only trivial solution

$$\langle \phi_u \rangle = \langle \phi_d \rangle = \langle \phi_s \rangle = 0$$

But: @ high density, instantons are screened away

+ large N arguments:

$$T_d \simeq T_{U(1)}$$

$\Downarrow$

D. Gross
R. Pisarski
L. Yaffe

When density grows,

R. Pisarski
F. Wilczek

$$a \sim \int d^4x \langle T\{ G_{\mu\nu}\tilde{G}^{\mu\nu}(x), G_{\mu\nu}\tilde{G}^{\mu\nu}(0) \} \rangle$$

E. Shuryak
M. Velkovsky

. . . . . .

should decrease – evidence from lattice calculations

- Lattice?
  → fig

Does the behavior of the effective potential change?

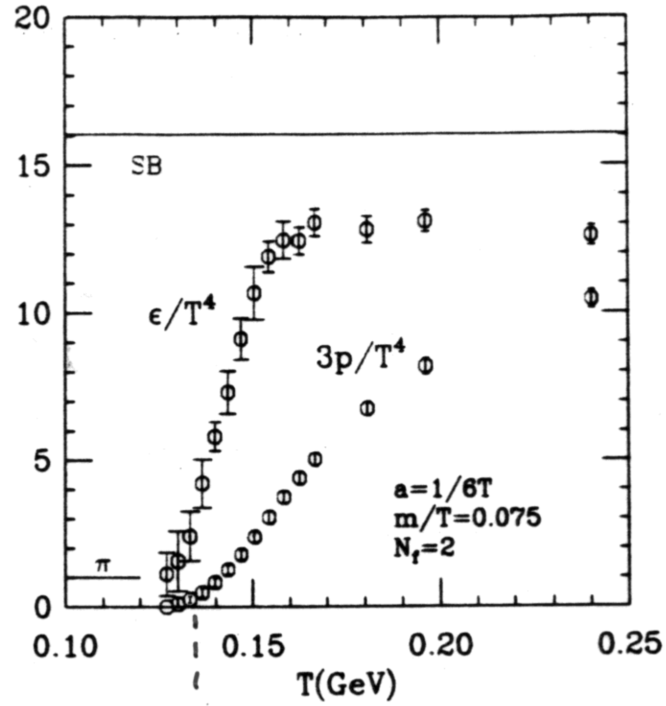
$a \to 0.4a$ @ $T_c - \epsilon$?

YES

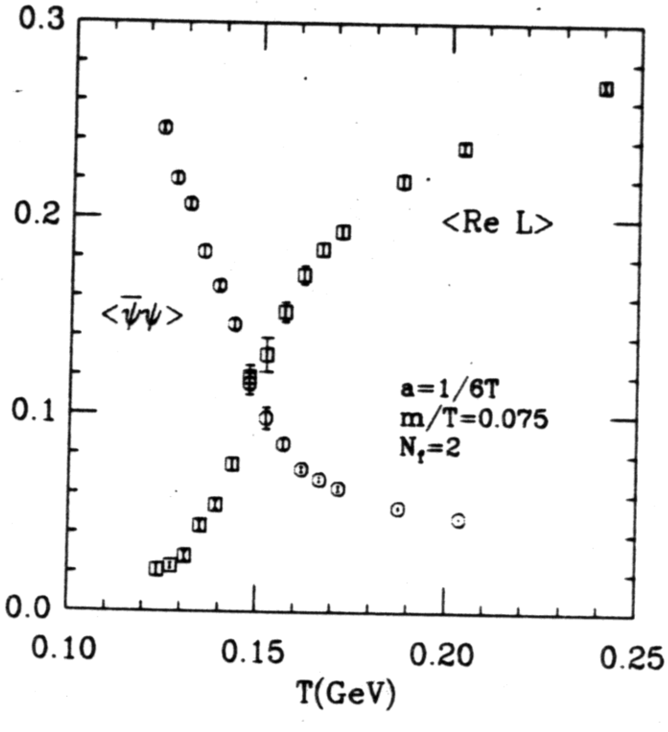
→ figure

T. Blum et al., PRD51(95)5153

2-flavor QCD



# of degrees of freedom: $\sim N_c^0$ | $\sim N_c^2$



$\sim e^{-E_q/T}$

$\langle\bar\psi\psi\rangle = \langle\bar\psi_L\psi_R\rangle + \langle\bar\psi_R\psi_L\rangle$

B. Allés
M. D'Elia,
A. Di Giacomo
P.W. Stephenson
hep-lat/9808004

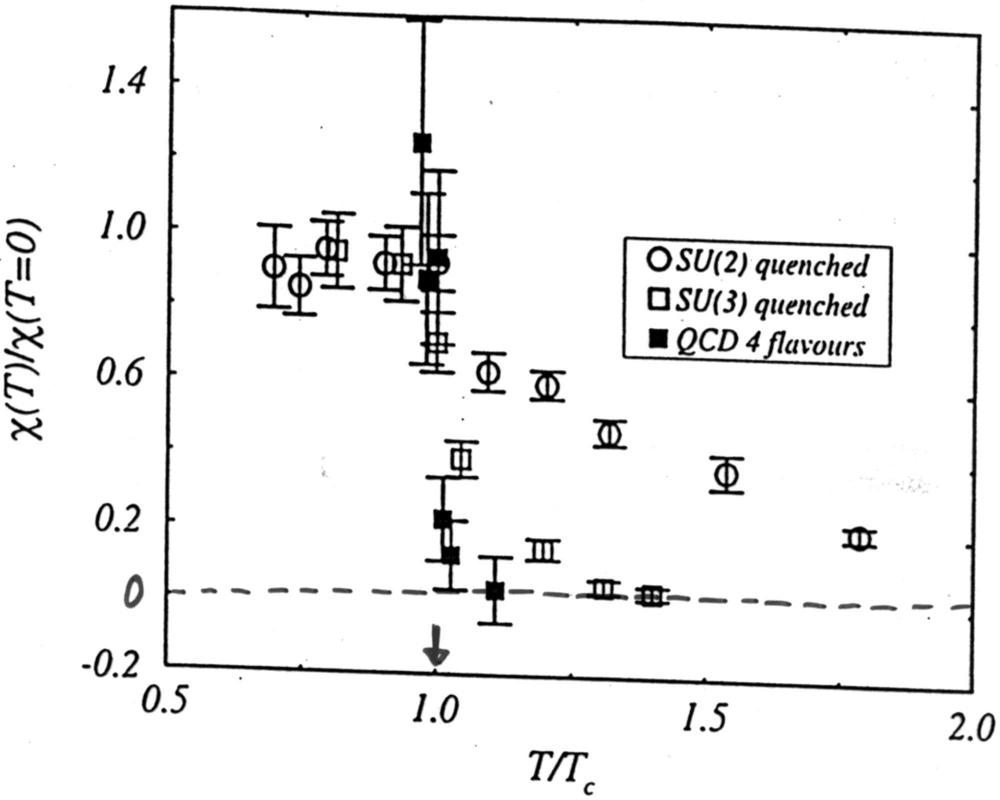


Figure 3. Behaviour of the topological susceptibility as a function of the normalized temperature T/T_c.

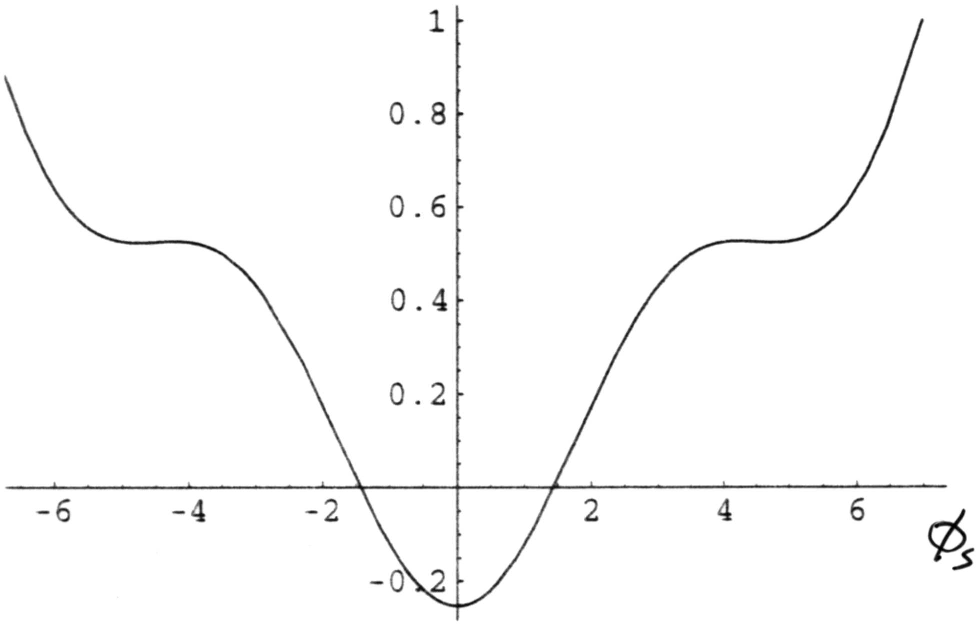
<u>large N_c</u>:

- below T_c, interactions are suppressed by $1/N_c$,
  # of degrees of freedom $\sim N_c^0$
  $T_c \sim N_c^0$
  => "cold" gas of glueballs and mesons
- above T_c, # of degrees of freedom $\sim N_c^2$

=> huge change of the free energy at T_c

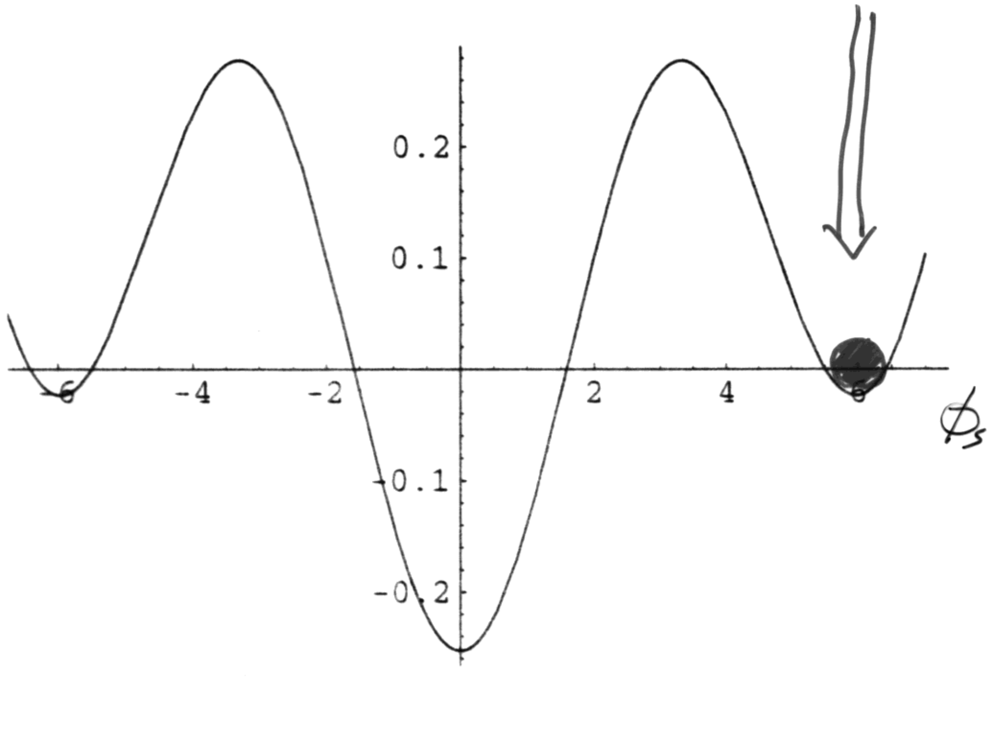
$\Downarrow$

<u>any</u> phase transition occurs at T_c



1
0.8
0.6
0.4
0.2
-0.2
-6
-4
-2
2
4
6
ϕ_s

Metastable,
CP & P odd, vacuum!



The additional minima are <u>local</u>;
they have the energy density $\epsilon > \epsilon_{\text{true vacuum}}$,
so they do not contribute to the partition function in the $V \to \infty$.

$\Rightarrow$ $\boxed{\text{does not contradict to Vafa-Witten theorem}}$

<u>But</u>: they describe metastable, "false" vacua which can be excited (at RHIC, for example.)

These metastable vacua contain $\eta - \eta'$ condensate $J^{PC} = 0^{-+}$

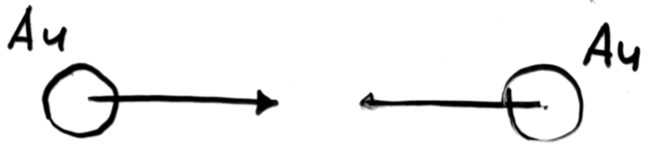
$$\Downarrow$$

Massive violation of P, CP, and ~~(possibly)~~ isospin

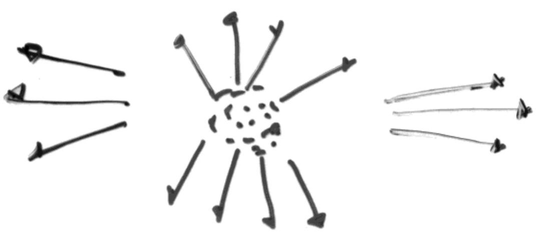
Signatures

- indirect – enhanced yields of η, η' mesons (coupled to $U_A(1)$ anomaly)

- direct –



P-even initial state



P-odd final state?

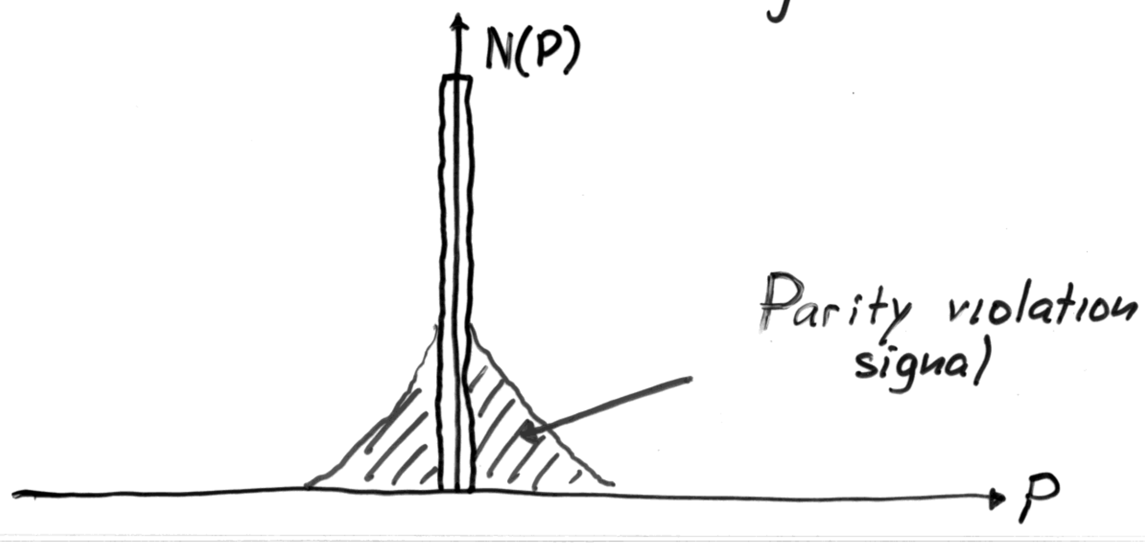
1) Measure P-odd global pionic observables, like

$$\Rightarrow \quad P = \sum_{\pi^+\pi^-} \frac{[\vec{p}_{\pi^+} \times \vec{p}_{\pi^-}] \cdot \hat{\vec{z}}}{|\vec{p}_{\pi^+}| \cdot |\vec{p}_{\pi^-}|}$$

e.g., beam axis

sum over all $\pi^+\pi^-$ pairs <u>in a given event</u>

2) Plot the number of events with a given P:



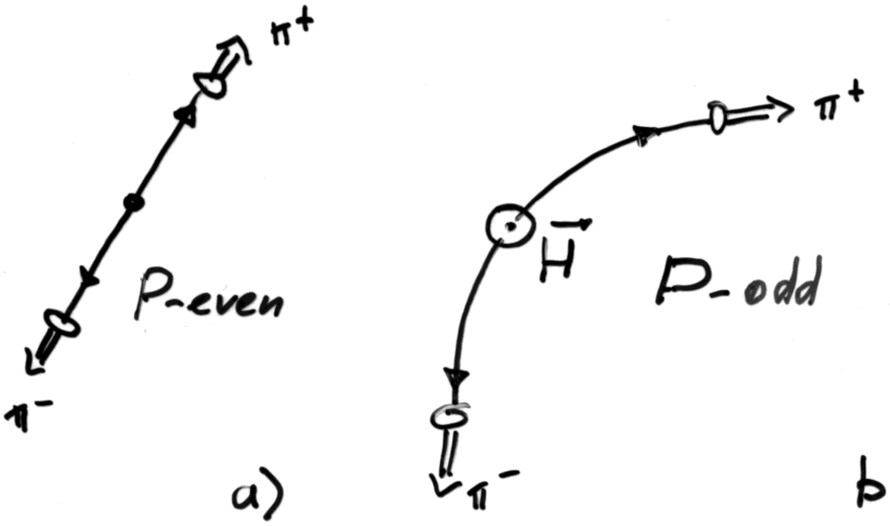
Consider a set of vectors $\vec{a}^{n} \in R^3$

- How to construct P–odd invariants?

H. Weyl '46 : All of them can be represented as a sum of terms, each of which contains one binary vector product

- Physical picture:

P-odd bubbles contain $\langle G\tilde{G}\rangle \sim \langle \vec{E}\cdot\vec{H}\rangle \neq 0$



a)

no field;

$[\bar{p}_+ \times \bar{p}_-] = 0$

b)

in the presence of the chromo-magnetic field,

$[\bar{p}_+ \times \bar{p}_-] \neq 0$

P-odd observables:

- $$J = \sum_{\pi^+,\pi^-} (\hat{\vec{p}}_{\pi^+} \times \hat{\vec{p}}_{\pi^-}) \cdot \hat{\vec{z}}$$

  $\hat{\vec{z}}$ – e.g., beam direction

  - NA49: $|J| \lesssim 10^{-3}$ (!) – is it possible to increase accuracy?

- $$K_- = \sum_{\pi^+,\pi^-} (\hat{\vec{p}}_{\pi^+} \times \hat{\vec{p}}_{\pi^-}) \cdot \hat{k}_-$$

  better observable?

  $$\vec{k}_- = \sum_{\pi^+} \vec{p}_{\pi^+} - \sum_{\pi^-} \vec{p}_{\pi^-}$$

  $$\hat{k}_- = \frac{\vec{k}_-}{k_-}$$

- + M. Gyulassy's "twist tensor"

Estimate of P-odd effects

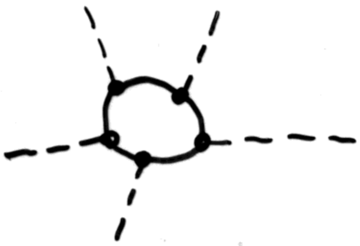
- How does the P-odd bubble decay?

We found stationary points of the effective action. We assumed that they are constant in space and time.

However, to describe the bubble decay, we must consider time-dependent anomalous terms in the effective action

=> Wess-Zumino-Novikov-Witten term

(the only possibility — E. D'Hoker, S. Weinberg)



$$S_{WZW} = -i \frac{N_c}{240\pi^2} \int d^5x \; \varepsilon^{\alpha\beta\gamma\delta\sigma} \cdot \mathrm{tr}\,(R_\alpha R_\beta R_\gamma R_\delta R_\sigma); \qquad R_\alpha = U^+ \partial_\alpha U$$

$K\bar{K} \leftrightarrow \phi \leftrightarrow 3\pi$

$P = (-)^{N_B} P_0$

P_0 - odd term!

for $U = \exp(iu)$, $\partial u \ll 1$ $\qquad N_c = 3$

=>

$$S_{WZW} \simeq \frac{2}{5\pi^2} \int d^4x \; \varepsilon^{\alpha\beta\gamma\delta} \, \mathrm{tr}\,(u \partial_\alpha u \partial_\beta u \partial_\gamma u \partial_\delta u)$$

time-dependent!

The field in the bubble has 3 components: ϕ_u, ϕ_d, ϕ_s

How do they transform into charged pions?

$$S_{WZW} \simeq \frac{2}{5\pi^2} \int dt \int d^3r \; \phi_u \, \partial_r \phi_d \, \partial_0 \phi_s \, (\vec{p}_{\pi^+} \times \vec{p}_{\pi^-}) \cdot \hat{r}$$

time integral $\int dt \, \partial_0 \phi_s \sim \delta \phi_s = \phi_s$ since $\phi_s = 0$ in normal vacuum

space integral $\int d^3r \, \partial_r \phi_d \, (\bar{p}_{\pi^+} \times \bar{p}_{\pi^-}) \cdot \hat{r} \sim \int d\Omega \int R^2 dr \, \partial_r \phi_d \, (\bar{p}_+ \times \bar{p}_-) \cdot \hat{r} \simeq$

$$\simeq \int d\Omega \, R^2 \, \phi_d \, (\bar{p}_{\pi^+} \times \bar{p}_{\pi^-}) \cdot \hat{r}$$

Since for the condensate field $|p| \sim \frac{1}{R}$, the dependence on R drops out

$\Downarrow$

$$S_{WZW} \simeq \frac{2 \phi_u \phi_d \phi_s}{5\pi^2} \int d\Omega \, (\hat{\vec{p}}_{\pi^+} \times \hat{\vec{p}}_{\pi^-}) \cdot \hat{r}$$

$|S_{WZW}| \sim 10^{-3}$ — independent of the size, lifetime, and width of the bubble!

N.B. The number of pions produced, of course, depends on the bubble size:

$$\Delta \mathcal{E}_{vac} \simeq 25 \, n^2 \; \mathrm{MeV/fm^3}$$

$\Rightarrow$ $N_\pi \simeq 100 \, n^2$ pions for $R \sim 5\,\mathrm{fm}$

Summary

1. Due to the non-abelian anomalies of QCD, excited vacuum states can be P-odd

2. In terms of (classical) pion fields, they correspond to configurations with non-trivial topology

$\Downarrow$

Search for parity violation in experiment!!!

(Study global P-odd observables on the EbyE basis)

Parity home page: http://www.rhic.bnl.gov/~jhthomas/parity