Measuring Histograms of the Number of Pairs of Particles, Constructing Ratios of Histograms, and the relationship to Correlations – PART 1

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OUTLINE

- Define two-particle correlations and density ratios of the number of pairs of particles.
- Describe sibling and mixed pairs of particles and number of pair densities and histograms.
- Application to η,ϕ space

- **Technical issues including:**
  - Kinematic projections
  - Normalization
  - Charge dependence
  - Notation conventions

Correlations & Density Ratios for the Number of Pairs of Particles:

Two-particle correlation in momentum space

$$C_{(2)}(\vec{p}_1, \vec{p}_2) \equiv \underbrace{\rho_{(2)}(\vec{p}_1, \vec{p}_2)}_{\text{Two-particle density}} - \rho_{(1)}(\vec{p}_1)\rho_{(1)}(\vec{p}_2)$$

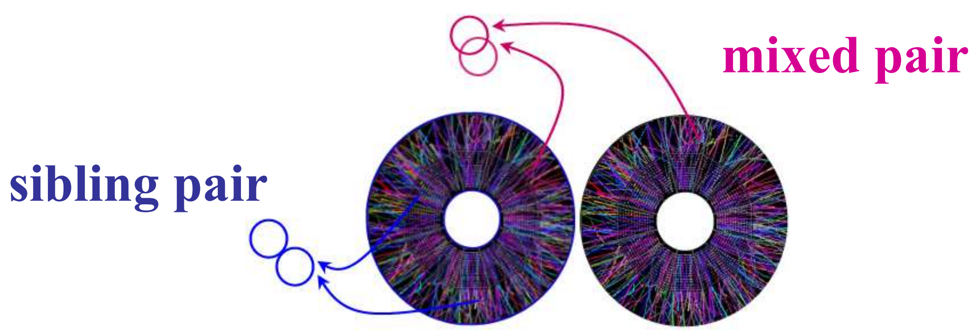
Single-particle densities: product equals <u>uncorrelated</u> two-particle density

$$\rho_{(2)}(\vec{p}_1, \vec{p}_2) \equiv \rho_{sibling}(\vec{p}_1, \vec{p}_2)$$

density of "sibling" pairs - each pair of particles is taken from the same event, hence the term *sibling*; pairs from many events are summed.

$$\rho_{(1)}(\vec{p}_1)\rho_{(1)}(\vec{p}_2) \equiv \rho_{mix}(\vec{p}_1, \vec{p}_2)$$

density of "mixed" pairs - one particle for a mixed-event pair is taken from one event and the second particle is taken from a different, but similar event; corresponding pairs from many events are summed.



Correlations & Density Ratios for the Number of Pairs of Particles: (continued)

Combining:

$$C_{(2)}(\vec{p}_1, \vec{p}_2) = \rho_{mix}(\vec{p}_1, \vec{p}_2)[r(\vec{p}_1, \vec{p}_2) - 1],$$

Define ratio of densities of the number of pairs of particles:

$$r\left(\vec{p}_1, \vec{p}_2\right) \equiv \frac{\rho_{sibling}\left(\vec{p}_1, \vec{p}_2\right)}{\rho_{mixed}\left(\vec{p}_1, \vec{p}_2\right)}$$

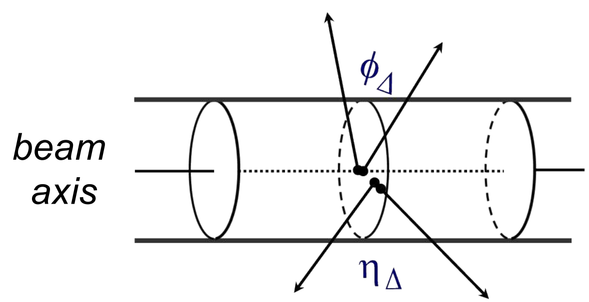
Measuring the Ratio of Histograms of Number of Pairs of Particles:
Projections onto η,ϕ - or – ***Axial*** momentum space

Sort events within a centrality bin according to:

(1) Location of primary vertex along beam line (z-axis)

(2) Number of primary particles (N_{ch}) within p_t,η,ϕ acceptance.

For all accepted particles in this subset of events compute $(\phi_1-\phi_2)$ and $(\eta_1-\eta_2)$ for every sibling and mixed pair.



Where we define:

$\phi_\Delta = \phi_1 - \phi_2$

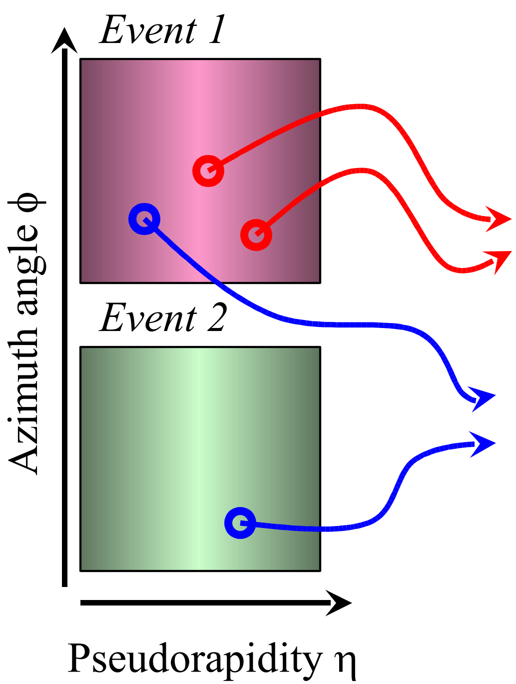
$\eta_\Delta = \eta_1 - \eta_2$

(see last slide in this tutorial)

Compute histograms for the number of sibling and mixed pairs of particles using only those subsets of events within 7.5 cm along the z-axis and N_{ch} within 50. Repeat for all event subsets in a centrality bin.

Measuring the Ratio of Histograms of Number of Pairs of Particles:

Projections onto η,ϕ – or - ***Axial*** momentum space (continued)



Sum the number of sibling pairs in each bin in $(\phi_1-\phi_2)$ vs $(\eta_1-\eta_2)$ for all events in all subsets

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Similarly, sum the number of mixed-event pairs in each bin in $(\phi_1-\phi_2)$ vs $(\eta_1-\eta_2)$ for all events in all subsets

Define the bin-by-bin ratio:

$$r(\eta_1-\eta_2,\phi_1-\phi_2)$$

This is an example of projecting the six-dimensional correlation distribution onto two-dimensional, relative pair-wise momentum coordinates.
We refer to such projections as the **joint** (*i.e.* two things) – **autocorrelation** (*i.e.* on relative coordinates)

Normalized Ratio of Histograms - $\hat{r}$

$$\hat{r}(\eta_1 - \eta_2, \phi_1 - \phi_2) \equiv \frac{N_{mixed-pairs}}{N_{sibling-pairs}} r(\eta_1 - \eta_2, \phi_1 - \phi_2)$$

where:

$N_{sibling-pairs} =$ Total number of sibling pairs for all events in the centrality bin.

$N_{mixed-pairs} =$ Total number of mixed-event pairs used among the "similar" events within each Z_{vertex} and N_{ch} bin, summed over all such subsets of events in the centrality bin.

Charge Independent and Charge Dependent Ratios:

Define normalized ratios of histograms for the four combinations of charge signs (++,--,+-,-+), for like-sign (LS) pairs and unlike-sign (US) pairs, and for all charge-sign pairs as in the preceding discussion.

We have verified that $\hat{r}_{++} = \hat{r}_{--}$ and $\hat{r}_{+-} = \hat{r}_{-+}$ within statistics.

Charge independent (CI) normalized ratios of histograms are computed exactly as described in the preceding slides using all charged particles.

$$\hat{r}^{CI}(\eta_\Delta, \phi_\Delta) \equiv \frac{N_{mixed-pairs}}{N_{sibling-pairs}} r(\eta_\Delta, \phi_\Delta)$$

Charge dependent (CD) normalized ratios of histograms are computed by applying the steps described in the preceding slides for all like-sign pairs of charged particles (++ and --) and for all unlike-sign pairs of charged particles (+- and -+) and forming the difference:

$$\hat{r}^{CD}(\eta_\Delta, \phi_\Delta) \equiv \hat{r}_{LS} - \hat{r}_{US}$$

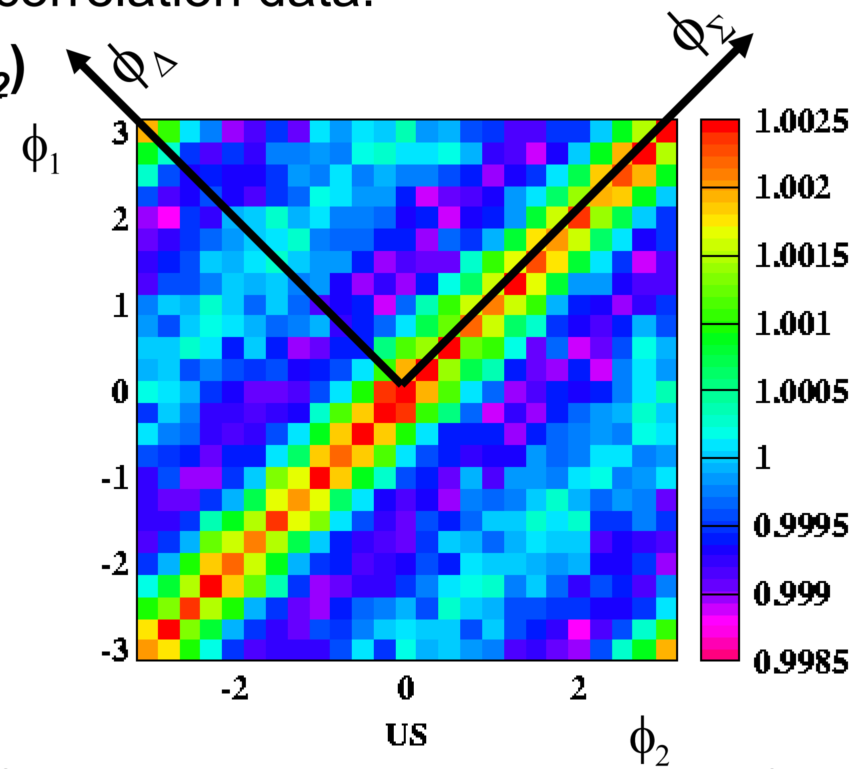
Why does the Event Structure PWG use $\boldsymbol{x}_\Delta$, rather than the more common $\Delta \boldsymbol{x}$ for difference quantity $\boldsymbol{x_1}$-$\boldsymbol{x_2}$?

The reason is related to the fact that we often refer to rotated coordinate systems which take advantage of the observed symmetries in the two-point correlation data.

Rotated axes lie along the sum **($\boldsymbol{x_1+x_2}$)** and difference **($\boldsymbol{x_1-x_2}$)** directions.

Notation $\boldsymbol{x}_\Sigma$,$\boldsymbol{x}_\Delta$ follow naturally.

On the other hand definition $\Delta \boldsymbol{x} = \boldsymbol{x_1-x_2}$ calls for the corresponding notation "$\Sigma \boldsymbol{x}$" which would be confused with summation. For example the ϕ_1 versus ϕ_2 two-point ratio data are shown here along with the ϕ_Σ ,ϕ_Δ coordinate system.



Application of these methods in STAR papers:

- “**Hadronization geometry and charge-dependent number autocorrelations on axial momentum space in Au-Au collisions at sqrt(s_{NN}) = 130 GeV,”**
  Submitted June 30, 2004
  e-Print Archives (nucl-ex/0406035).

- “**Minijet deformation and charge-independent particle correlations on axial momentum space in Au-Au collisions at sqrt(s_{NN}) = 130 GeV,”**
  Presently on starpapers-l *(August, 2004).*

- “**Transverse momentum correlations and minijet dissipation in Au-Au collisions at sqrt(sNN) = 130 GeV,”**
  Submitted August 11, 2004
  e-Print Archives (nucl-ex/0408012)