

# Early stage of RHIC collisions and its equilibration: a story told by correlations

BNL Nuclear Physics Seminar

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## **The STAR Collaboration:**

Argonne NL, U.Bern, U.Birmingham, Broohaven NL, Caltech, UC Berkeley, UC Davis, UCLA, Carnegie Mellon U., Creighton U., NPI AS Chech Republic, LHI JINR, PPL JINR, U. Frankfurt, IP Bhubaneswar, IIT Mumbai, Indiana U., IRS Strasbourg, U. Jammu, Kent State U., Lawrence Berkeley NL, MIT, Max-Plank-Instutut, MSU, MEPHI, City College of NY, NIKHEF, Ohio State U, Panjab U., Penn State U., IHEP Protvino, Purdue U., U. Rajasthan, Rice U., U. Sao Paolo, UST China, Shanghai IAP, SUBATECH, Texas A&M U., U. Texas, Tsinghua U., Valparaiso U., VECC Kolkata, Warsaw UT, U. Washington, Wayne State U, IPP CCNU (HZNU) Wuhan, Yale U., U. Zagreb

# 1 Content of the talk

- Equilibration

Arguably the central issue in hadronic sector at RHIC. Is it taking place ? What is the mechanism ?

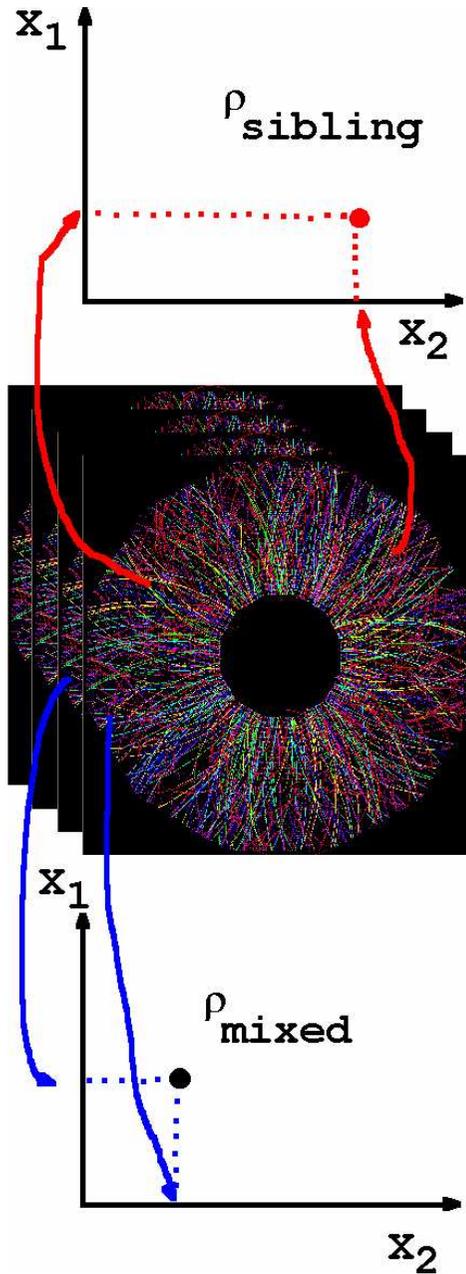
- Methods

Initial state: a “calibrated source” of correlations. Watch their evolution into final state in time → system size. Use two particle correlations and Discrete Wavelet Transform.

- Observations

- Conclusions

## 2 Autocorrelation

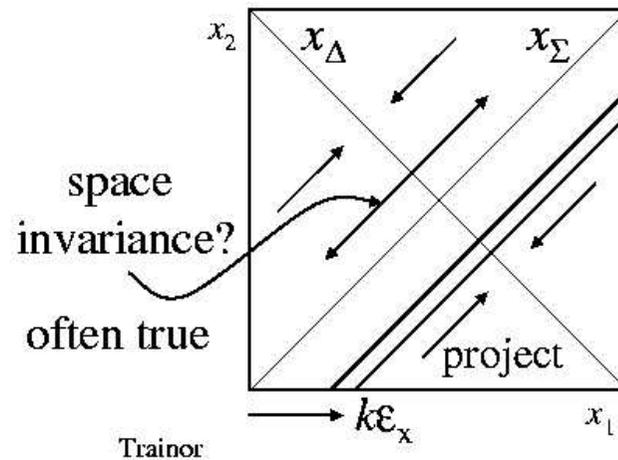


$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \rightarrow \begin{pmatrix} x_\Sigma \equiv x_1 + x_2 \\ x_\Delta \equiv x_1 - x_2 \end{pmatrix},$$

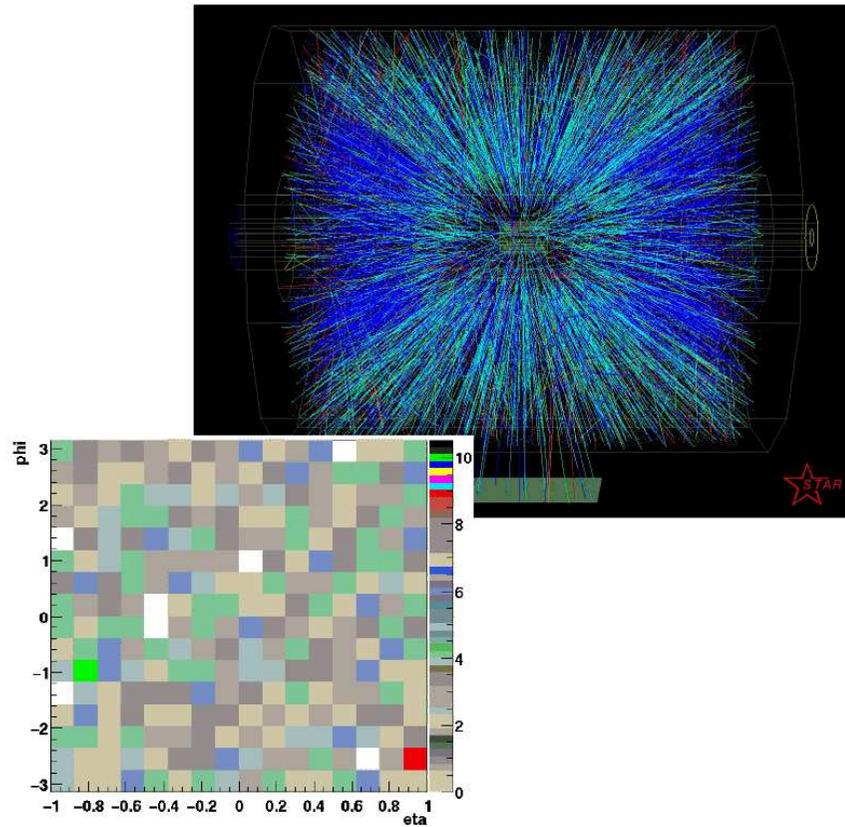
always a lossless transformation of data.

**Autocorrelation**  $A$  is a projection of a two-point distribution onto difference variable(s)  $x_\Delta$ , lossless for  $x_\Sigma$ -invariant (homogenous, stationary) problems.

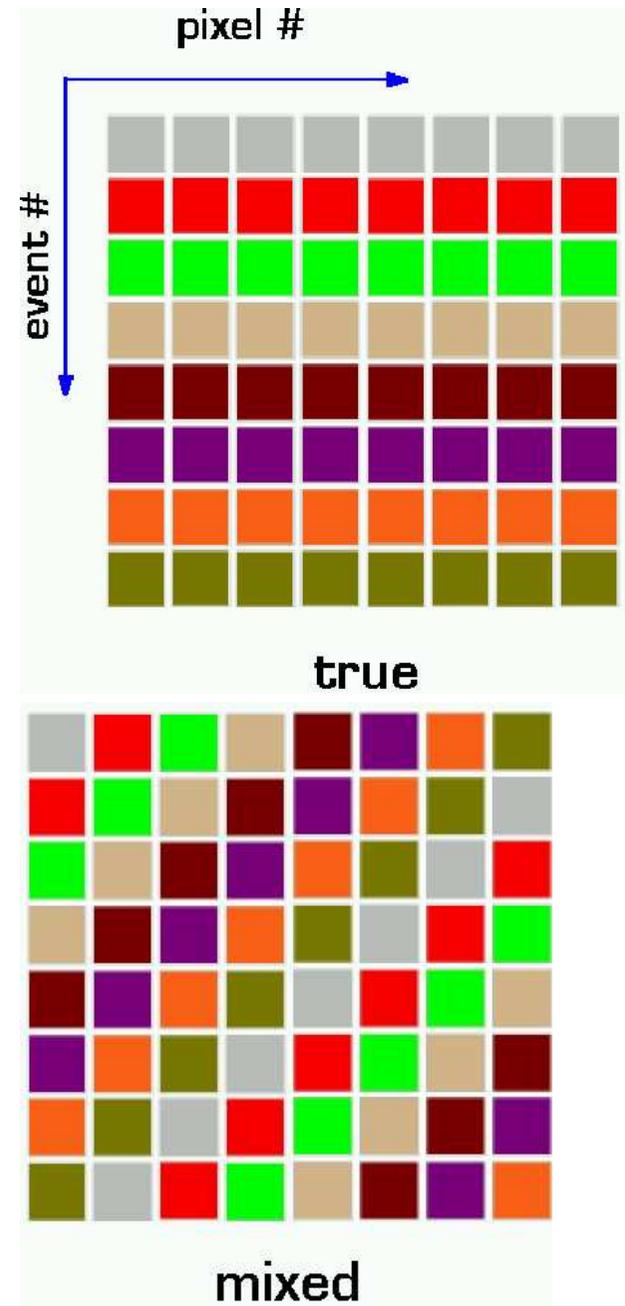
$$\Delta R(x_1, x_2) = \frac{\rho_{sibling}(x_1, x_2)}{\rho_{mixed}(x_1, x_2)} - 1$$



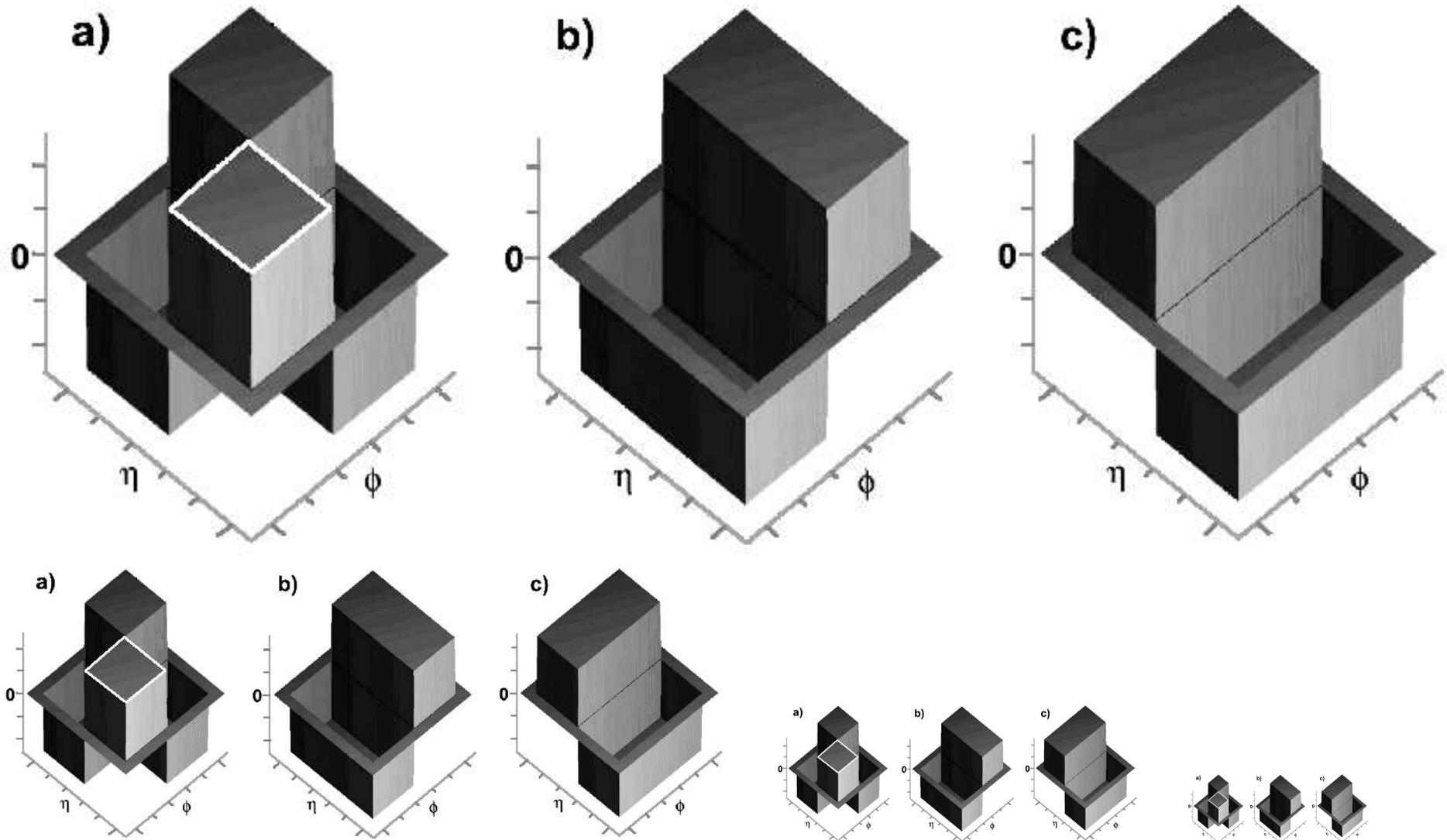
### 3 Uncorrelated event reference for DWT



mixed events: no pixel used twice;  
 $\leq 1$  pixel from any event in the same mixed event; no mixing of events with largely different multiplicity and vertex.

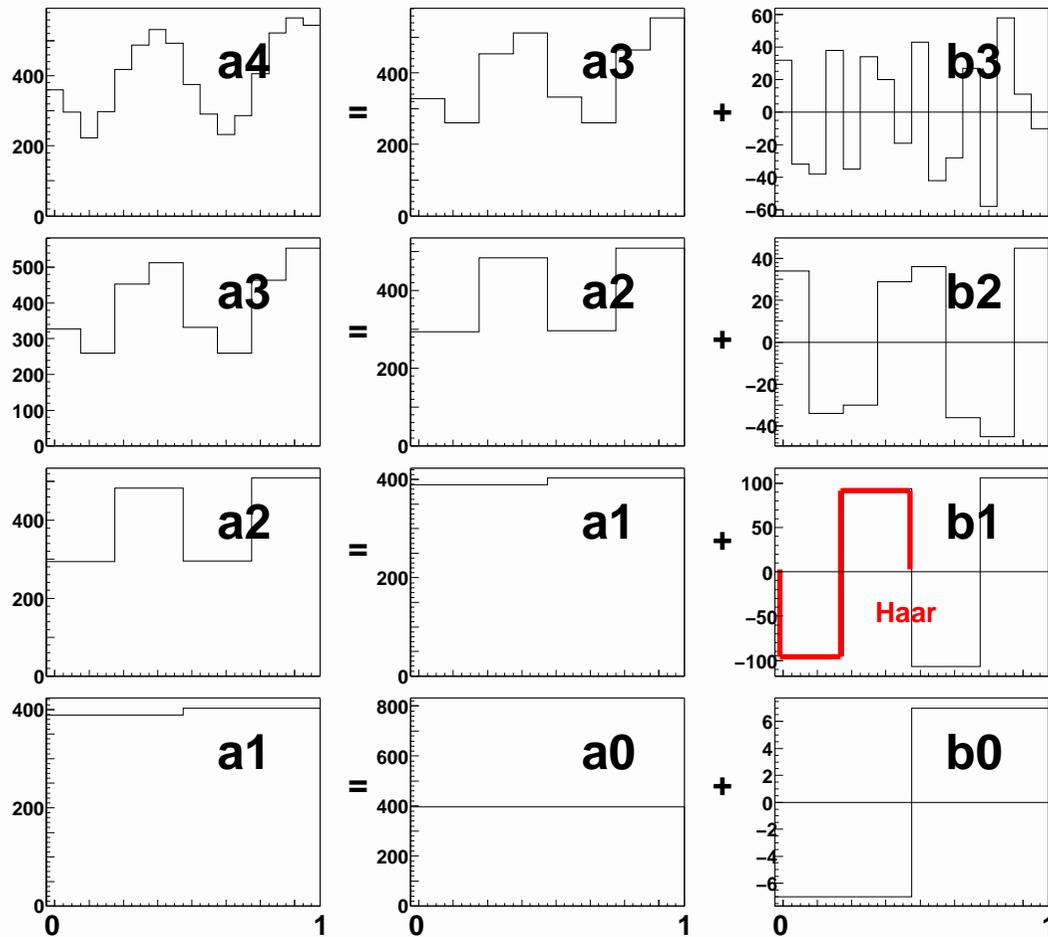


## 4 Discrete Wavelet Transform (DWT)

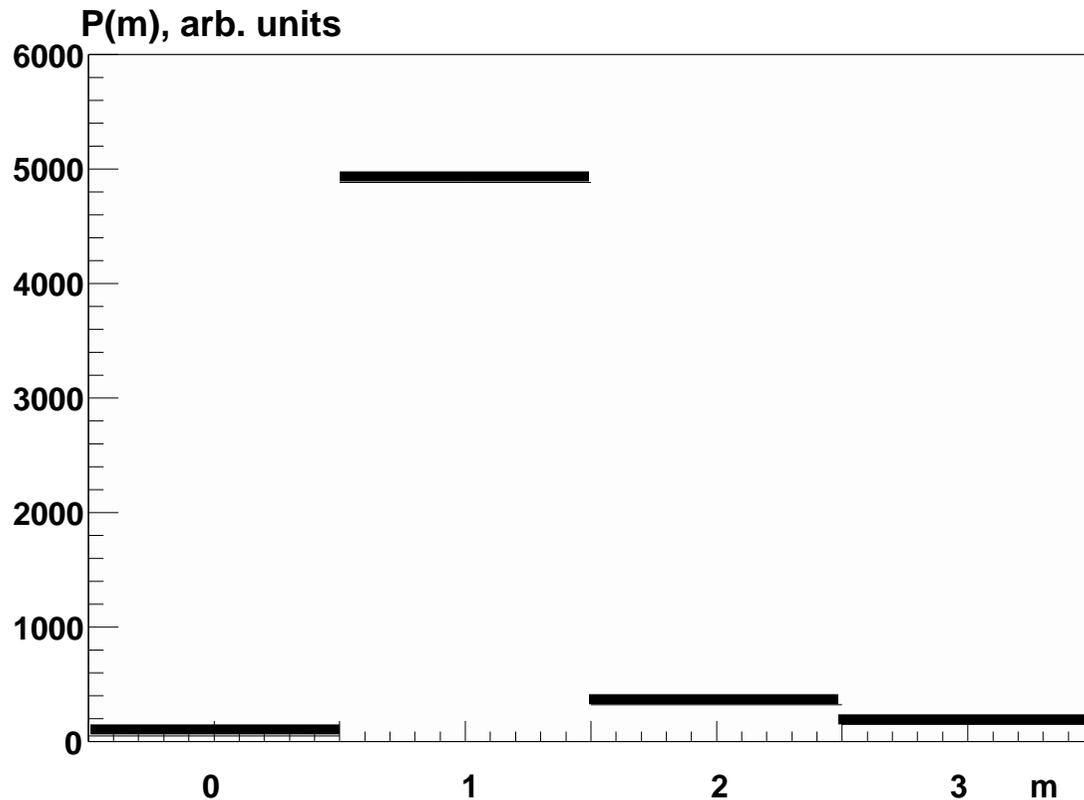


$F_{m,i,j}^\lambda(\phi, \eta)$ —Haar wavelet **orthonormal basis** in  $(\phi, \eta)$ : scale fineness ( $m$ ), directional modes of sensitivity ( $\lambda$ ), track density  $\rho(\eta, \phi, p_T)$ , locations in 2D  $(i, j)$ . **DWT** is an expansion in this basis.

## 5 A flow-inspired example



Elliptic flow-inspired example:  
 $x$  axis – an angle in “natural units” ( $2\pi = 1$ ),  $y$  axis – multiplicity. The multiresolution theorem:  $a_4 = a_0 + b_0 + b_1 + b_2 + b_3$ , can have better fineness.

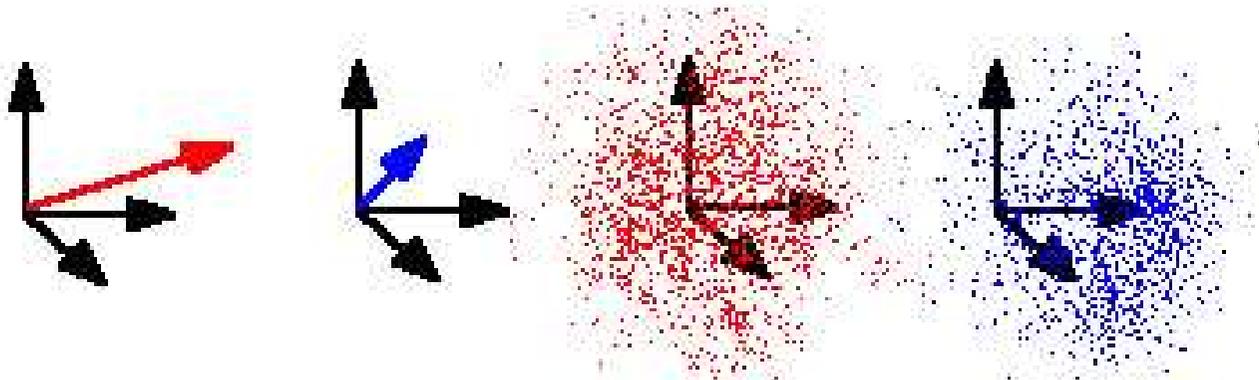
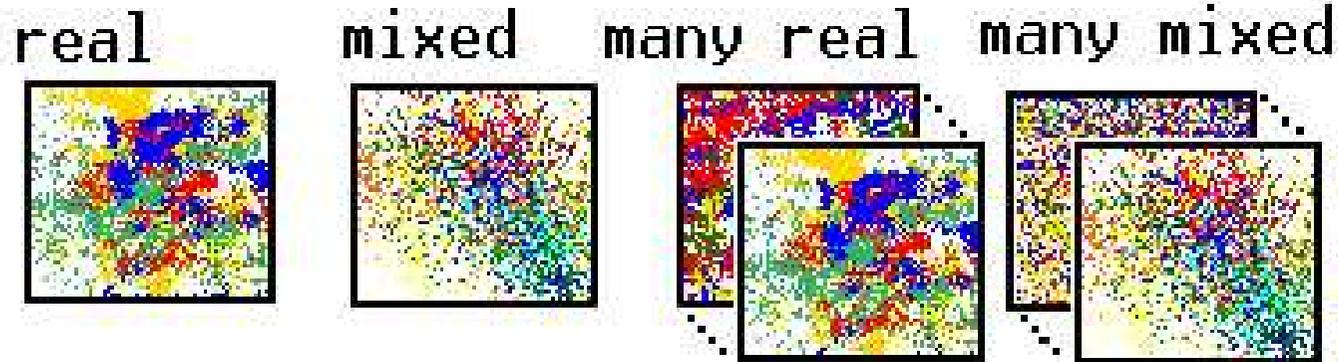


Power spectrum of that flow event as a function of “fineness”  $m$ . The dominant contribution is  $m = 1$  (the “ $v_2$ ” harmonic, **b1**). Statistical fluctuations also contribute.

$$P(m) = 2^{-m} \sum_i \langle \rho, F_{m,i} \rangle^2.$$

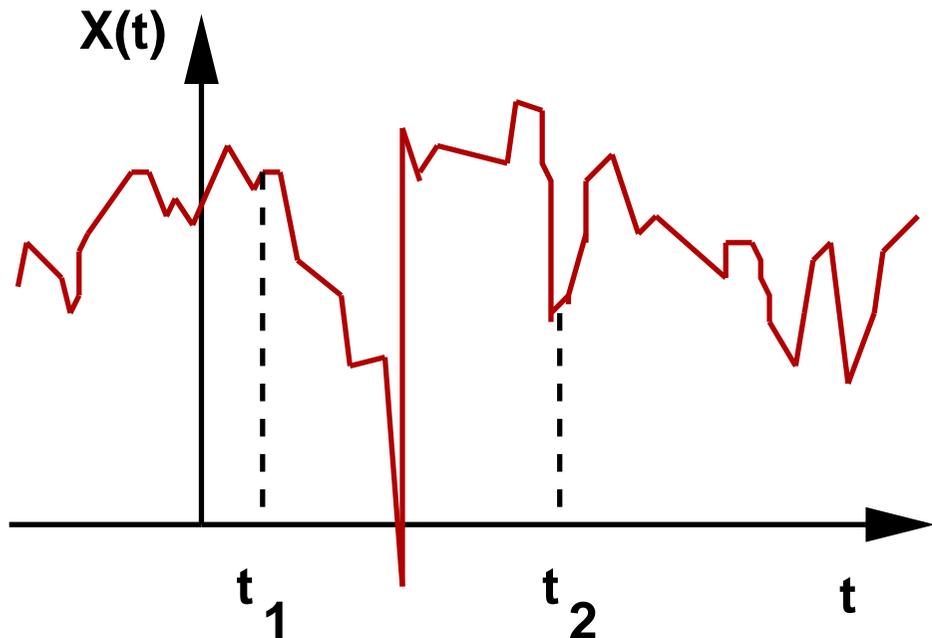
Computational complexity  $O(N)$ !

## 6 DWT Power Spectra and the Hilbert Space



Power of local fluctuations, mode  $\lambda$ :  $P^\lambda(m) = 2^{-2m} \sum_{i,j} \langle \rho, F_{m,i,j}^\lambda \rangle^2 \propto \text{norm}^2$   
 in the DWT subspace. “Dynamic texture”  $P_{dyn}^\lambda(m) \equiv P_{true}^\lambda(m) - P_{mix}^\lambda(m)$ .

## 7 DWT Power Spectra and Correlations



### Autocorrelation

$$A(\tau) = \int_{-\infty}^{\infty} X(t)X(t + \tau) dt$$

(1)

where  $\tau = t_2 - t_1$ , and  $X(t)$  is “homogeneous random field”.

Wiener-Khinchin theorem relates  $A(\tau)$  with the local fluctuation power spectrum  $P(\omega)$  via Fourier transform  $\mathcal{F}$ .

$$\mathcal{F}_{\omega \rightarrow \tau}(P(\omega)) = A(\tau) \quad (2)$$

$$P(\omega) = \mathcal{F}_{\tau \rightarrow \omega}(A(\tau)) \quad (3)$$

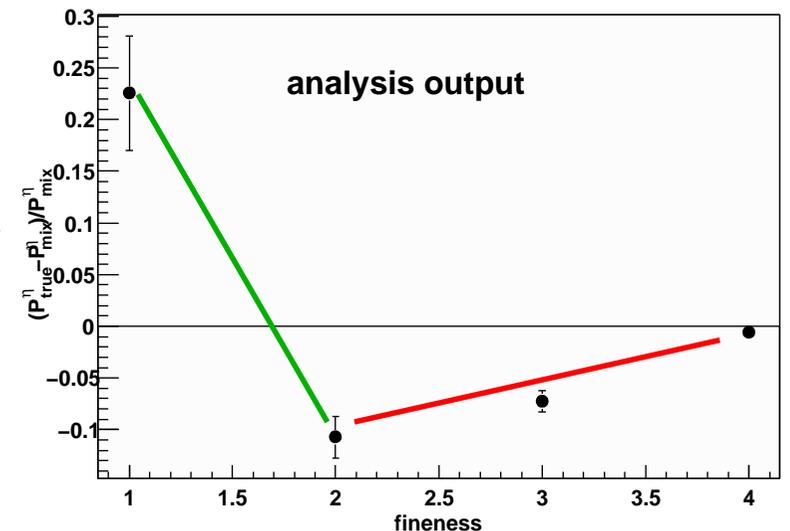
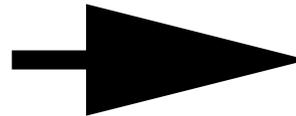
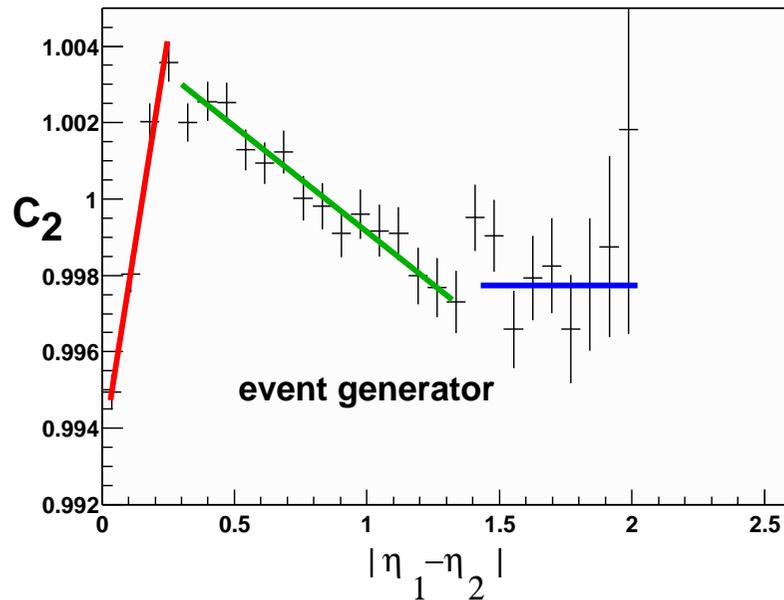
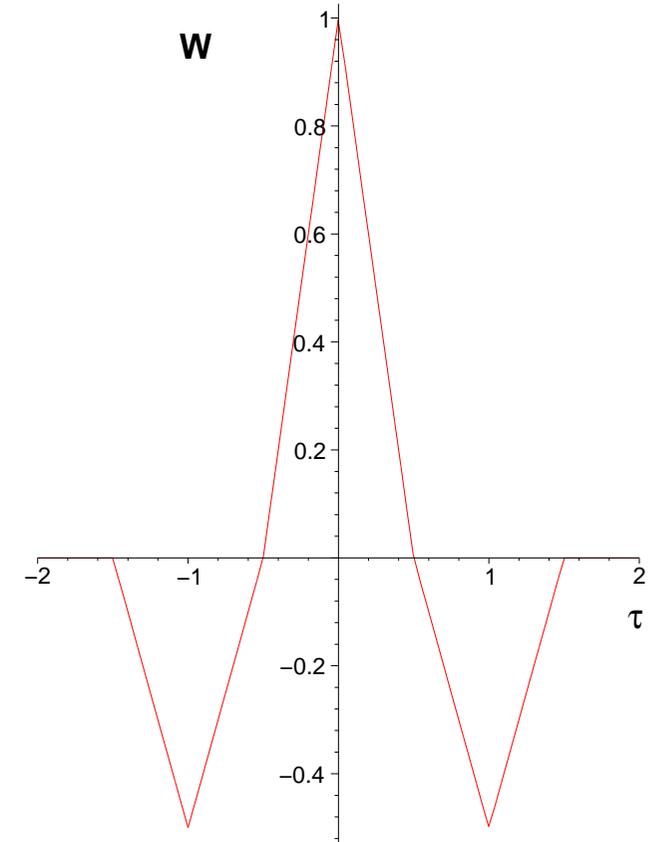
$$O(N) \rightarrow O(N^2) \quad (4)$$

Prefer  $O(N)$  for initial data processing for CPU reasons.

In the **Haar discrete wavelet** basis, the integral equation 3 looks different:

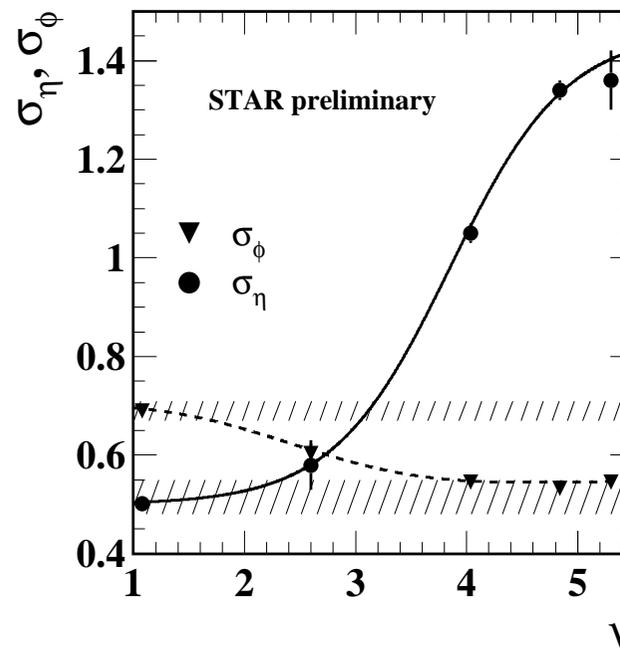
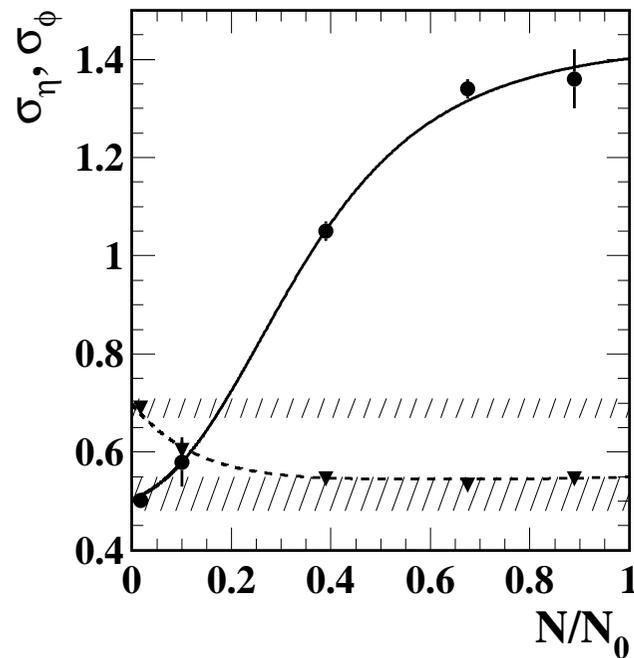
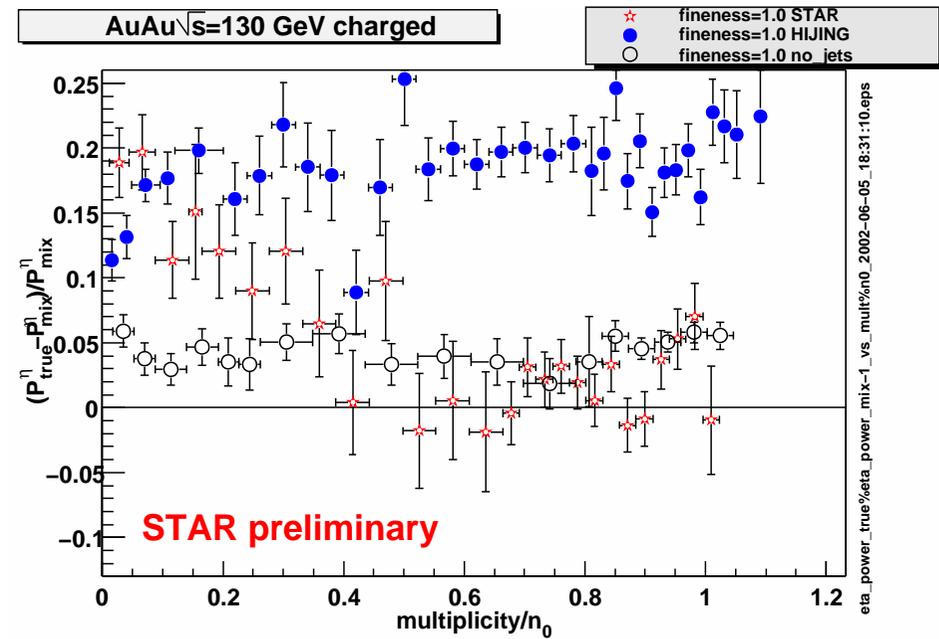
$$P(m) = \overline{\int_{-\infty}^{\infty} X(\tau/2)X(-\tau/2)W(\tau, m) d\tau},$$

where  $W$  is the weight function for the Haar wavelet.  $P(m)$  reflects differential structure on scale  $m$ . See example:

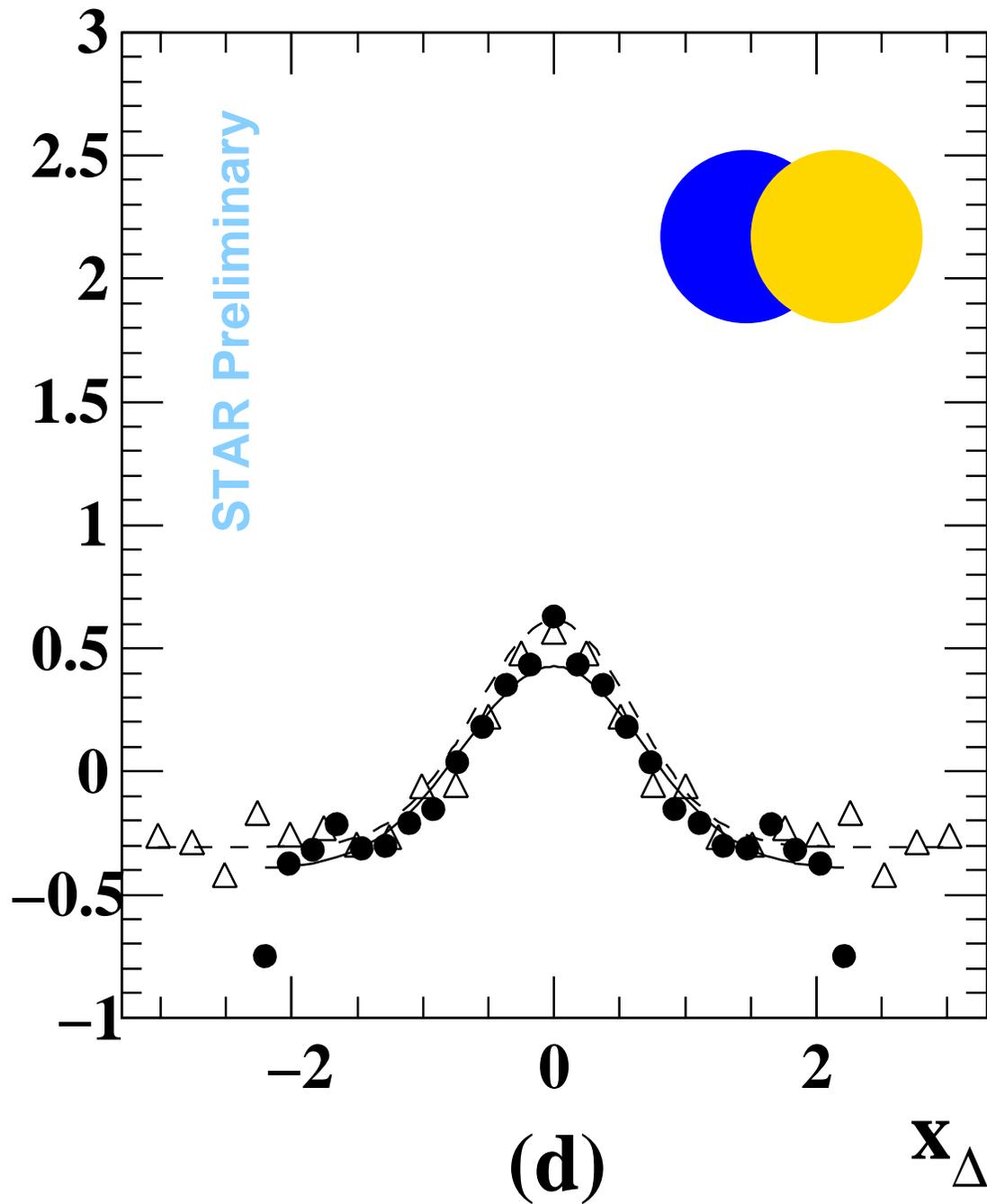


# 8 Centrality dependence

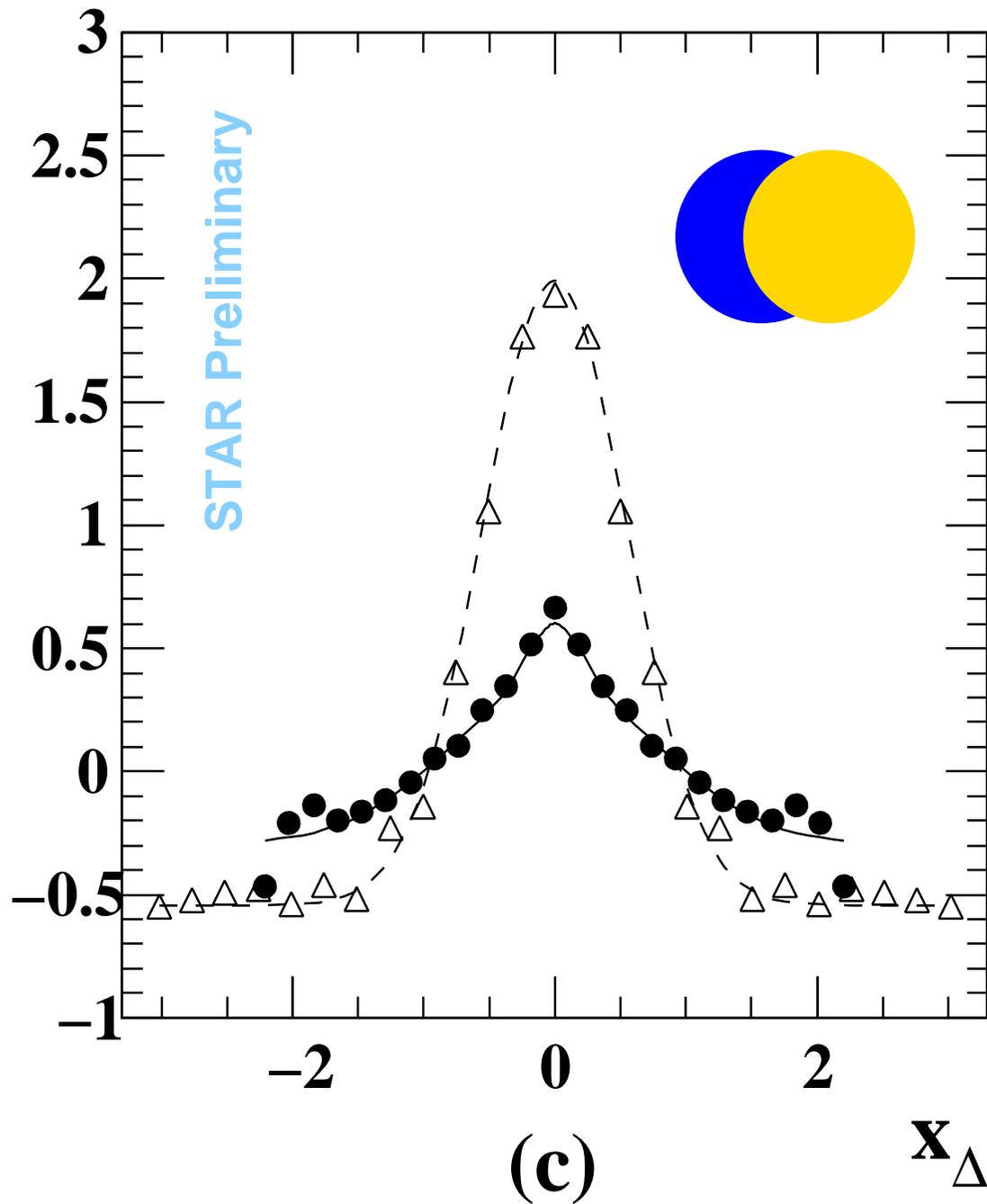
Istron Bay, Greece, June 2002, nucl-ex/0211015. STAR DWT analysis of charge-independent correlations, AuAu,  $\sqrt{s_{NN}} = 130$  GeV



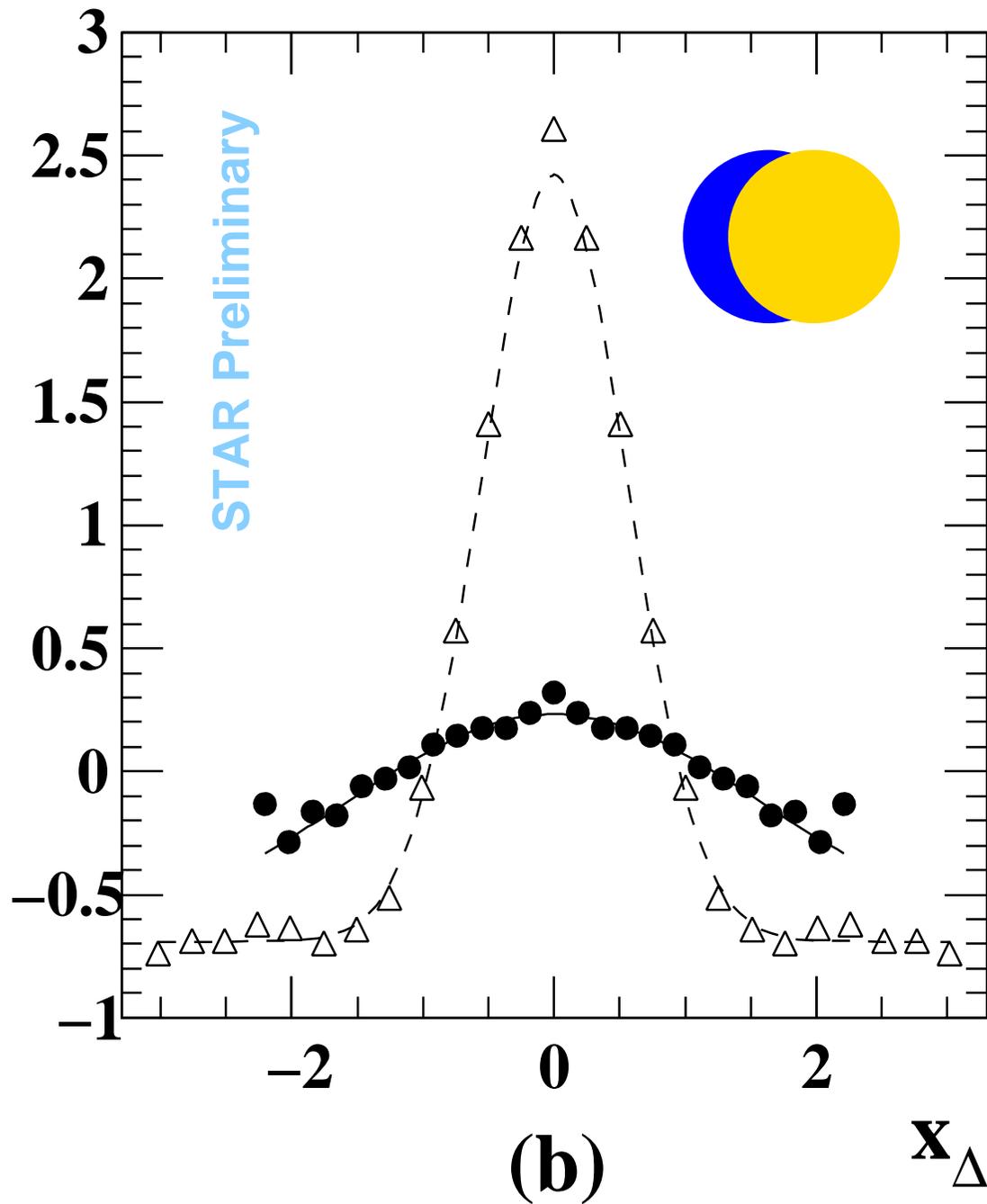
[STAR two-particle correlation analysis, to be submitted to PRL]  
The step-like character of the centrality dependence is elucidated using  $\nu = (N_{part}/2)^{1/3}$



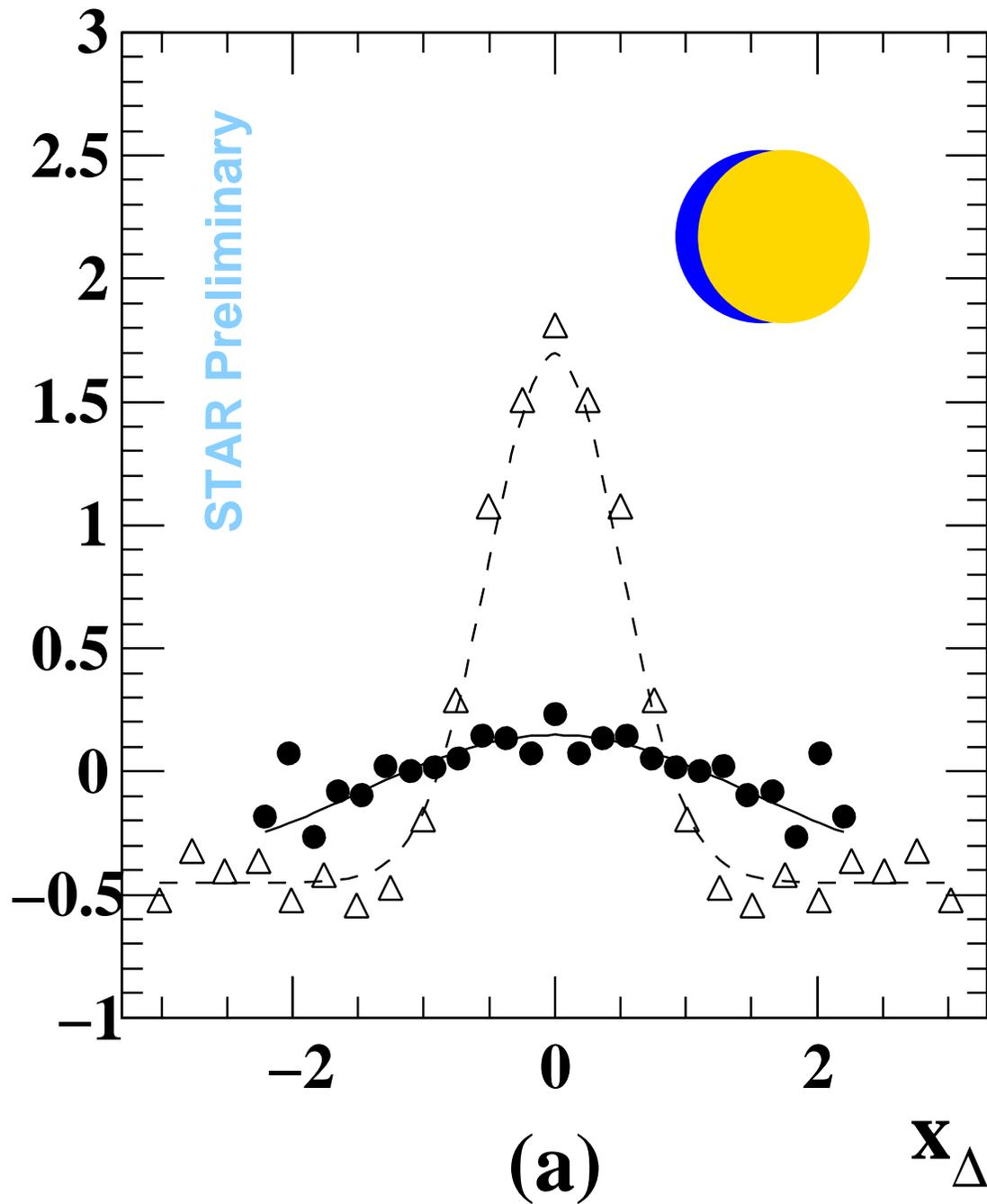
Per-particle-normalized charge-independent autocorrelations  $N\Delta R$  in difference variables  $\eta_\Delta$  (solid dots) and  $\phi_\Delta$  (open triangles).  $v_1$  and  $v_2$  are subtracted.



Per-particle-normalized charge-independent autocorrelations  $N\Delta R$  in difference variables  $\eta_\Delta$  (solid dots) and  $\phi_\Delta$  (open triangles).  $v_1$  and  $v_2$  are subtracted.

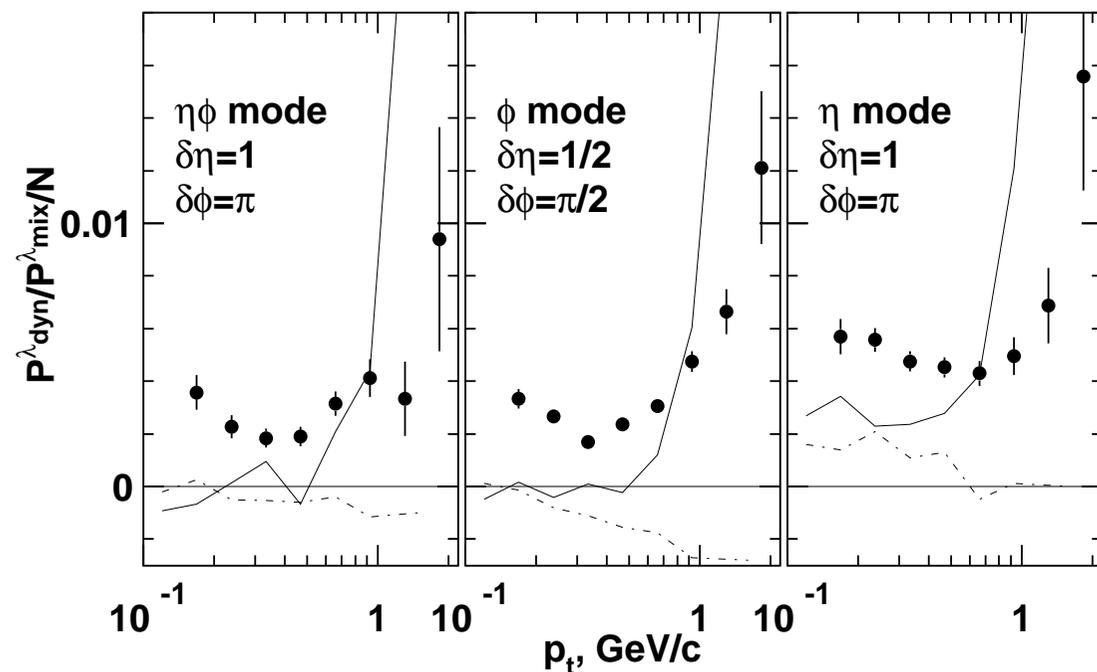


Per-particle-normalized charge-independent autocorrelations  $N\Delta R$  in difference variables  $\eta_\Delta$  (solid dots) and  $\phi_\Delta$  (open triangles).  $v_1$  and  $v_2$  are subtracted.



Per-particle-normalized charge-independent autocorrelations  $N\Delta R$  in difference variables  $\eta_\Delta$  (solid dots) and  $\phi_\Delta$  (open triangles).  $v_1$  and  $v_2$  are subtracted.

## 9 $p_t$ dependence: peripheral events, $\sqrt{s_{NN}} = 200$ GeV

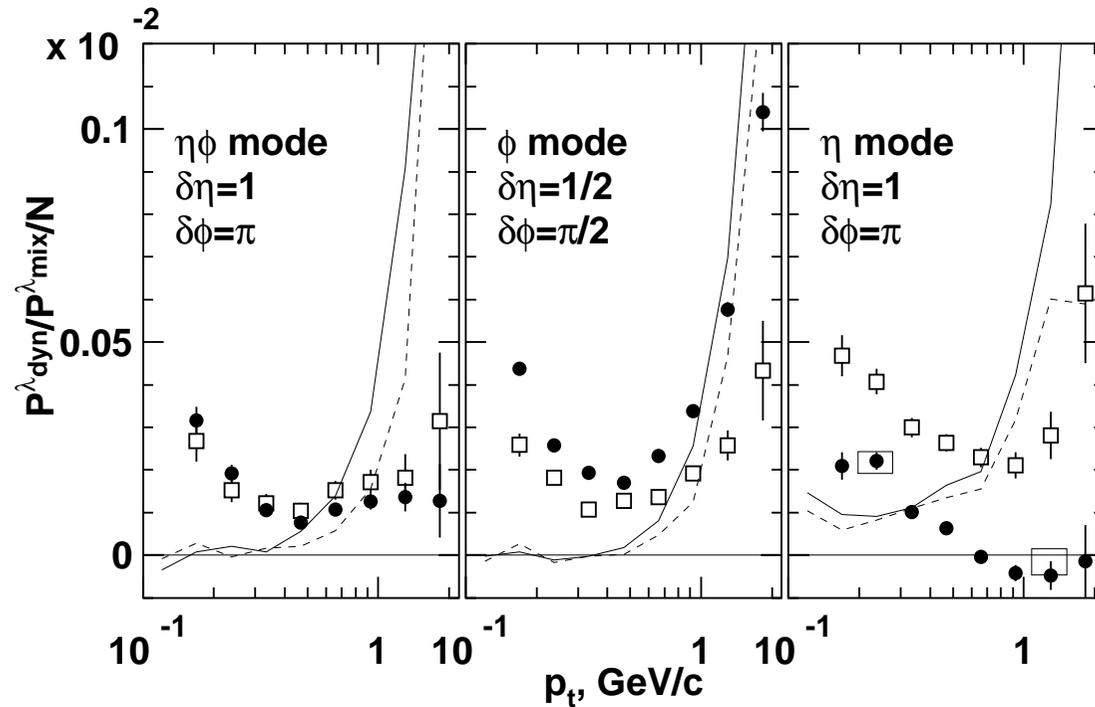


Peripheral events (60-84%): normalized dynamic texture for fineness scales  $m = 0, 1, 0$  from left to right panels, respectively, as a function of  $p_t$ . ● – STAR data; solid line – standard HIJING, dash-dotted line – HIJING without jets.

Qualitative trends in peripheral data are as expected. What signal to expect in the central data, if correlation does not change ?

$$\left( \frac{P_{true}}{P_{mix}} - 1 \right) \frac{1}{N} \Big|_{central} = \left( \frac{P_{true}}{P_{mix}} - 1 \right) \Big|_{peripheral} \frac{1}{N_{central}} \quad (5)$$

## 10 $p_t$ dependence: central events, $\sqrt{s_{NN}} = 200$ GeV



Central (top 4%) events: normalized dynamic texture for fineness scales  $m = 0, 1, 0$  from left to right panels, respectively, as a function of  $p_t$ . ● – STAR data; solid line – standard HIJING; dashed line – HIJING with jet quenching; boxes – peripheral STAR data just shown, renormalized as just described.

We are observing a modification of the minijet structure predominantly in the longitudinal,  $\eta$  direction. Longitudinal expansion of the hot and dense medium formed early in the collision makes this direction special and is likely to be part of the modification mechanism.

## 11 How to model correlations ?

via rejection/acceptance algorithm, according to a multiparticle probability density distribution. In general, for  $N$  particles denoted  $1, 2, \dots, N$  the differential probability density

$$P(1, 2, \dots, N) = P(1)P(2|1)P(3|1, 2)\dots P(N|1, 2, \dots, N - 1),$$

where  $P(2|1)$  and subsequent terms are conditional single particle probabilities.

Two-particle correlation

$$C(1, 2) = \frac{P(1, 2)}{P(1)P(2)} = \frac{P(2|1)}{P(2)}.$$

Add particle 2 to particle 1:

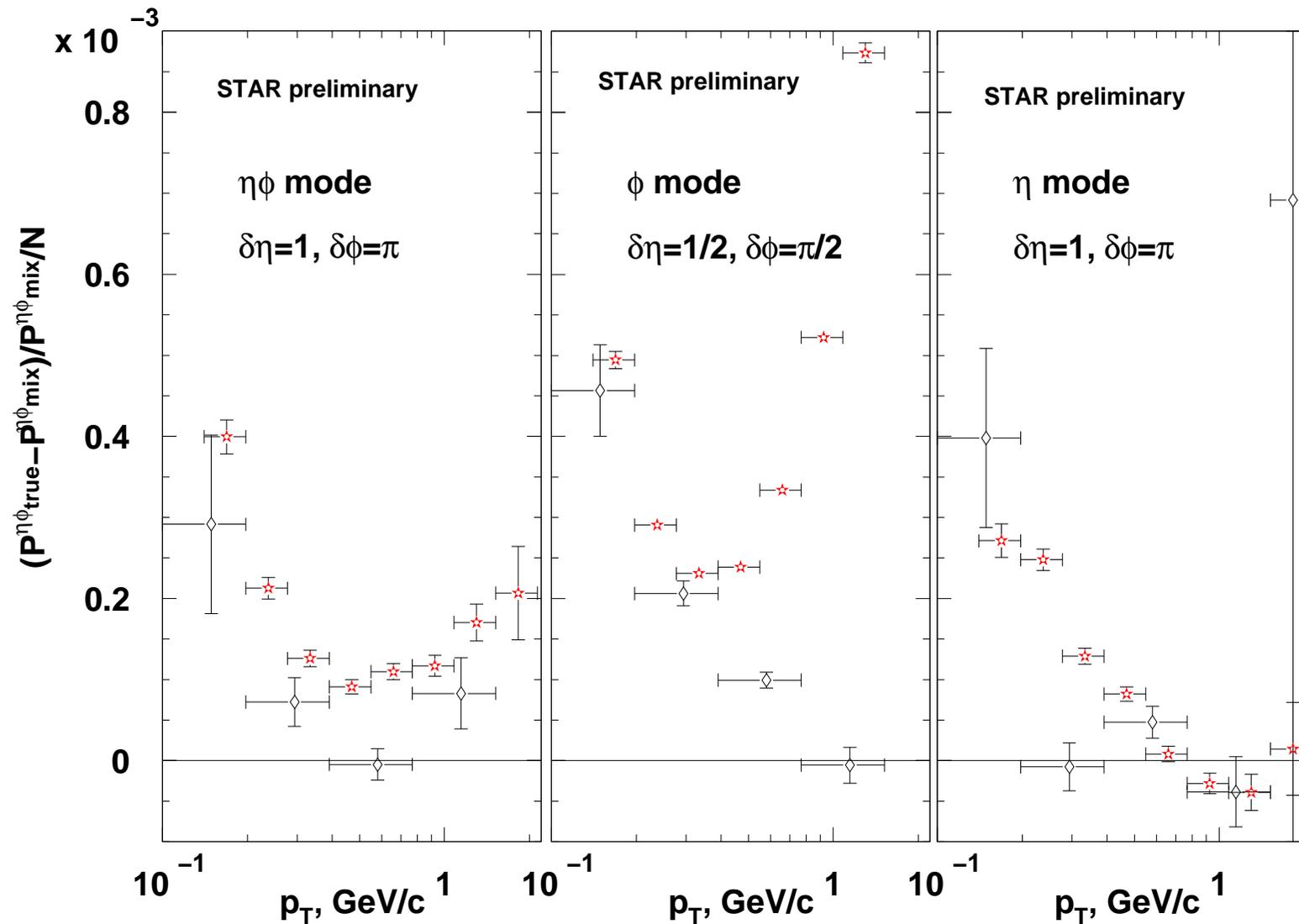
$$P(2|1) = P(2)C(1, 2)$$

A two-particle approximation: continue by induction

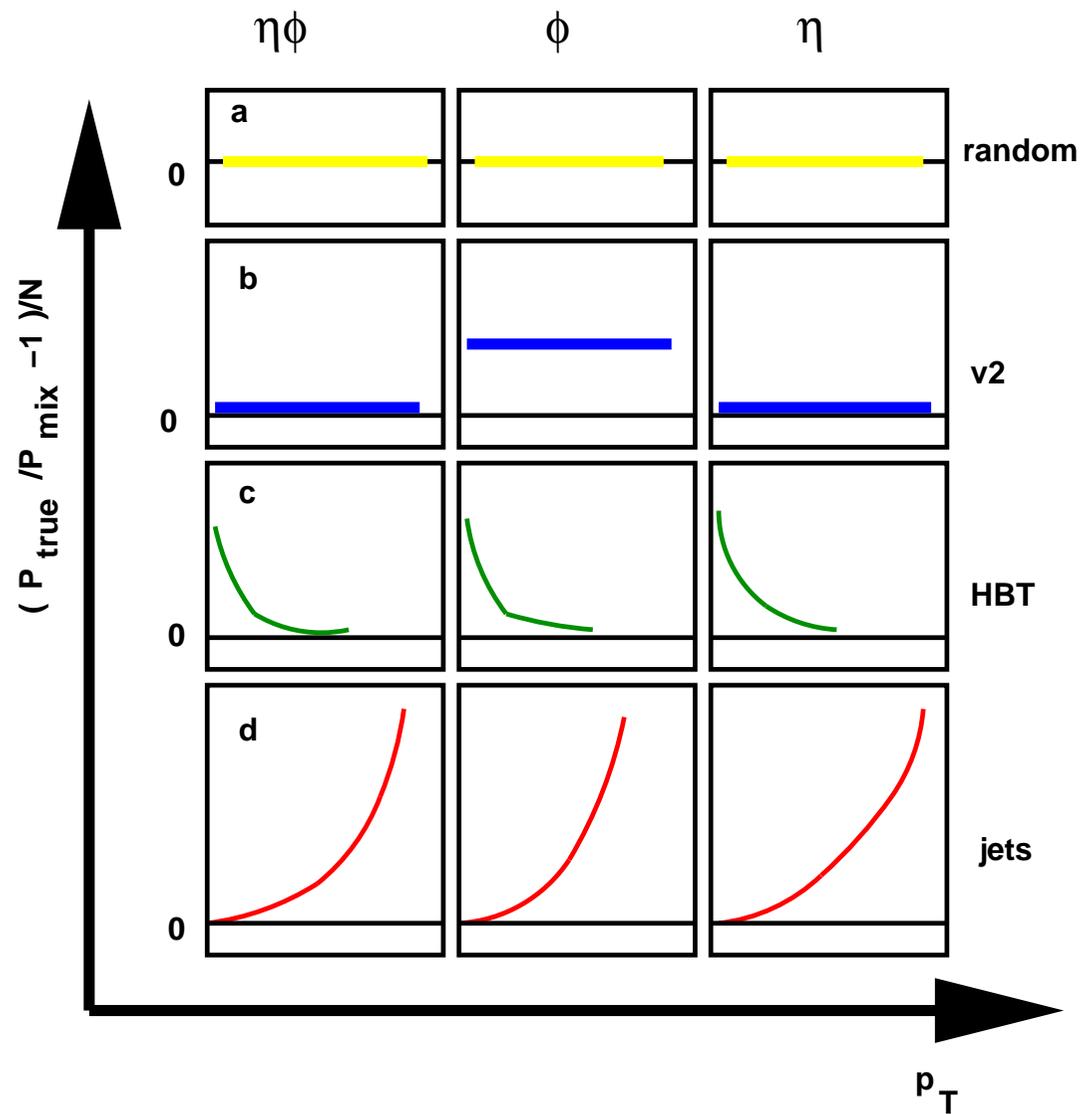
$$P(3|1, 2) = P((3|1)|2) = P(3|1)C(2, (3|1)) \approx P(3)C(3, 1)C(2, 3),$$

$$P(N|1, 2, \dots, N - 1) \approx P(N)\prod_{i=1}^{N-1} C(i, N)$$

# 12 Understanding low $p_t$



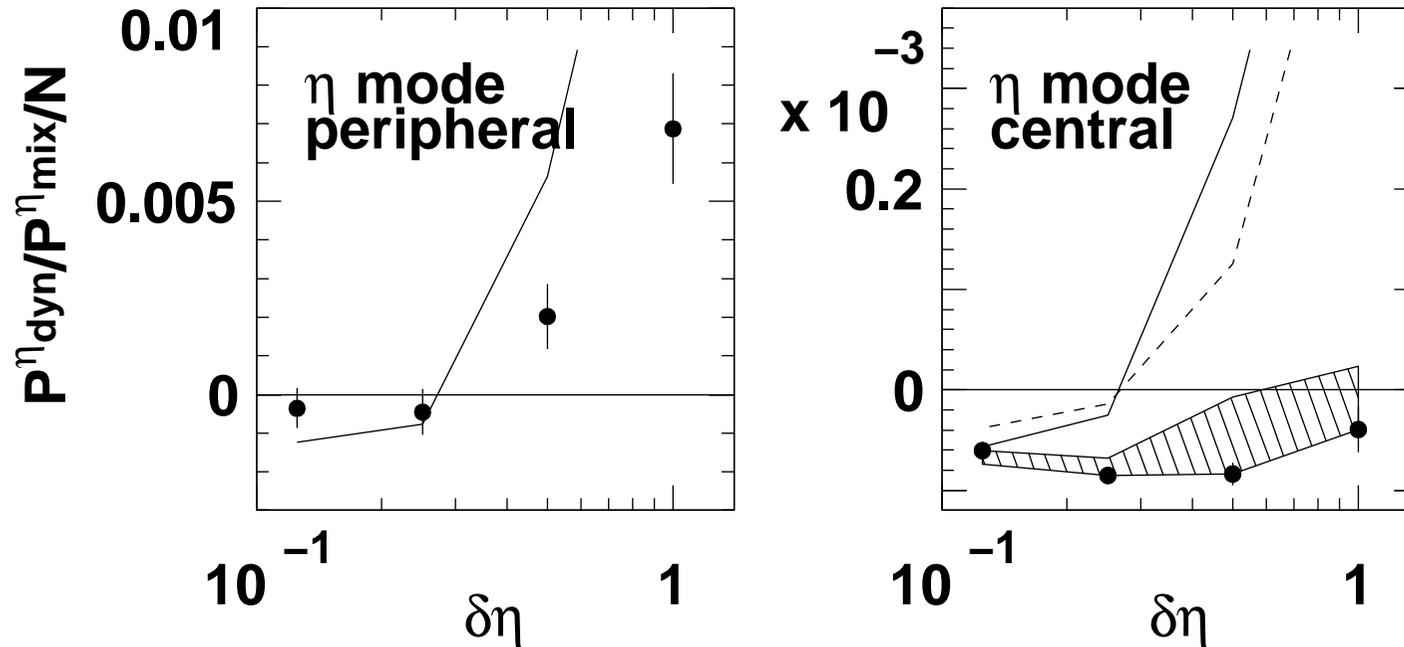
An event generator tuned to reproduce like- and unlike-sign correlations in  $Q_{inv}$ , reproduces the low  $p_t$  trends in the data. HBT, Coulomb and string fragmentation physics contribute at low  $p_t$ .



Dynamic texture response in various idealized situations (showing only one scale):

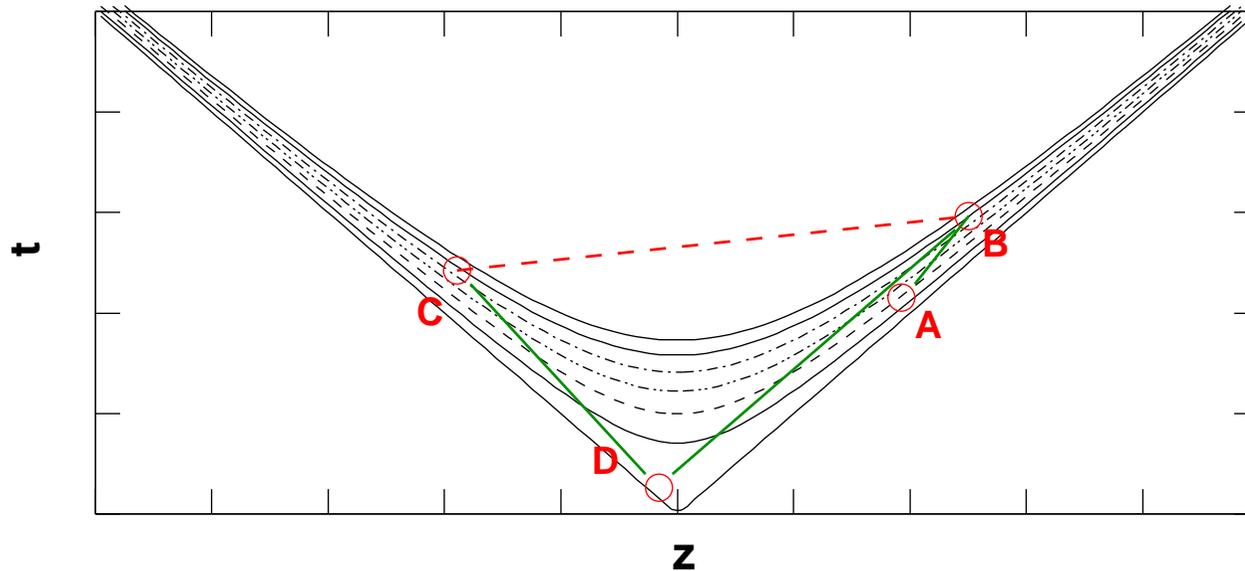
- (a) events of random (uncorrelated) particles
- (b)  $p_t$ -independent elliptic flow
- (c) Correlations at low  $Q_{inv}$  (Bose-Einstein correlations and Coulomb effect)
- (d) HIJING jets

## 13 Scale dependence

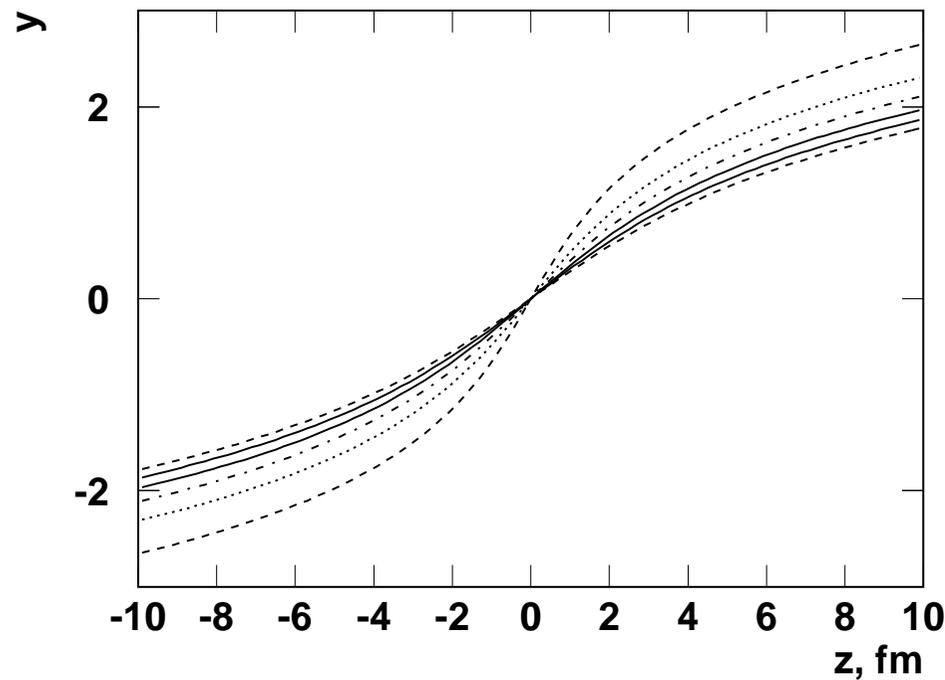


Scale dependence of the dynamic texture measure in peripheral and central events for  $1.1 < p_t < 1.5$  GeV/ $c$ . ● – STAR data; solid line – standard HIJING; dashed line – HIJING with jet quenching. An estimate of systematic error, mainly due to track merging, is shown as a hatched area.

# 14 Rapidity scale and collision history



Bjorken expansion:  
 $t = (z^2 + \tau^2)^{1/2}$ , CB is space-like; DC, DA, DB, AB contacts possible.



$y = \frac{1}{2} \ln \frac{t+z}{t-z}$  Rapidity and causality:  
 large  $\delta y \iff$  large  $\delta z$ . Large  $\delta y$  correlations reflect early state, otherwise acausal.

## 15 Summary

- DWT power spectra provide efficient ( $O(N)$ ) differential scale decomposition of the two-point correlation structure
- Fluctuations in local hadron density in the momentum space due to angularly correlated (mini)jets are observable with minimum-bias TPC tracks at  $p_t < 2$  GeV/c  $\Rightarrow$  can study equilibration !
- The minijet correlation structure is strongly modified with centrality; the effect appears to “turn on” around  $\nu = (N_{part}/2)^{1/3} \approx 3$ .
- Broadening of the correlation in  $\eta$  and weakening of  $P_{dyn}^\eta$  on the coarse scale are consistent descriptions of the effect. The modification is particularly strong at  $p_t > 0.8$  GeV. How does the coupling between longitudinal flow and minijets work ? What do we learn about the expanding fluid ? The fluid seems opaque and “dissipative”.
- The scale dependence of the effect points to the early stage as its source
- More info: nucl-ex/0407001

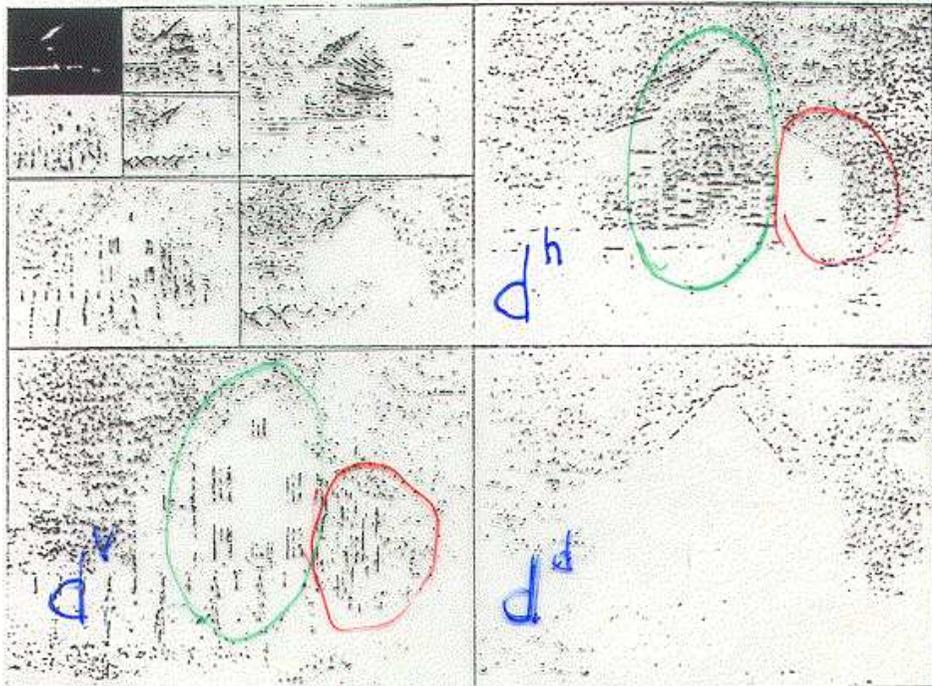
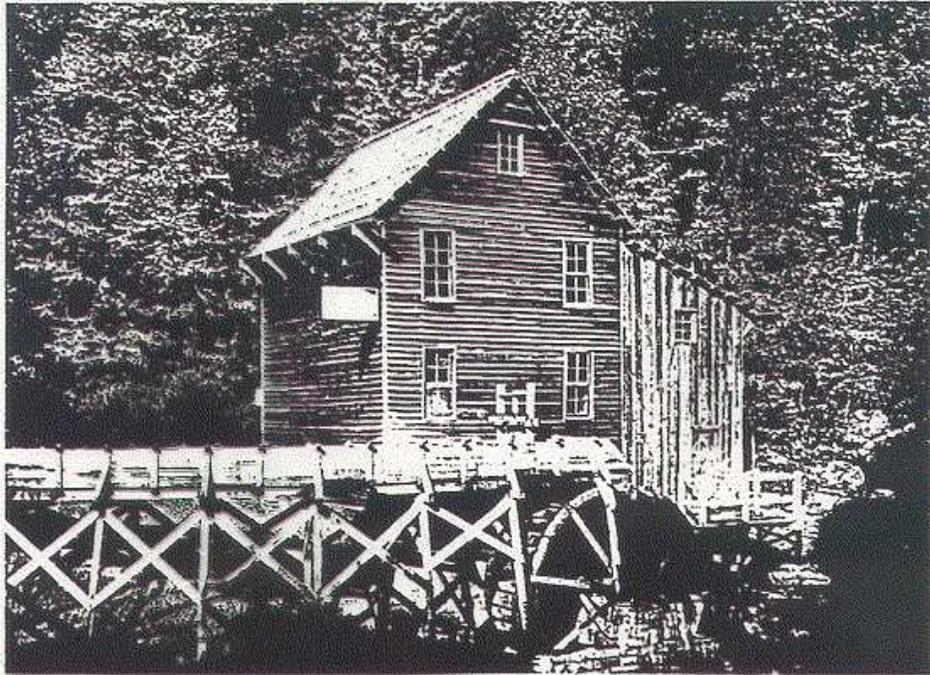
## 16 Extra slides

## 17 What is scale ?



"Fifty Abstract Paintings Which as Seen from Two Yards Change into Three Lenins Masquerading as Chinese and as Seen from Six Yards Appear as the Head of a Royal Bengal Tiger", S.Dali, 1963.

## 18 DWT of a photographic image



Reproduced from textbook:  
I. Daubechies, "Ten lectures on  
wavelets". The original caption: "A  
real image, and its wavelet  
decomposition into three  
multiresolution layers. On the  
wavelet components one clearly sees  
that the  $d^{j,v}$ ,  $d^{j,h}$ ,  $d^{j,d}$  emphasize,  
respectively, vertical, horizontal, and  
diagonal edges. In this figure, the  
bottom picture has been  
overexposed to make details in the  
 $d^{j,\lambda}$  more apparent. I would like to  
thank M. Barlaud for providing this  
figure." The colored marks are mine.