

# **Correlation structure of STAR events**

CINPP, Kolkata, India.

**Mikhail Kopytine for the STAR Collaboration**

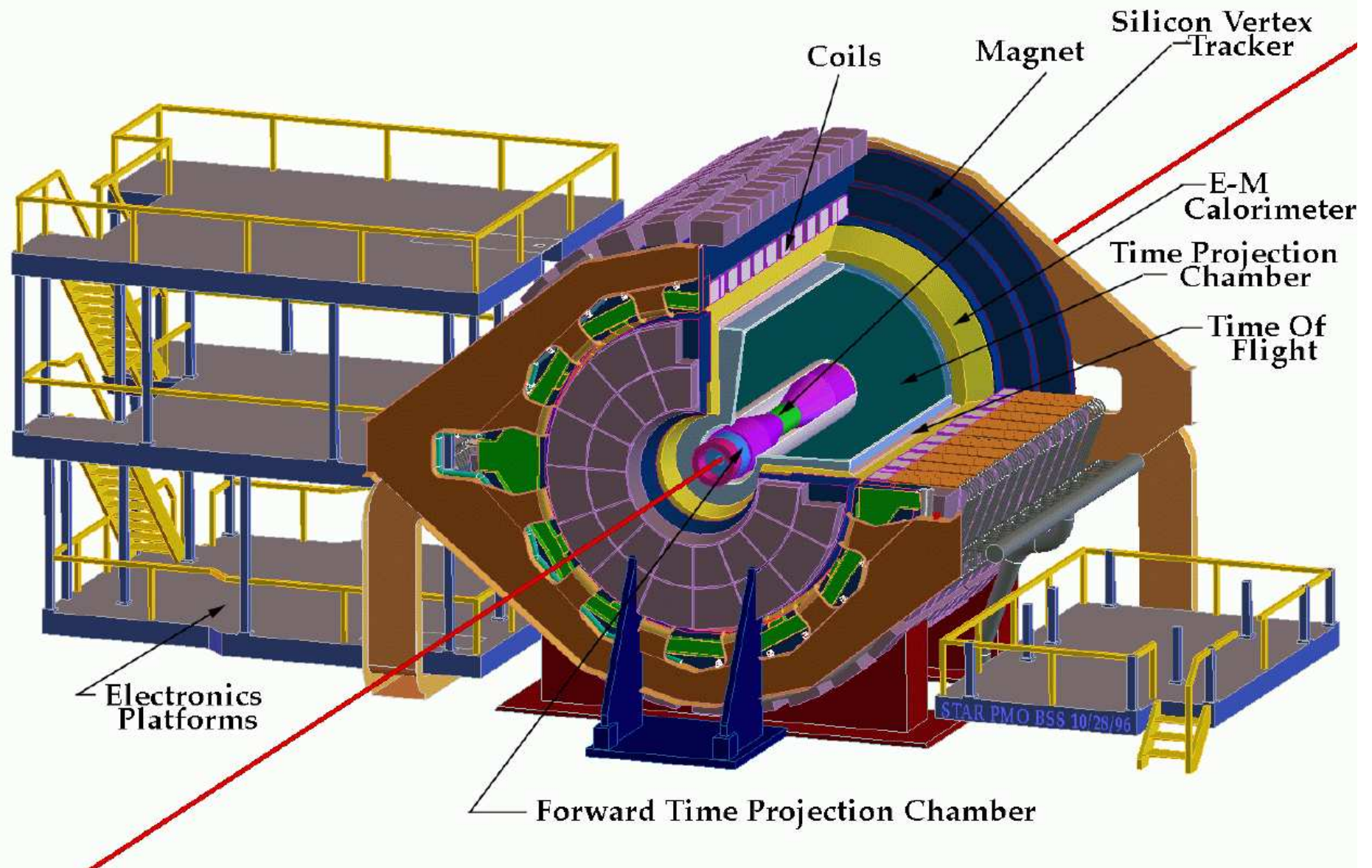
Kent State University

<http://www.star.bnl.gov/~kopytin/>

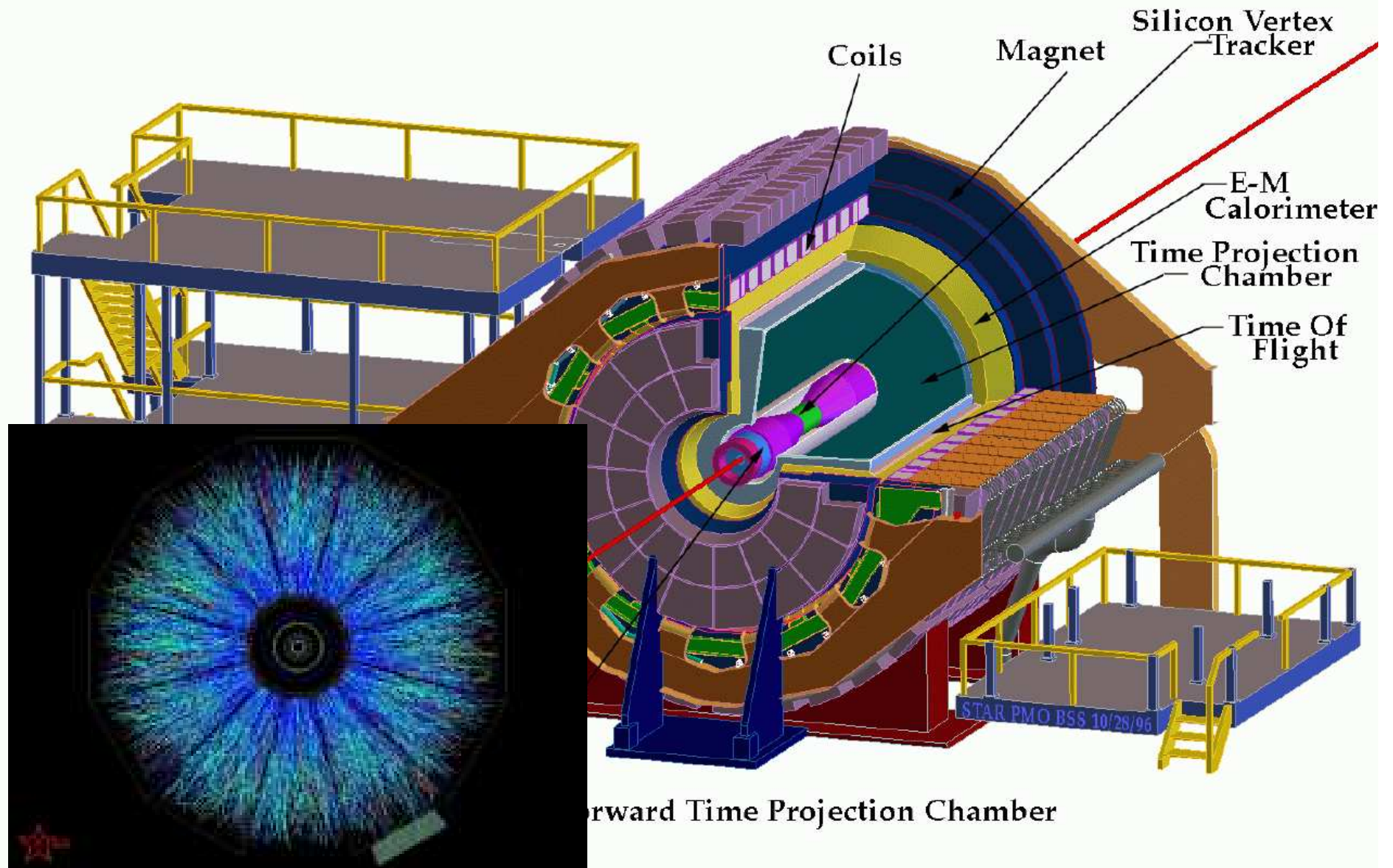
**February 4, 2005**



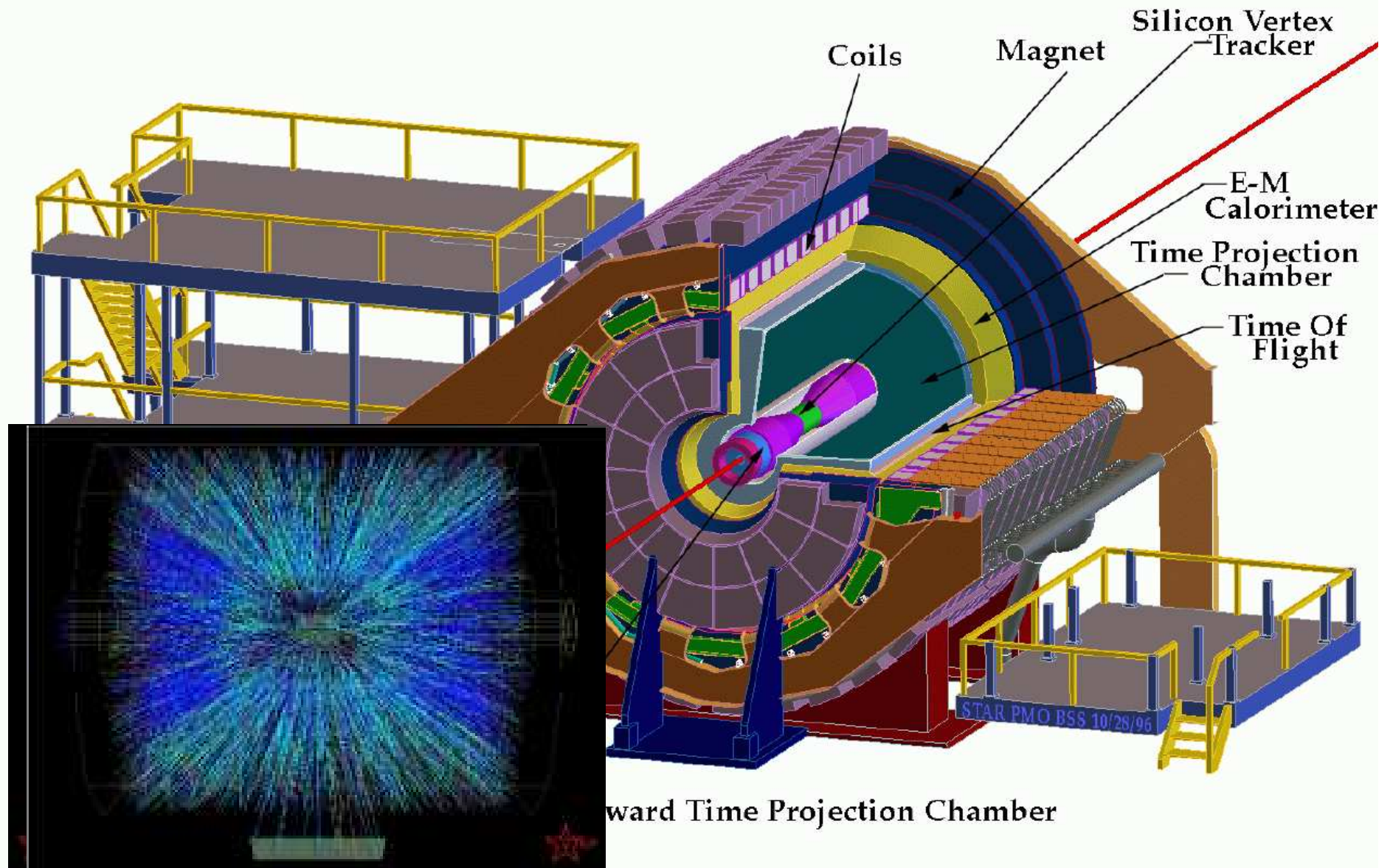
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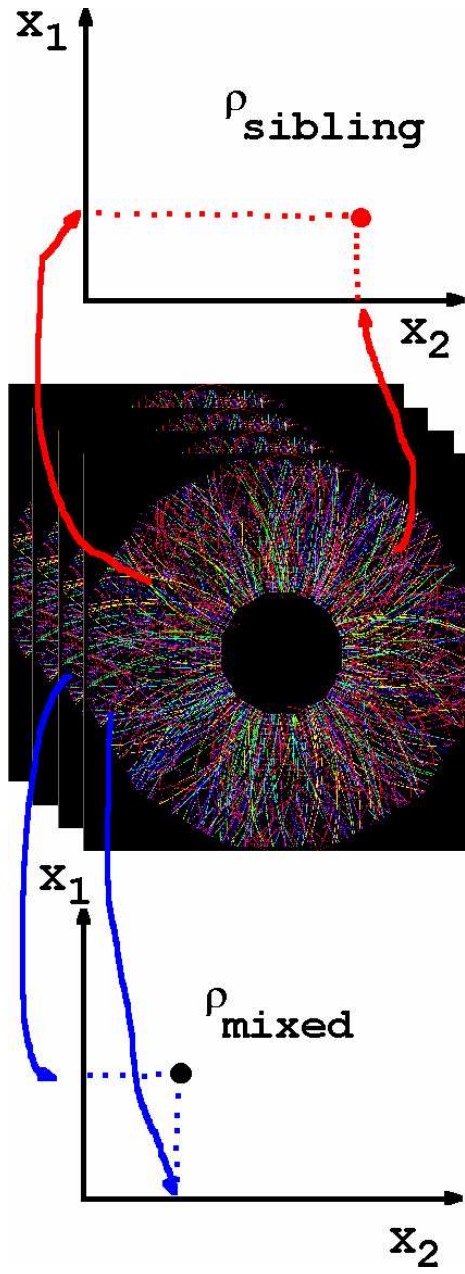
- Direct construction of a correlation function.
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- Observations

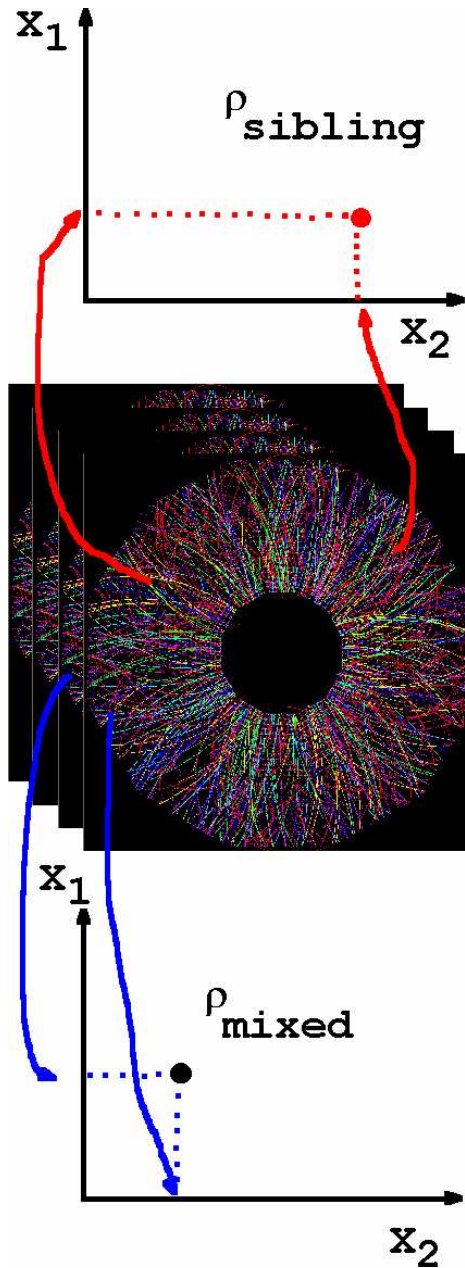
- Conclusions

## 2 Autocorrelation

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$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \rightarrow \begin{pmatrix} x_{\Sigma} \equiv x_1 + x_2 \\ x_{\Delta} \equiv x_1 - x_2 \end{pmatrix},$$

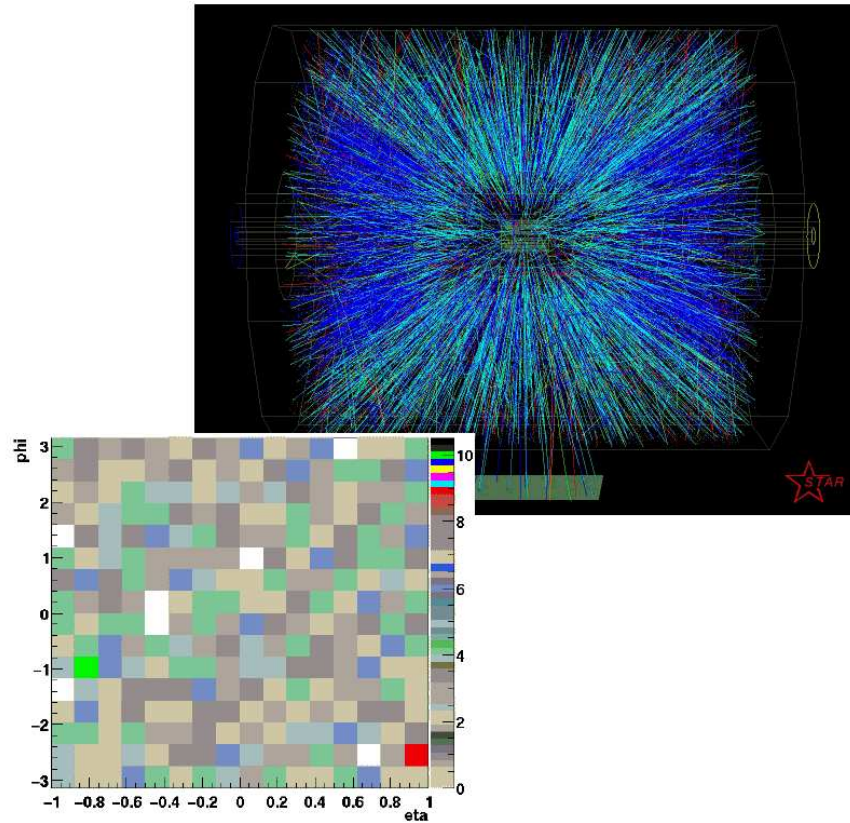
always a lossless transformation of data.

**Autocorrelation**  $A$  is a projection of a two-point distribution onto difference variable(s)  $x_{\Delta}$ , lossless for  $x_{\Sigma}$ -invariant (homogenous, stationary) problems.

$$\Delta R(x_1, x_2) = \frac{\rho(x_1, x_2)}{\rho_{\text{ref}}(x_1, x_2)} - 1$$

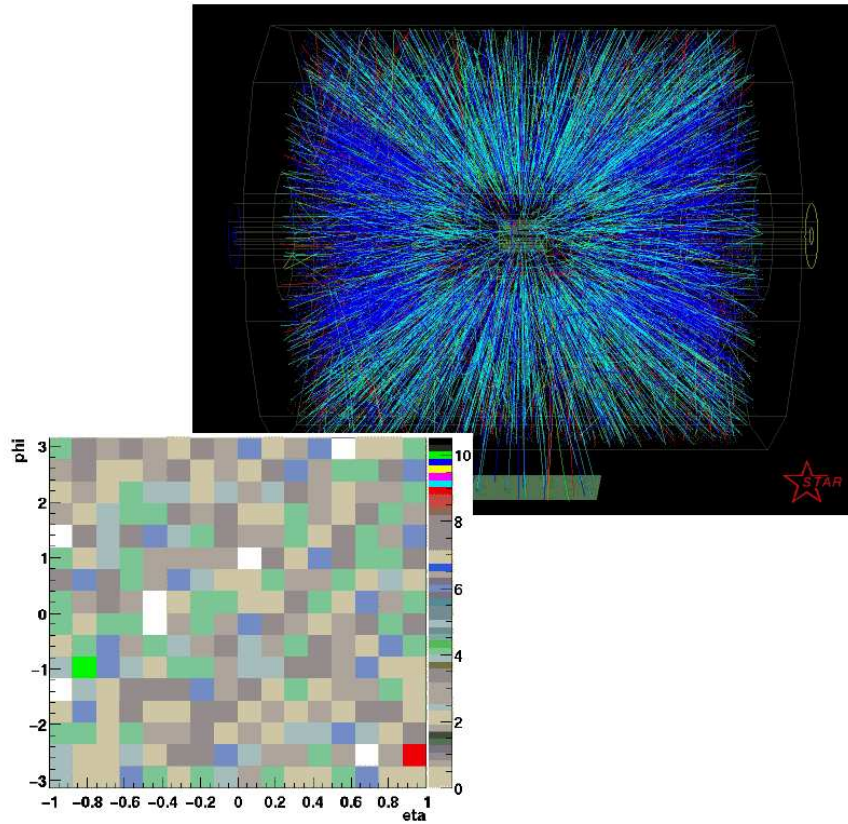
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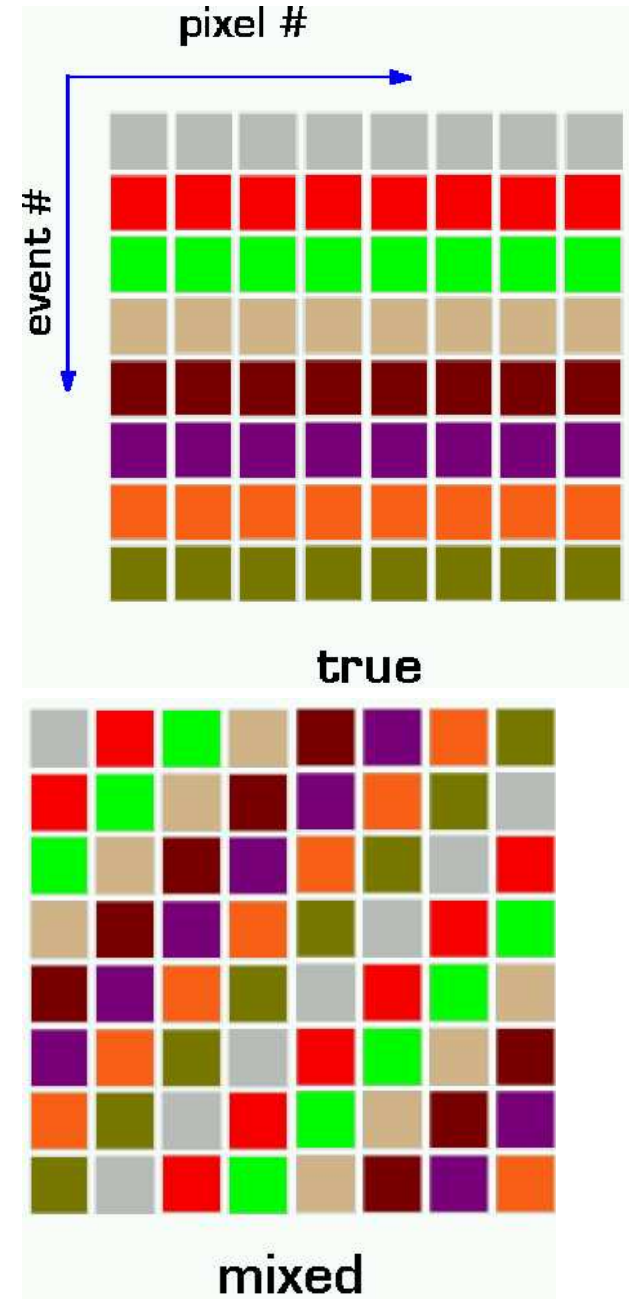


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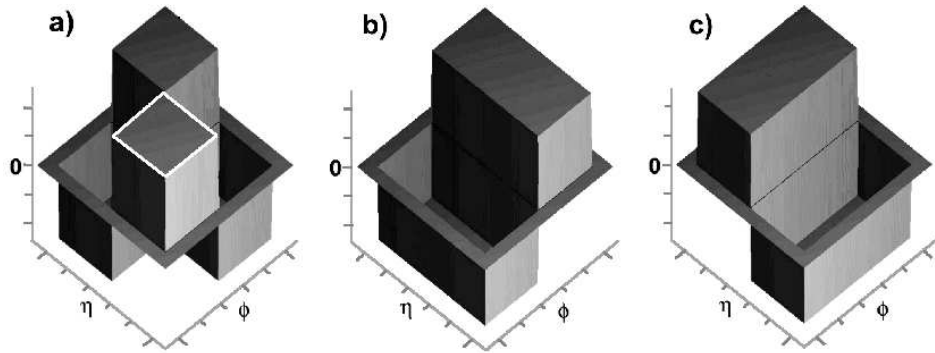
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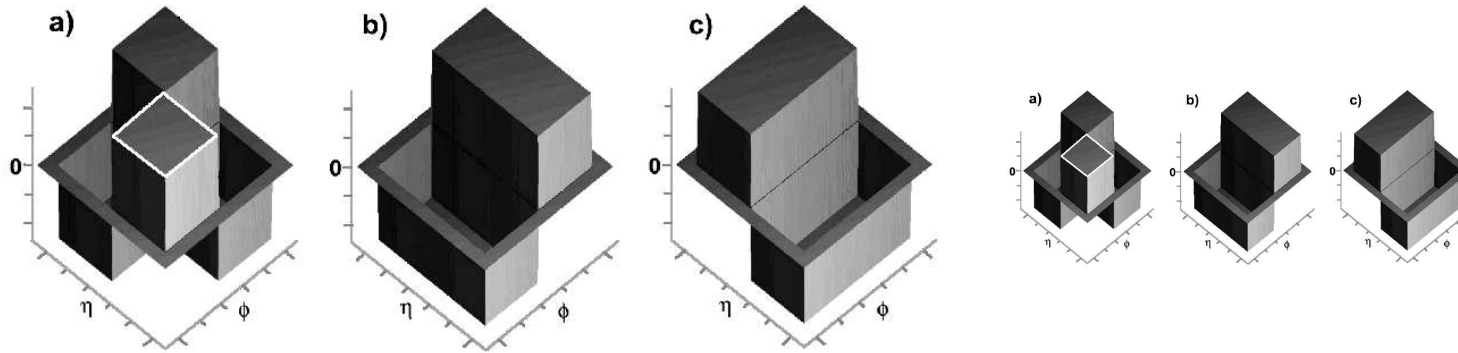


## 4 Local hadron density fluctuations and Discrete Wavelet Transform (DWT)

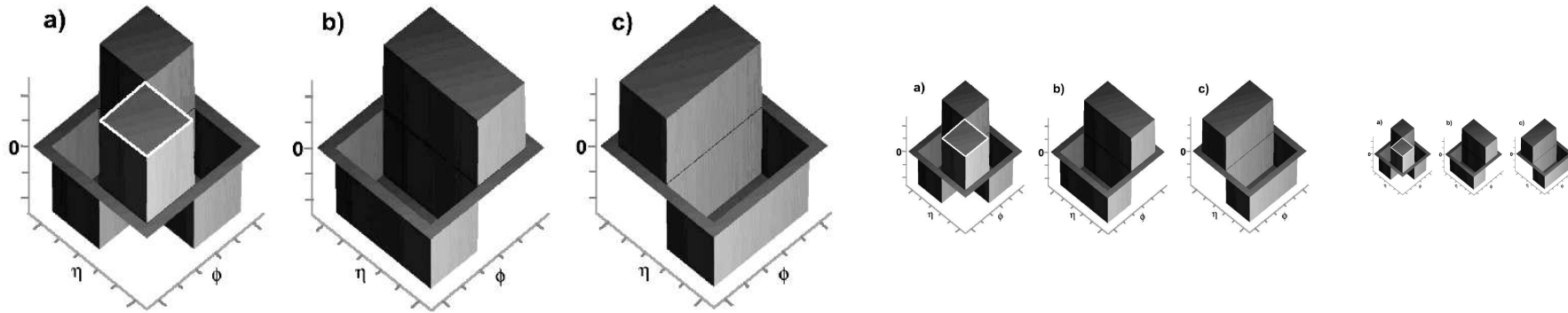
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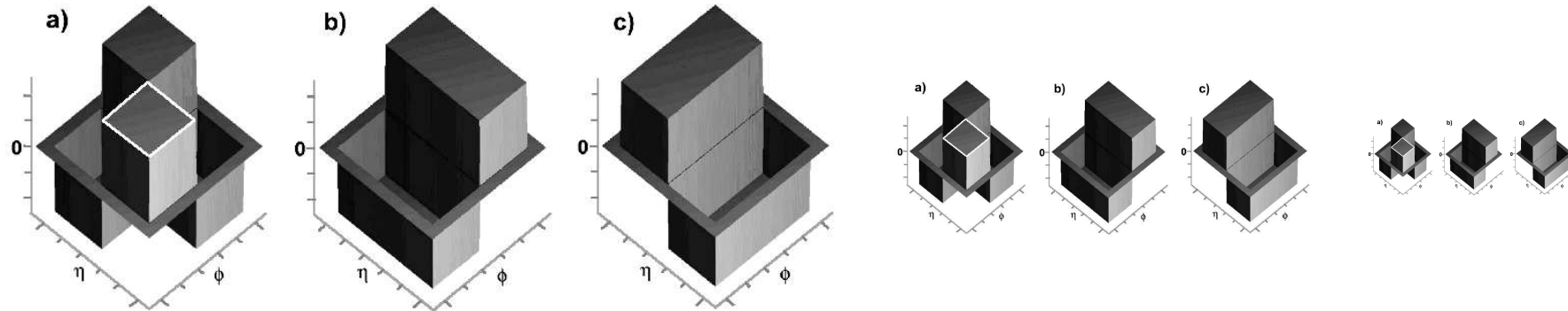
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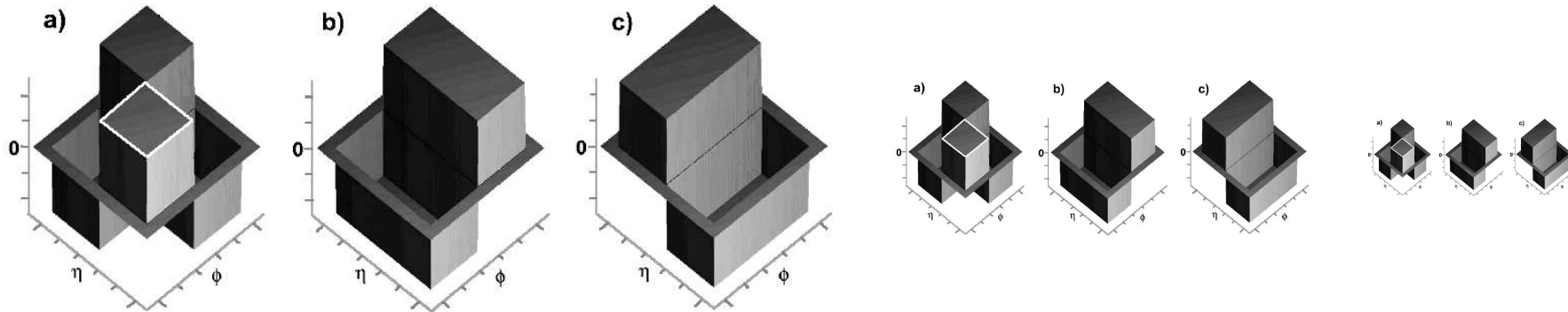


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$F_{m,l,k}^\lambda(\phi, \eta)$ —Haar wavelet **orthonormal basis** in  $(\phi, \eta)$ . scale fineness ( $m$ ), directional modes of sensitivity ( $\lambda$ ), track density  $\rho(\eta, \phi, p_t)$ , locations in 2D  $(l, k)$ . **DWT is an expansion in this basis.**

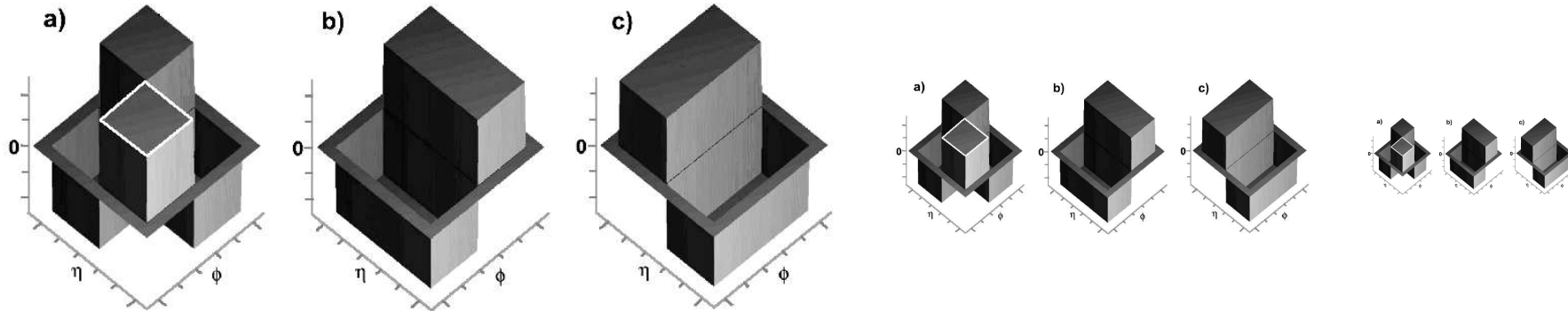
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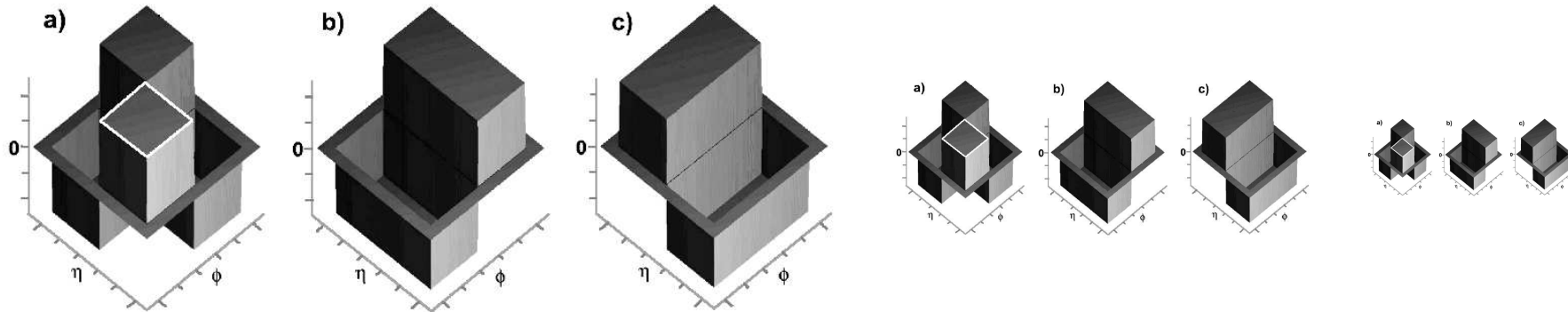
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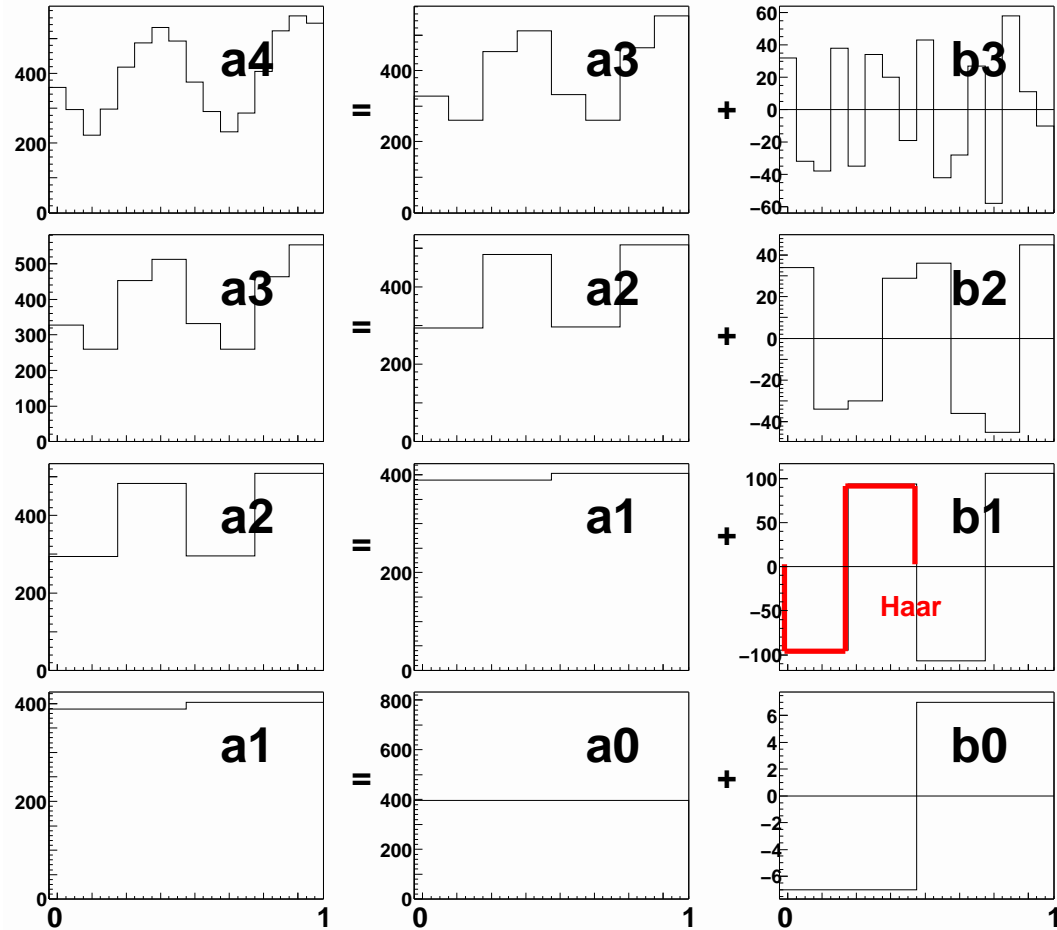
Normalized:

$$P_{\text{dyn}}^\lambda(m) / P_{\text{mix}}^\lambda(m) / n(p_t) \quad (3)$$

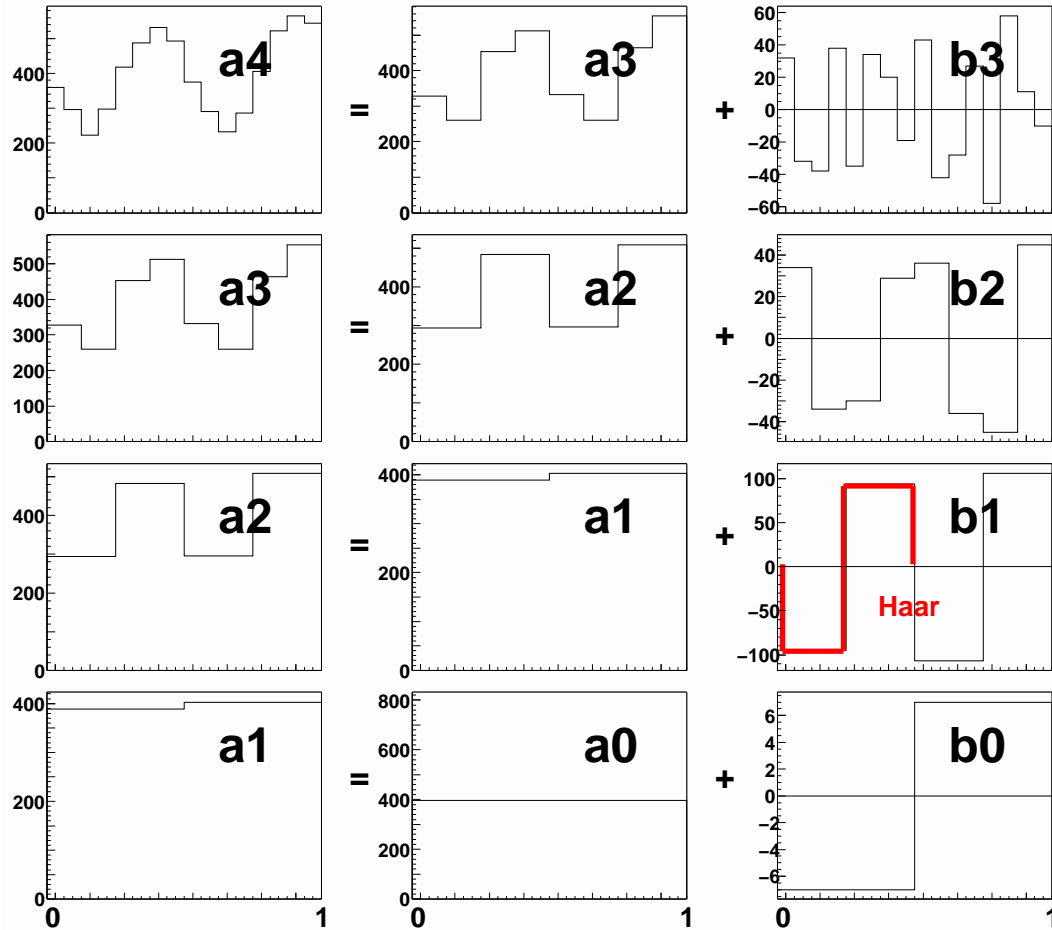


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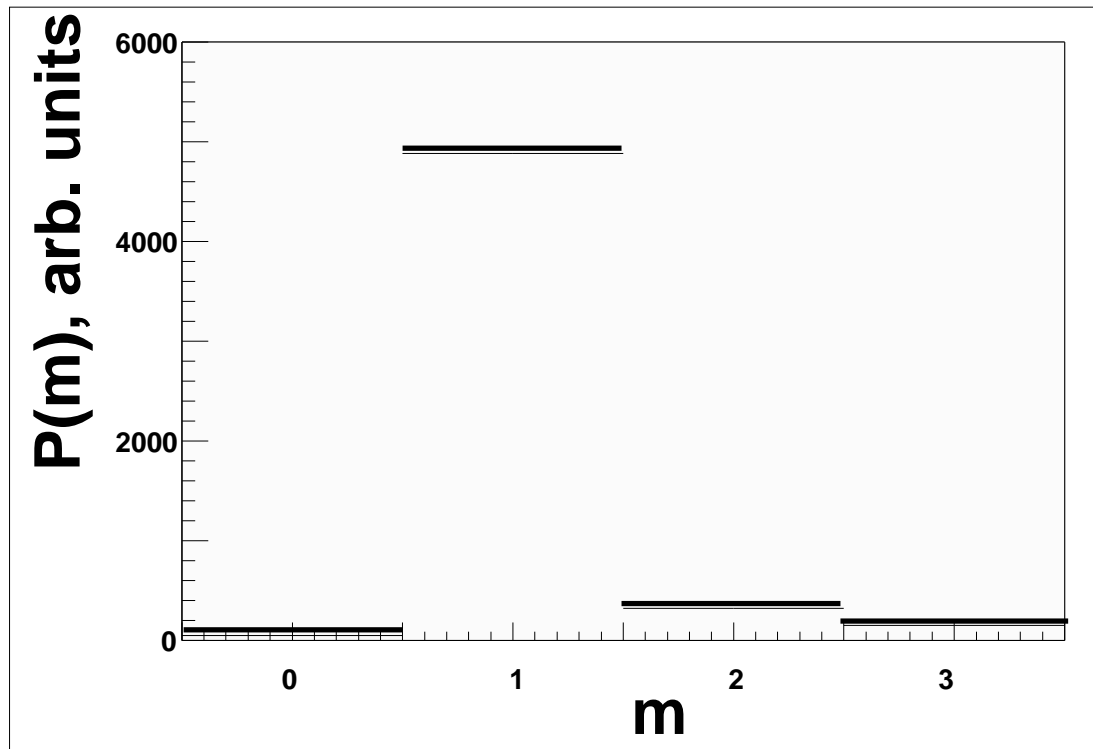


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Elliptic flow-inspired example:  
 $x$  axis – an angle in “natural units” ( $2\pi = 1$ ),  $y$  axis – multiplicity. The multiresolution theorem:  $a_4 = a_0 + b_0 + b_1 + b_2 + b_3$ , can have better fineness.

## 6 Example of a DWT power spectrum



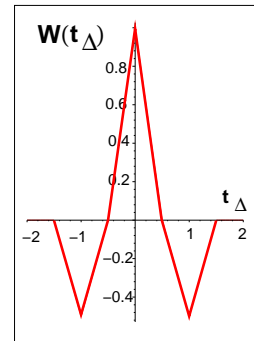
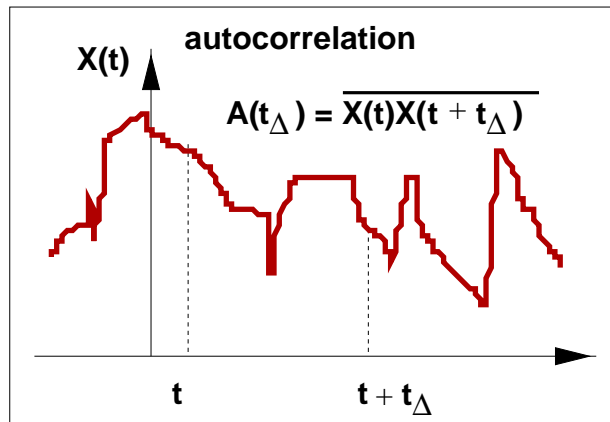
Power spectrum of that flow event as a function of “fineness”  $m$ . The dominant contribution is  $m = 1$  (the “ $v_2$ ” harmonic, **b1**). Statistical fluctuations also contribute.

$$P(m) = 2^{-m} \sum_i \langle \rho, F_{m,i} \rangle^2.$$

Computational complexity  $O(N)$ !

## 7 “Dynamic texture” as a nonparametric measure of the correlation shape

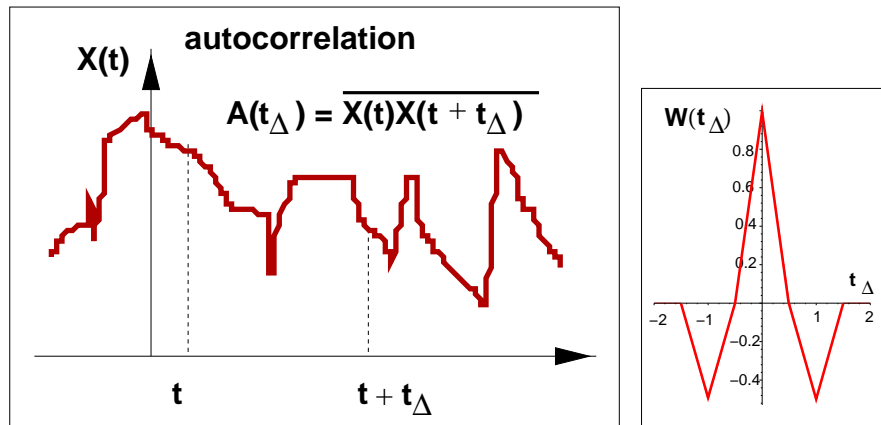
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$$P(m) = \overline{\int_{-\infty}^{\infty} X(\tau/2)X(-\tau/2)W(\tau, m) d\tau},$$

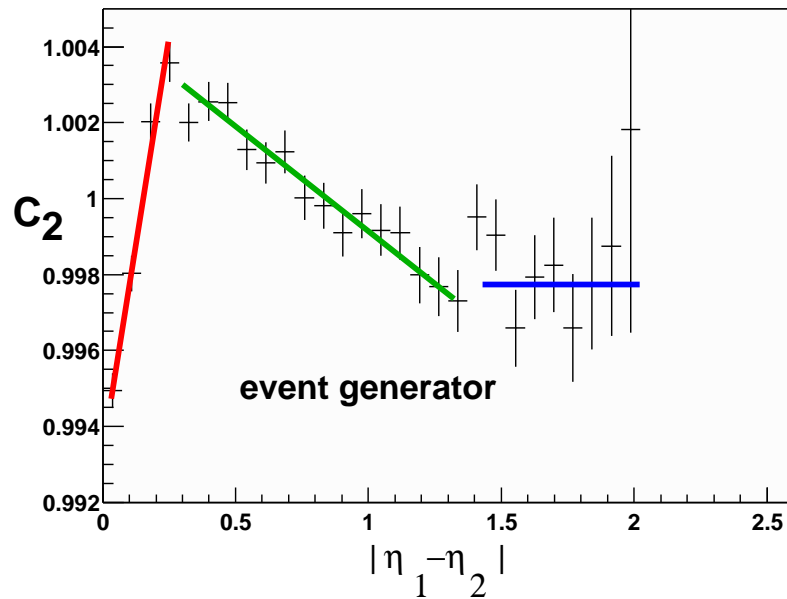
where  $W$  is the weight function for the Haar wavelet.  $P(m)$  reflects differential structure on scale  $m$ . See example:

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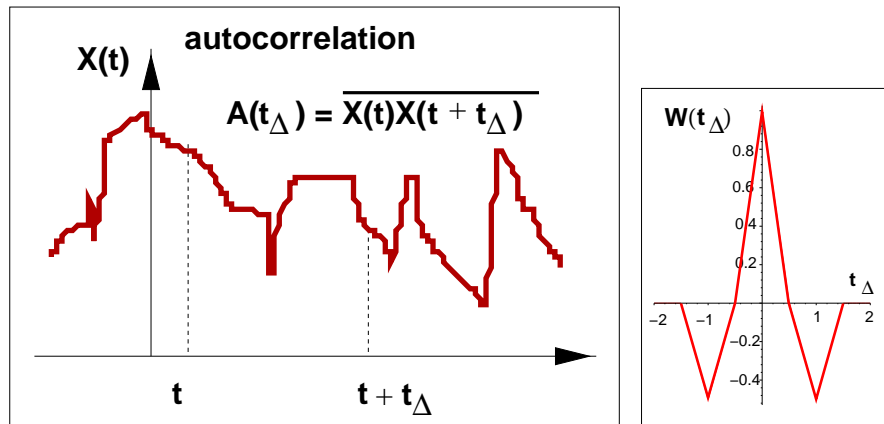


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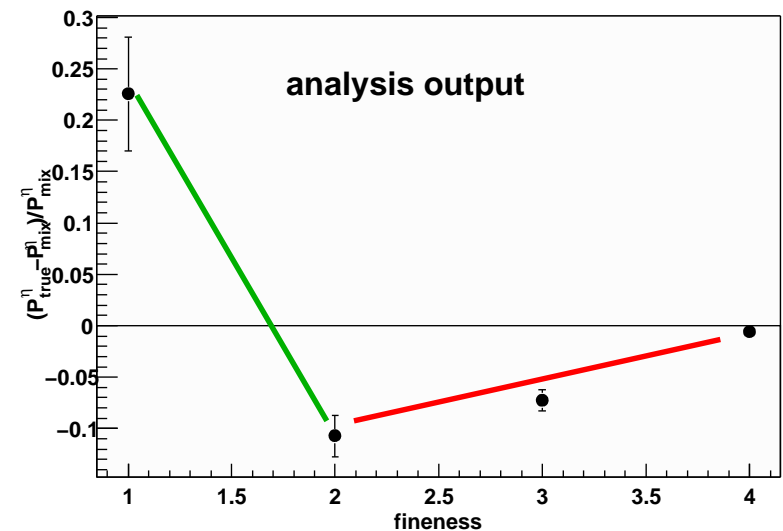
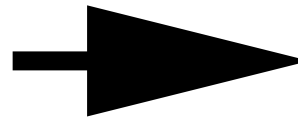
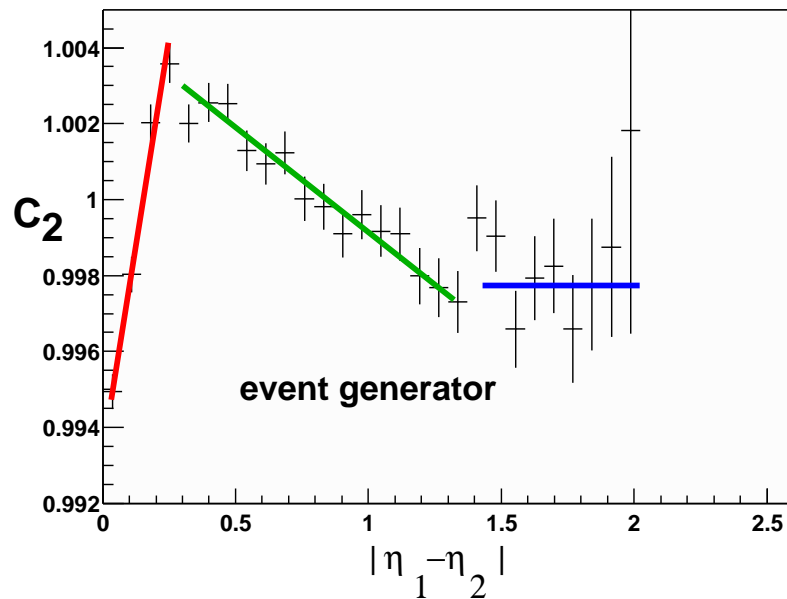


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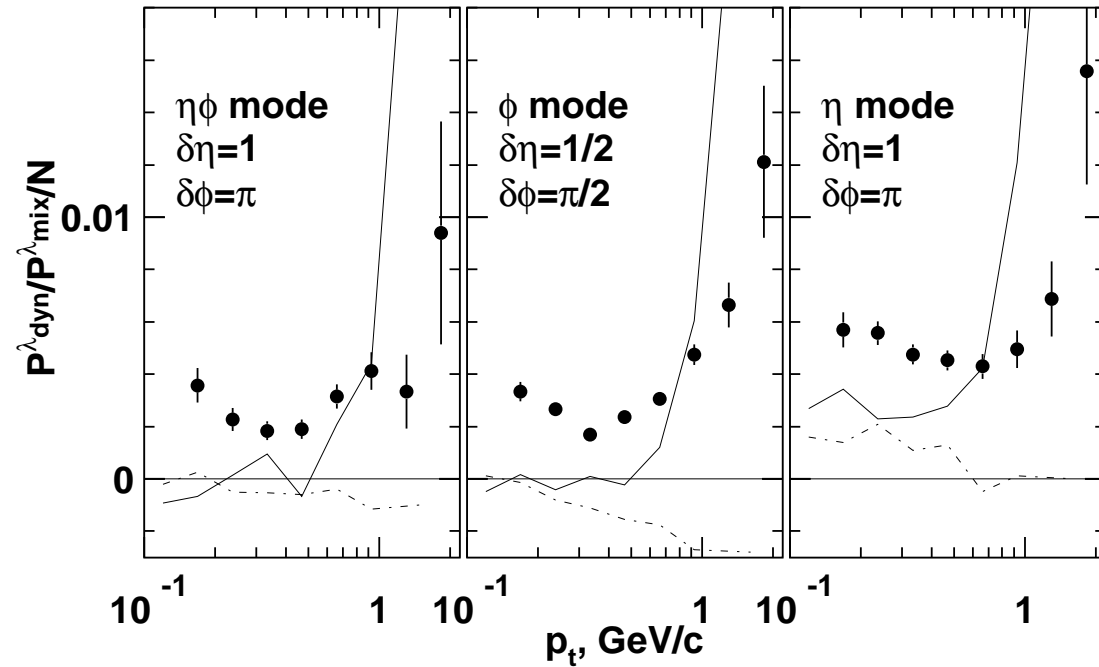
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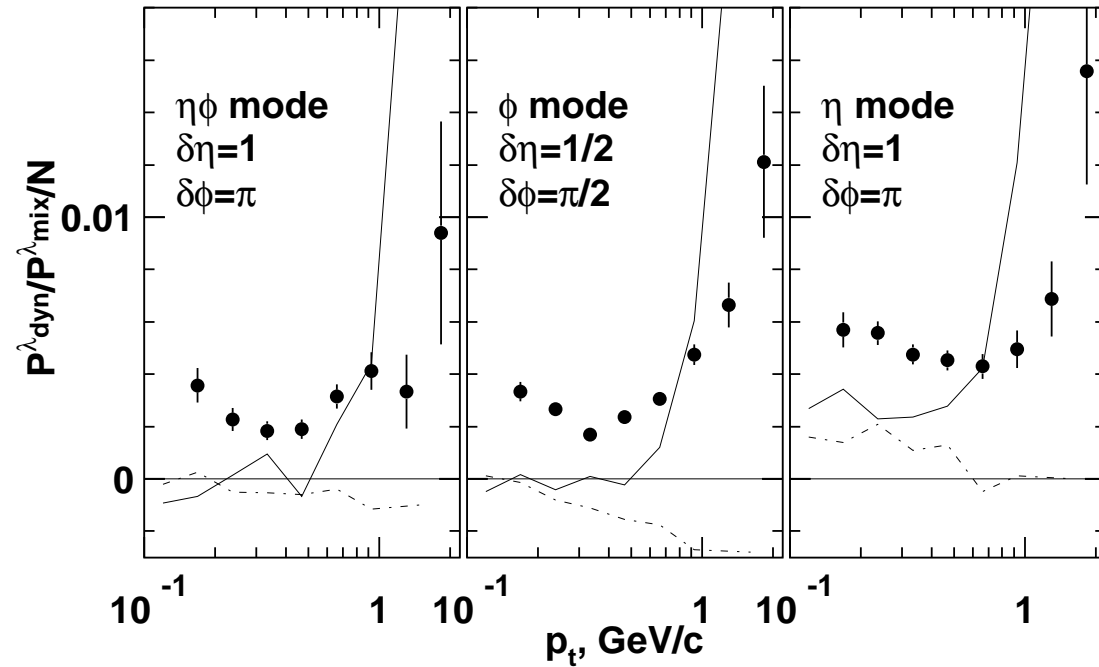


# 8 “Dynamic texture” $p_t$ dependence: peripheral events, $\sqrt{s_{NN}} = 200$ GeV



Peripheral (60-84%) events: normalized dynamic texture for fineness scales  $m = 0, 1, 0$  from left to right panels, respectively, as a function of  $p_t$ . ● – STAR data; solid line – standard HIJING, dash-dotted line – HIJING without jets.

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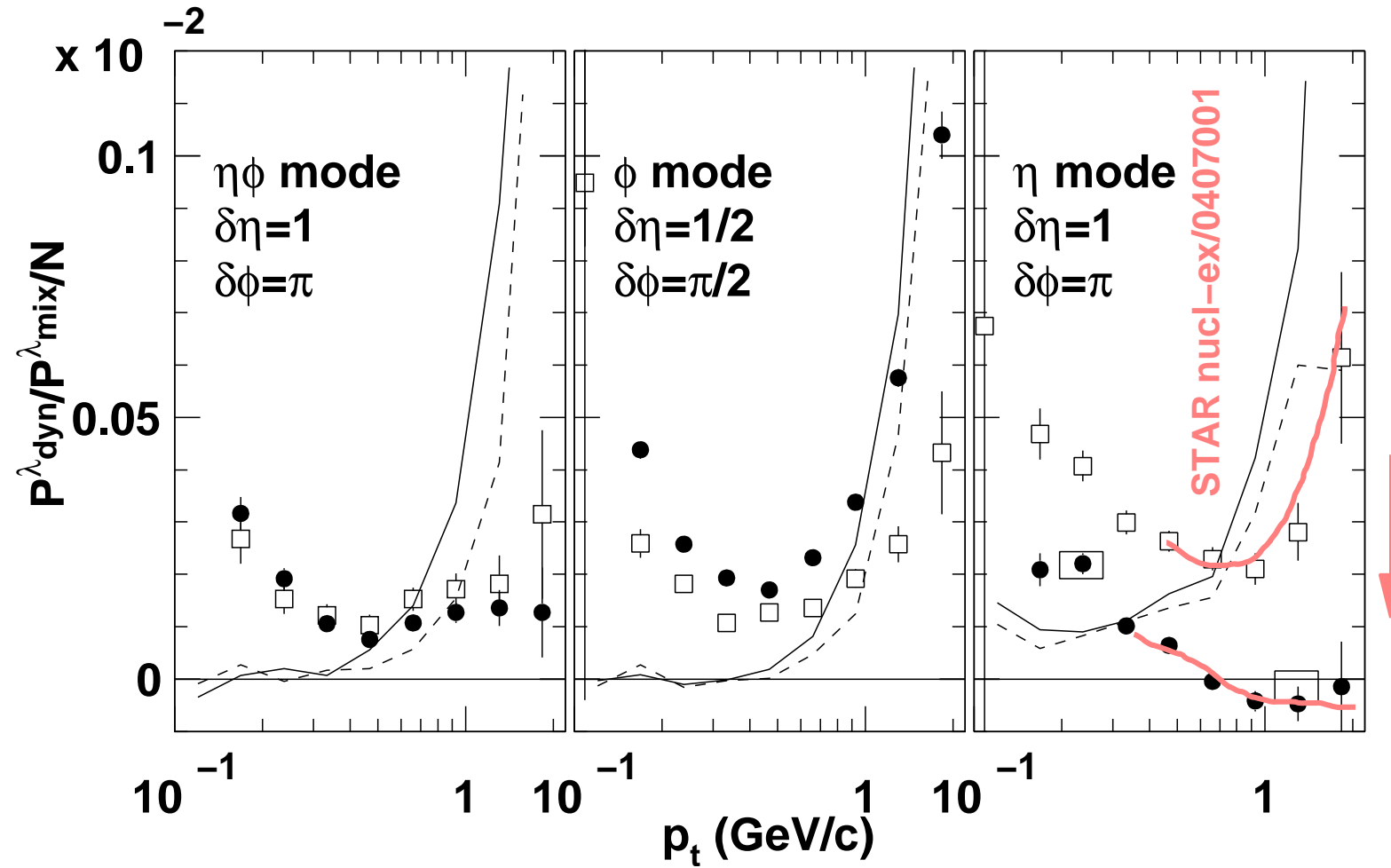
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Qualitative trends in peripheral data are as expected. What signal to expect in the central data, if correlation does not change ?

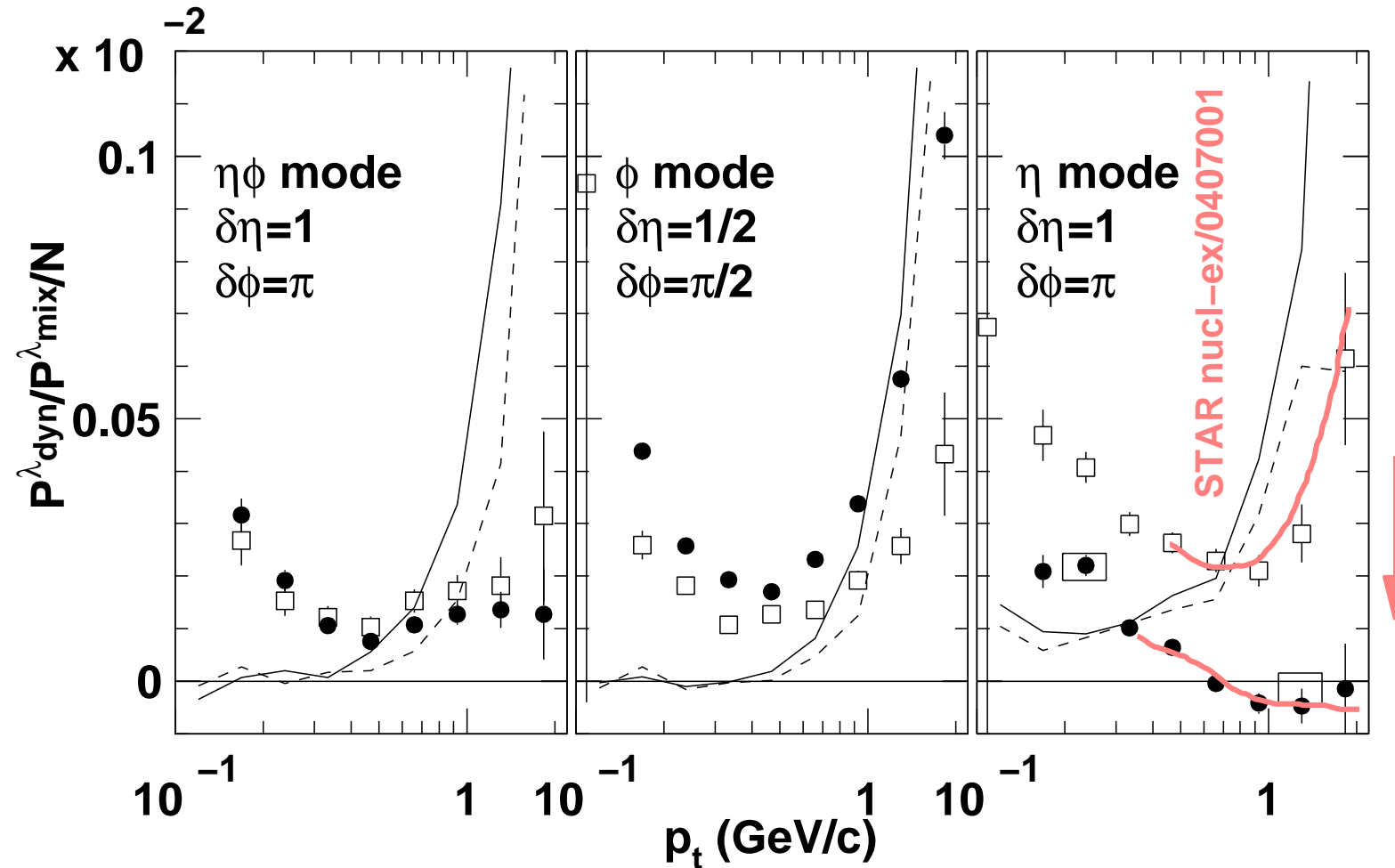
$$\left( \frac{P_{\text{true}}}{P_{\text{mix}}} - 1 \right) \frac{1}{N} \Big|_{\text{centr}} = \left( \frac{P_{\text{true}}}{P_{\text{mix}}} - 1 \right) \Big|_{\text{periph}} \frac{1}{N_{\text{centr}}} \quad (4)$$

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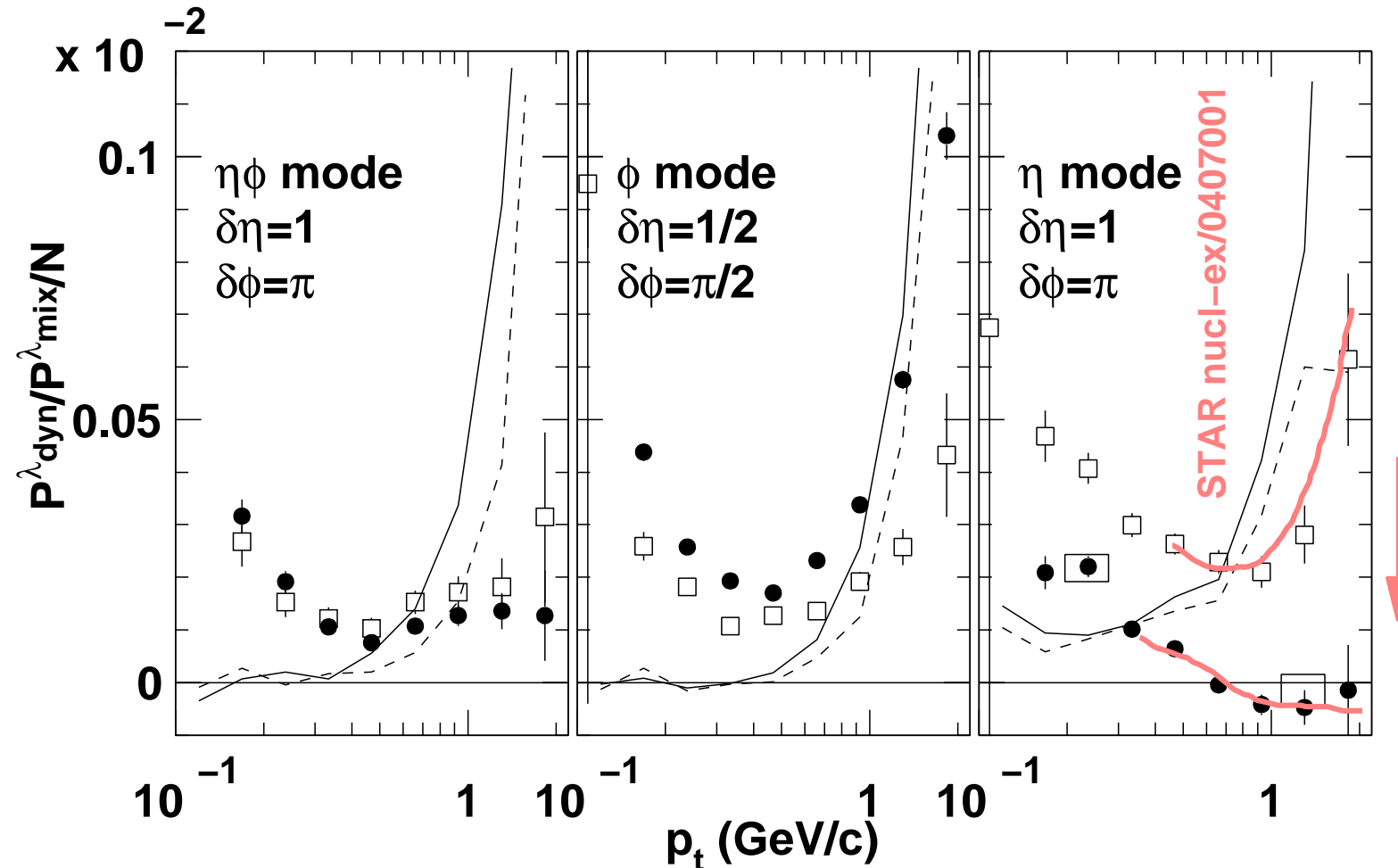
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Central (top 4%) events: normalized dynamic texture for fineness scales  $m = 0, 1, 0$  from left to right panels, respectively, as a function of  $p_t$ .

● STAR data; solid line – Hijing without jet quenching; dashed line – Hijing with quenching; □ peripheral STAR data renormalized to compare.

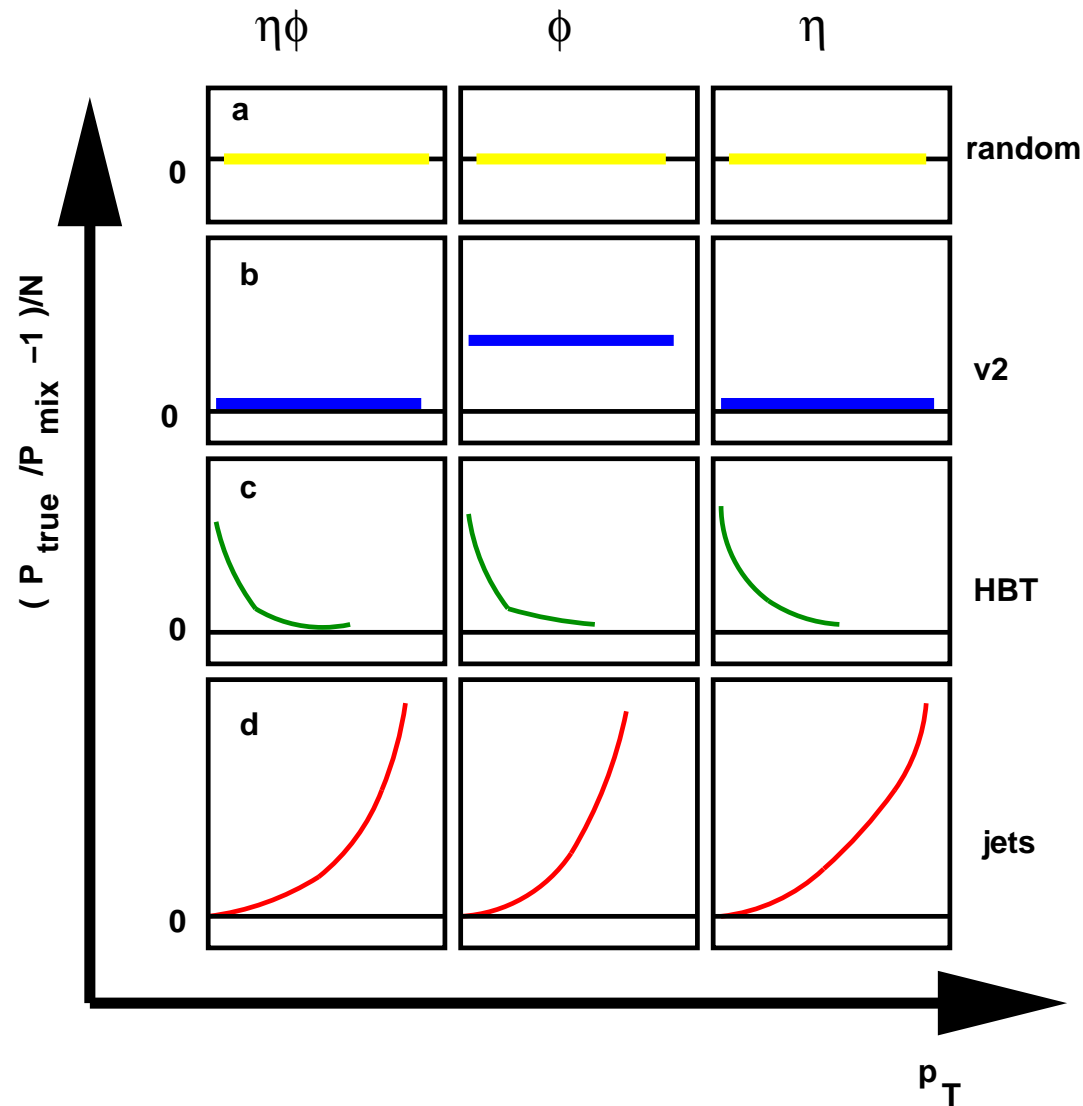
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● STAR data; solid line – Hijing without jet quenching; dashed line – Hijing with quenching; □ peripheral STAR data renormalized to compare. Minijet elongation  $\Rightarrow$  correlation broadening  $\Leftrightarrow$  reduced correlation gradient  $\Leftrightarrow$  reduced “texture”

# 10 “Dynamic texture” response



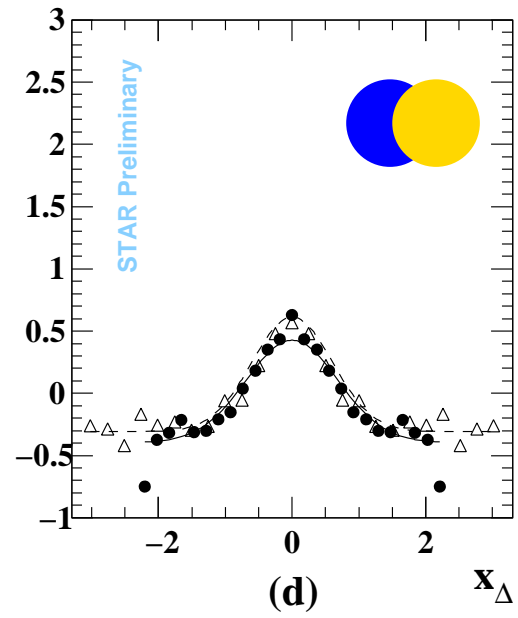
Dynamic texture response in various idealized situations (showing only one scale):

- (a) events of random (uncorrelated) particles
- (b)  $p_t$ -independent elliptic flow
- (c) Correlations at low  $Q_{\text{inv}}$  (Bose-Einstein correlations and Coulomb effect)
- (d) HIJING jets

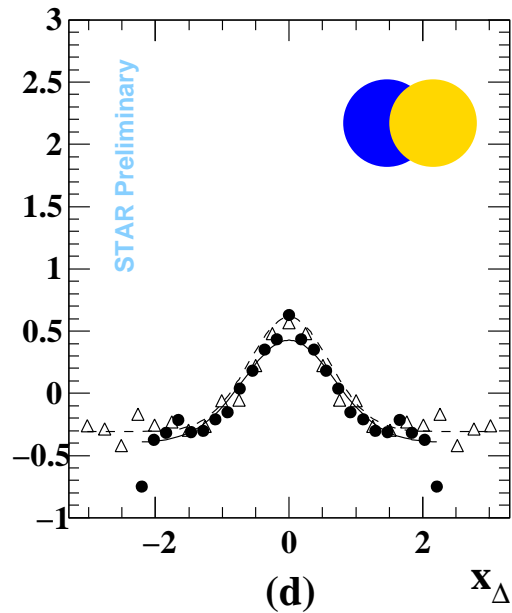
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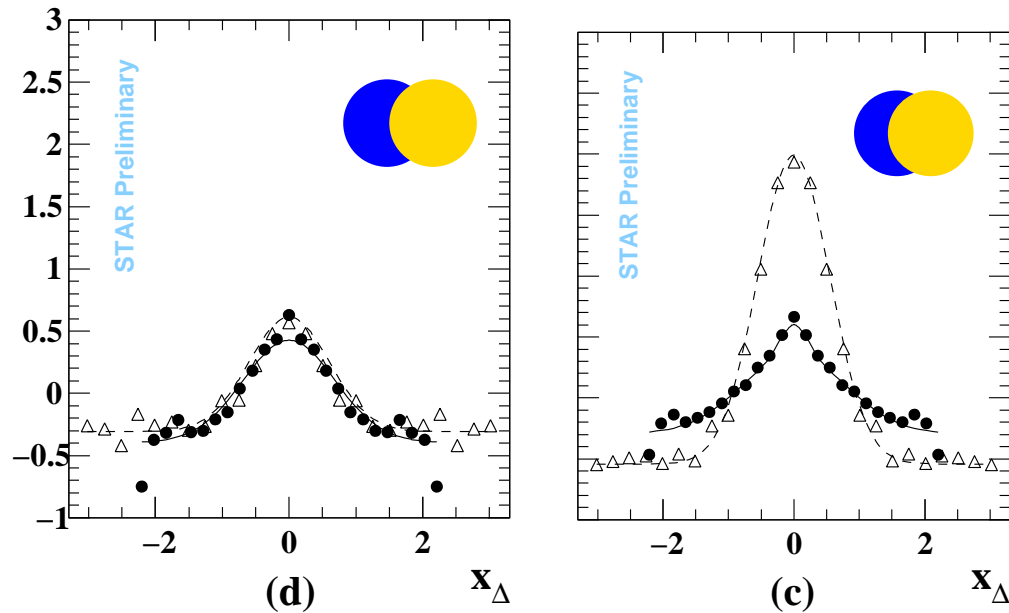


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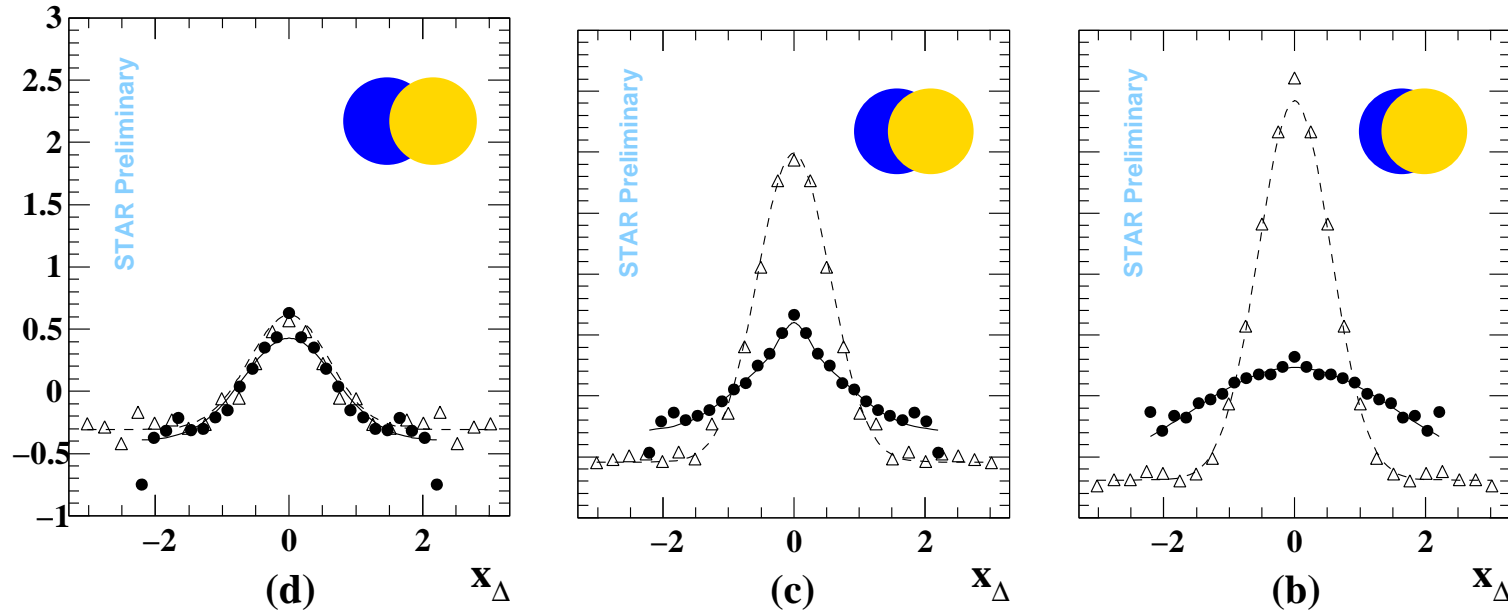
Projections of  $\bar{N}[\rho(\eta_\Delta, \phi_\Delta)/\rho(\eta_\Delta, \phi_\Delta)_{\text{ref}} - 1]|_{CI}$  on  $x_\Delta$  which is  $\phi_\Delta$  ( $\Delta$ ) or  $\eta_\Delta$  ( $\bullet$ ).  $v_1$  and  $v_2$  are subtracted.

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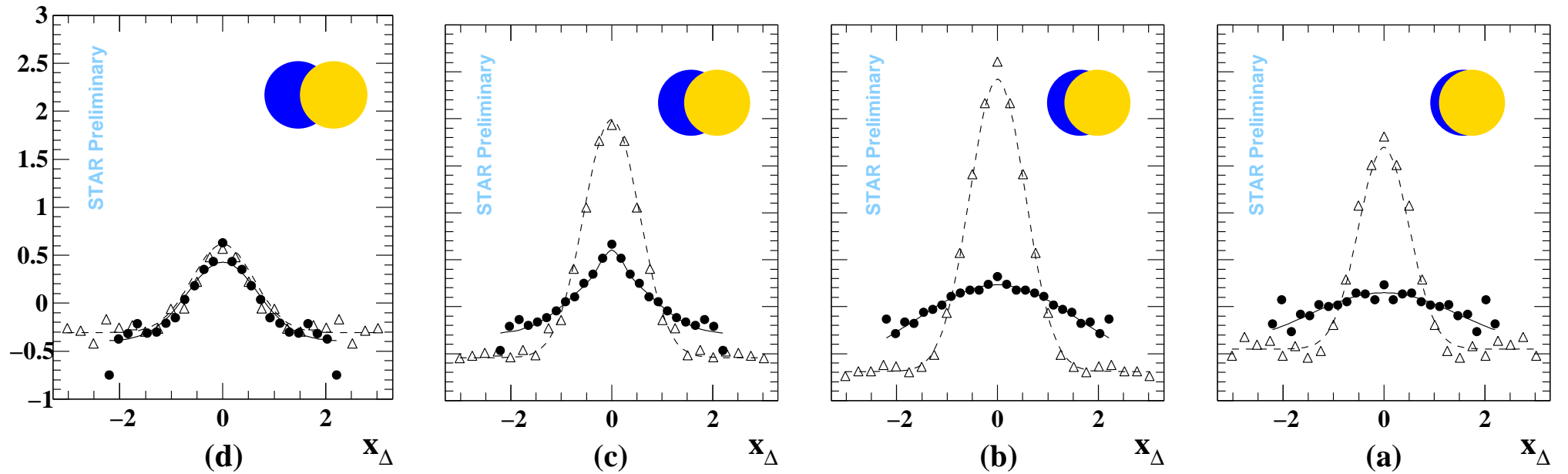
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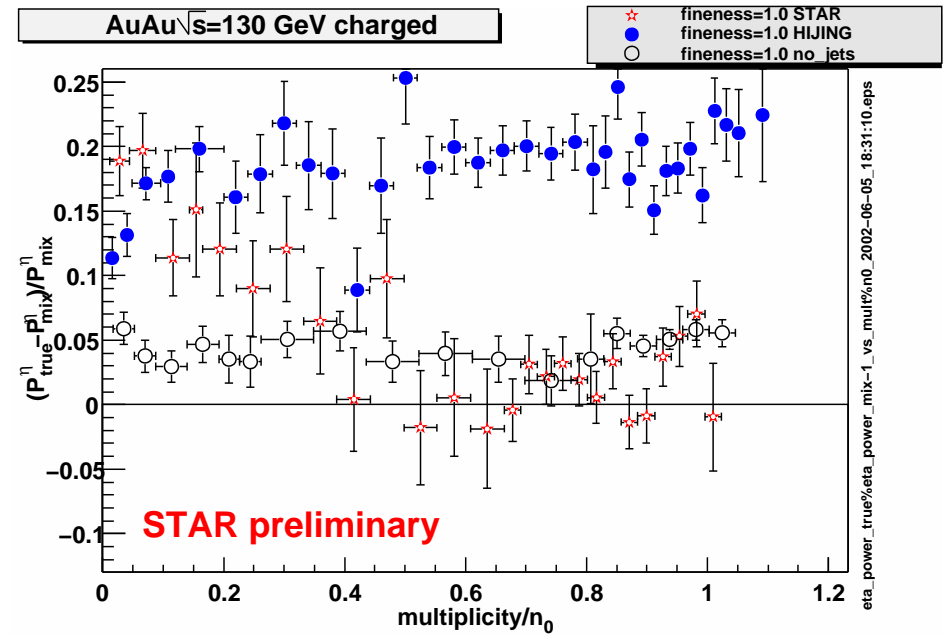


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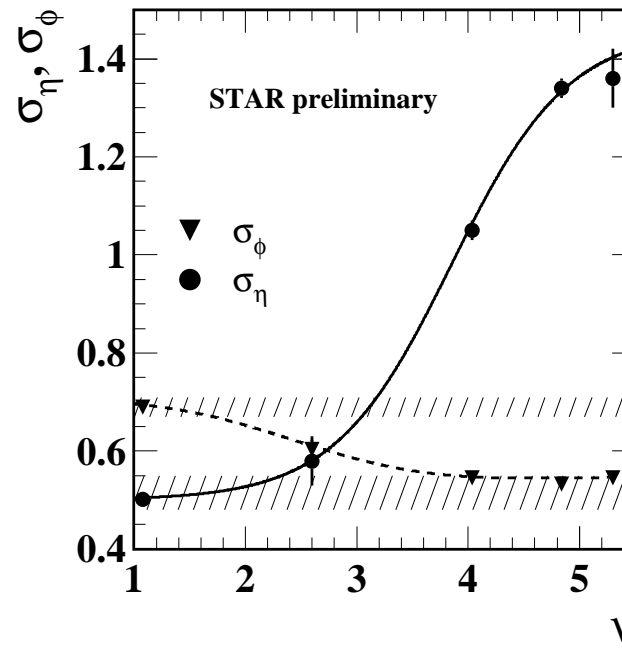
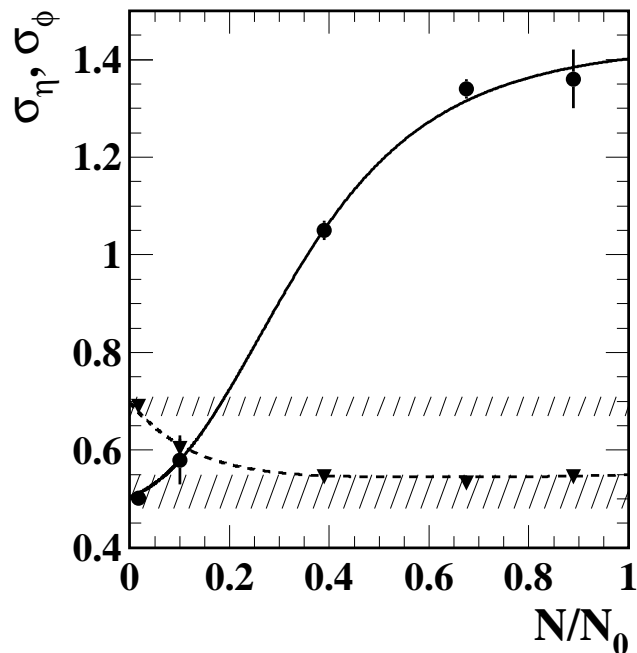
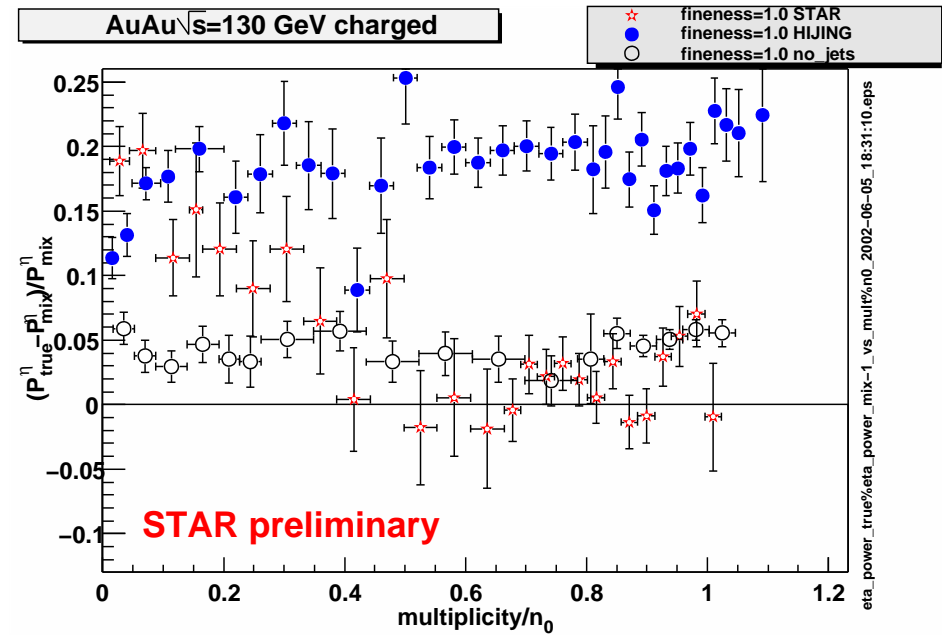
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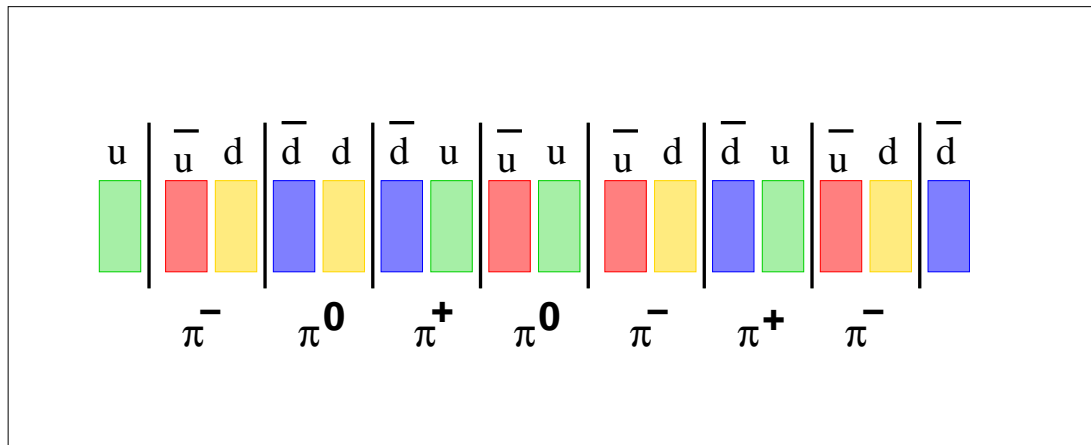
nucl-ex/0411003. STAR two-particle correlation analysis. The step-like character of the centrality dependence is elucidated using

$\nu = (N_{\text{part}}/2)^{1/3}$

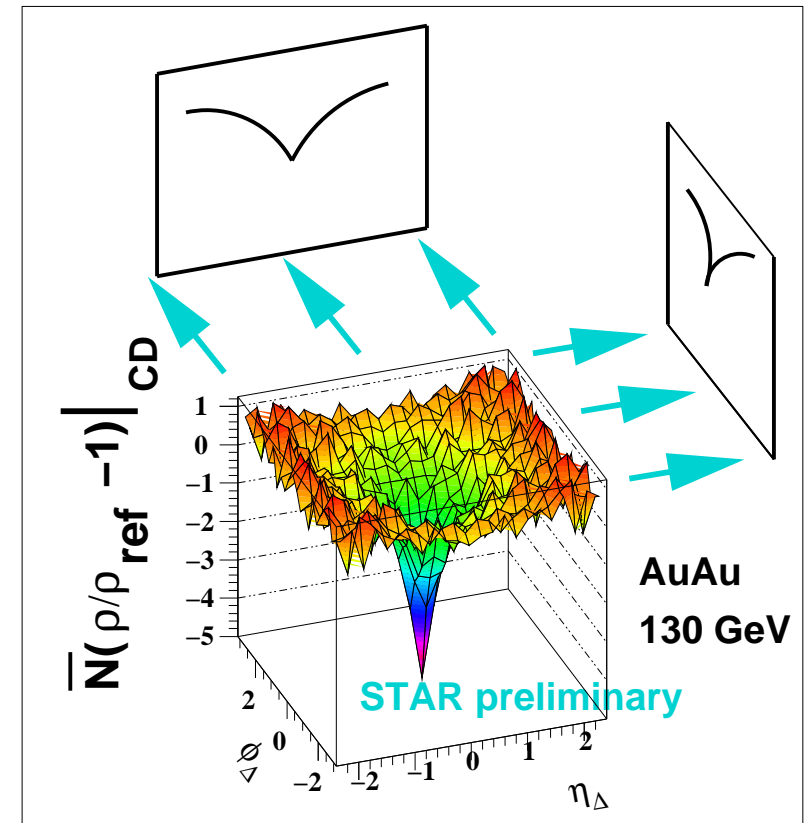
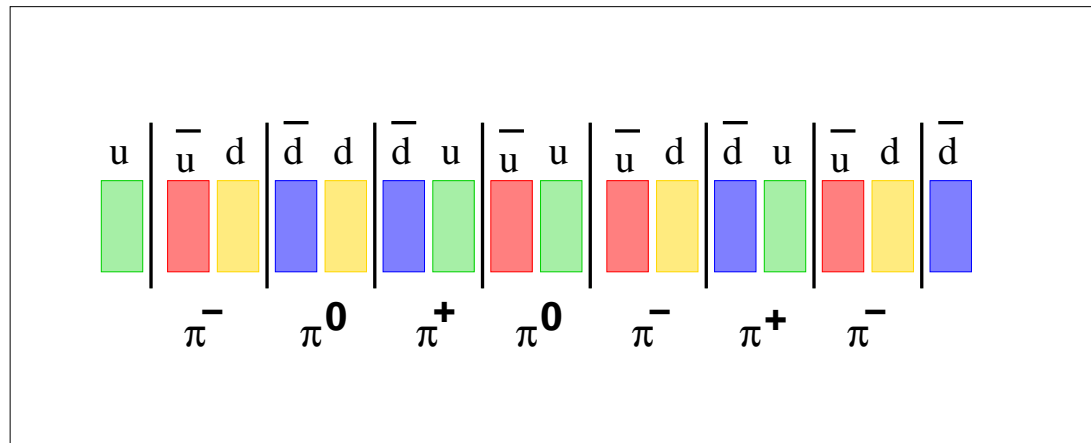


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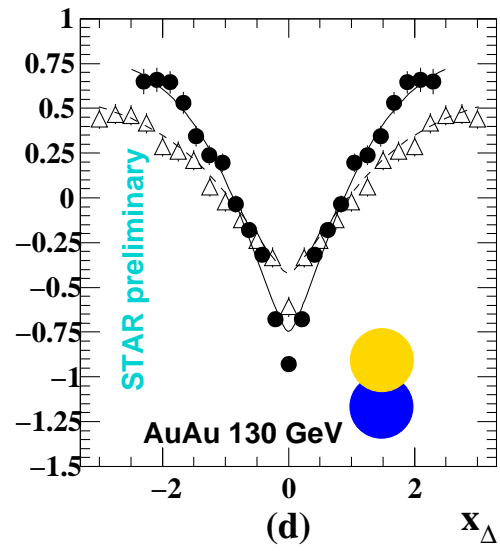
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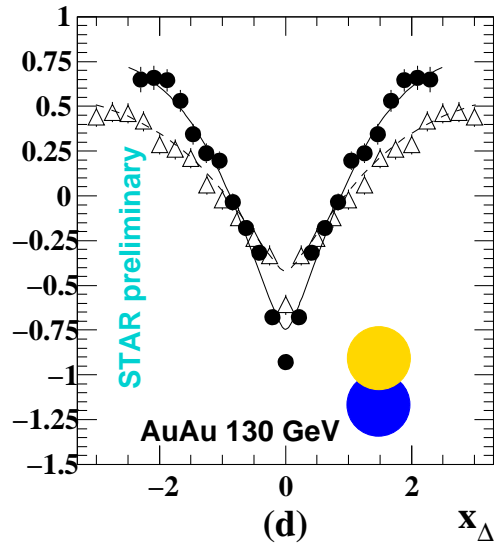
The driving physics: charge conservation in hadronization. Suppress short range correlations – BEC and conversion  $e^+e^-$  – by a kinematic pair cut. The  $\bar{N}_x$  is good when number of correlation sources  $\propto N$ .

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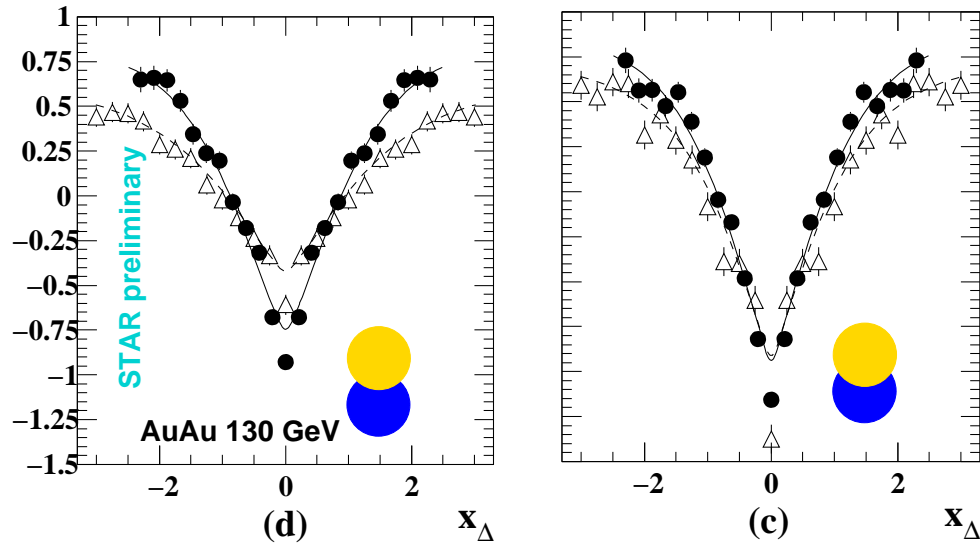


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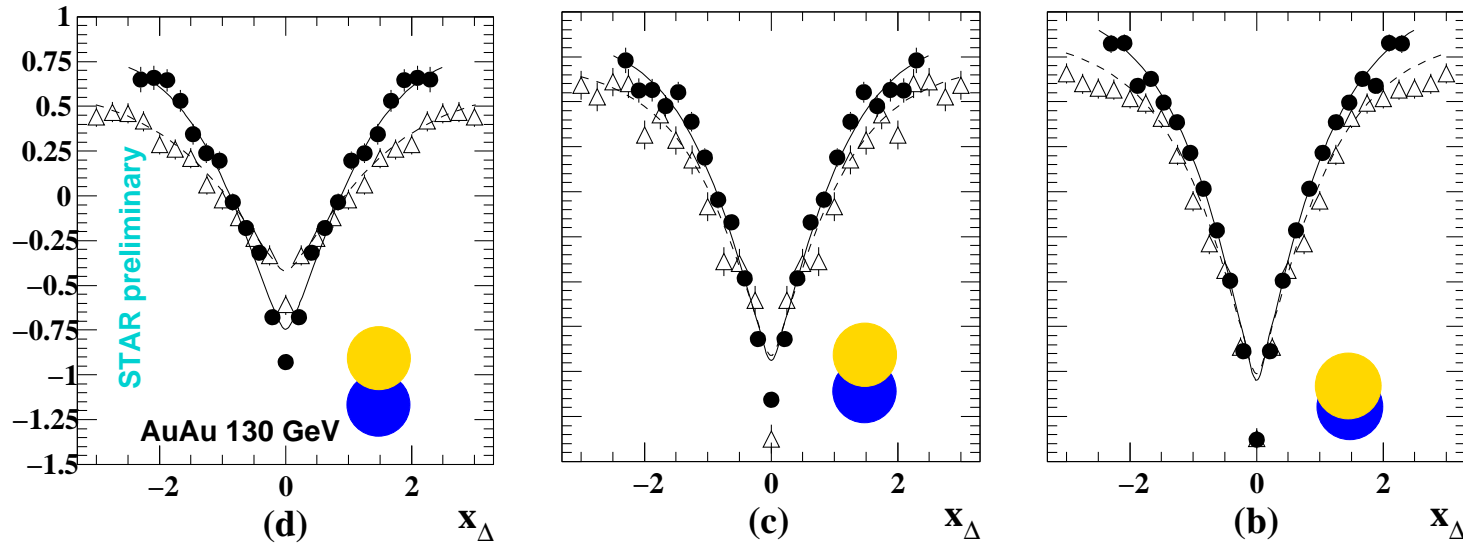
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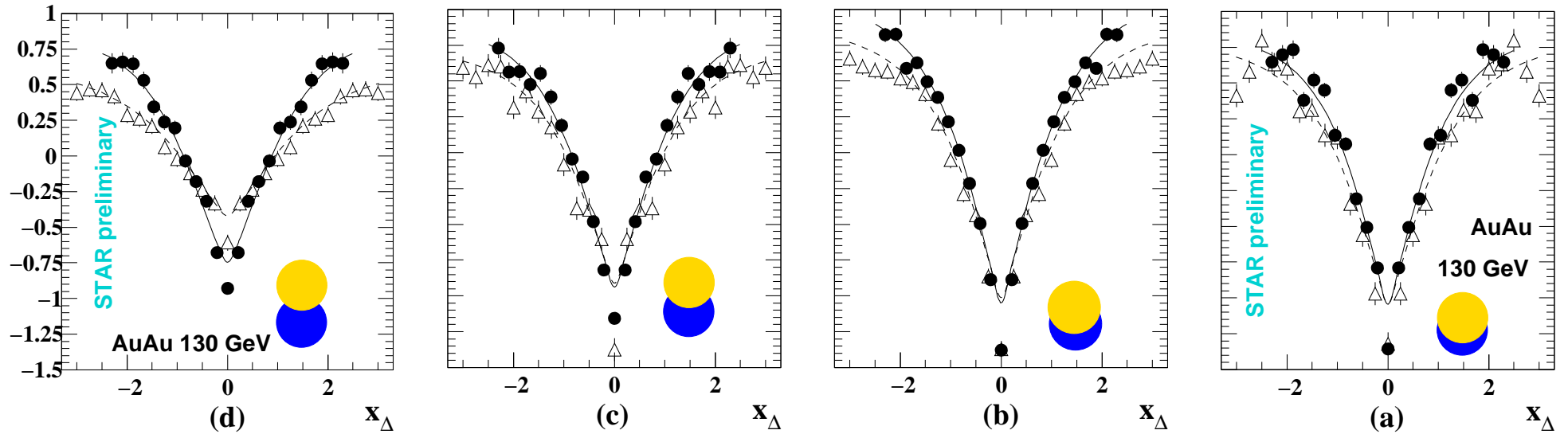
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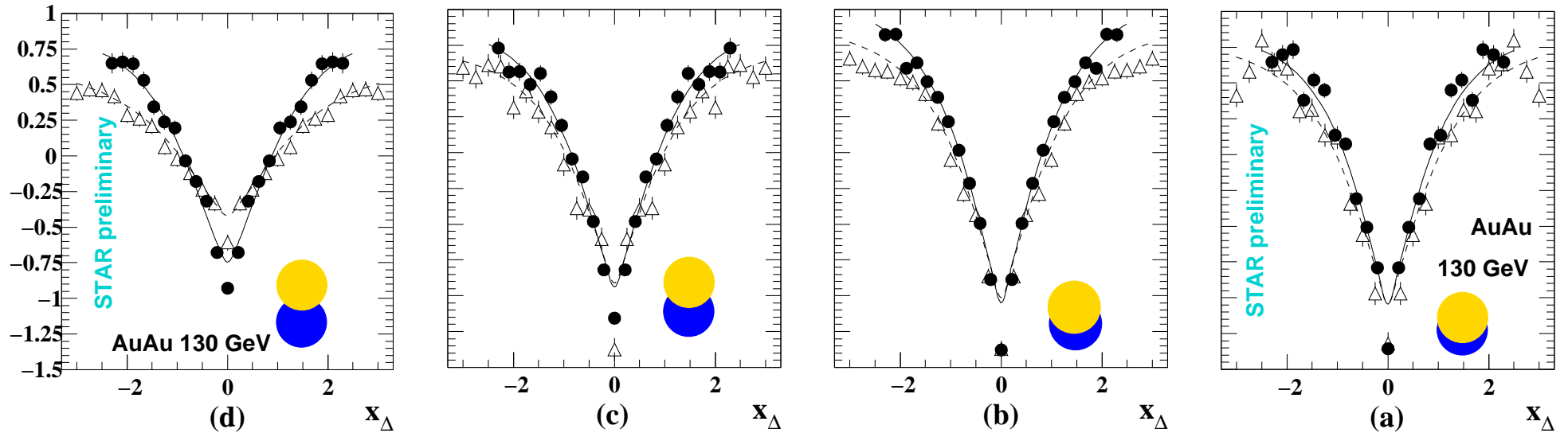


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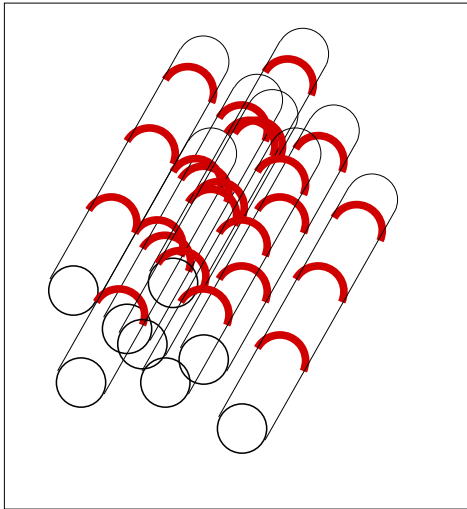


Projections of  $\bar{N}(\rho(\eta_\Delta, \phi_\Delta)/\rho(\eta_\Delta, \phi_\Delta)_{\text{ref}} - 1)|_{CD}$  on  $x_\Delta$  which is  $\phi_\Delta$  ( $\Delta$ ) or  $\eta_\Delta$  ( $\bullet$ ).  $\eta - \phi$  width disparity (d, peripheral) is gone in (a)  $\Rightarrow$  transition from (string) 1D to bulk ( $>2D$ ) fragmentation symmetrizes  $\eta$  and  $\phi$ .

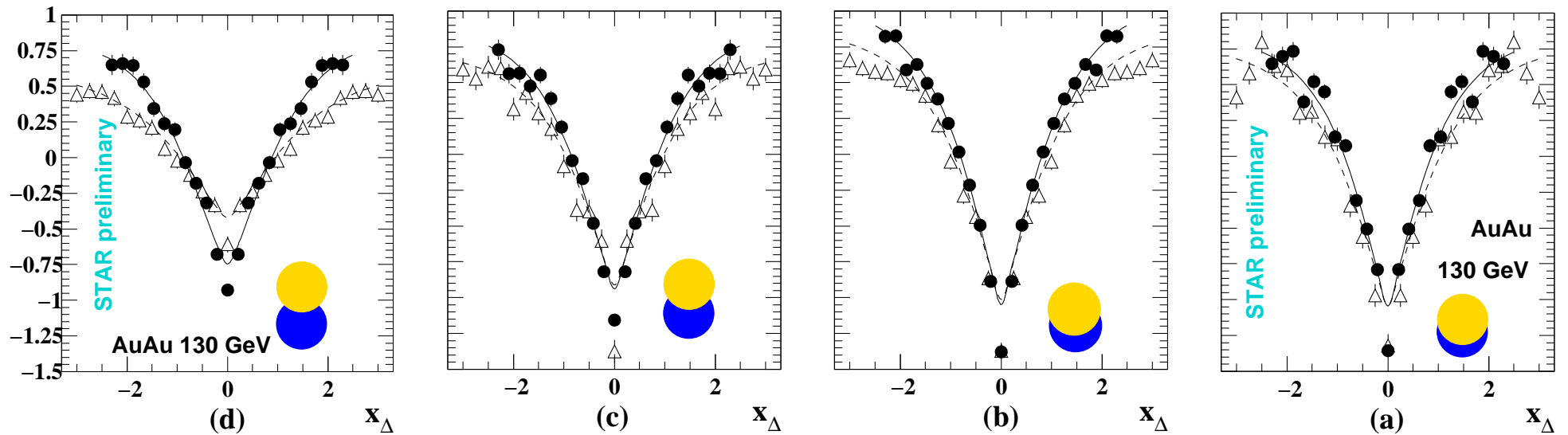
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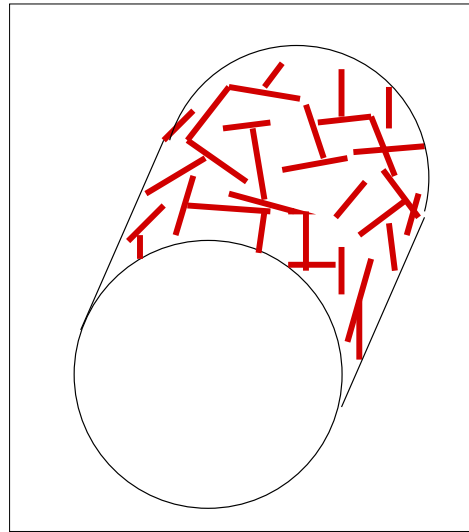
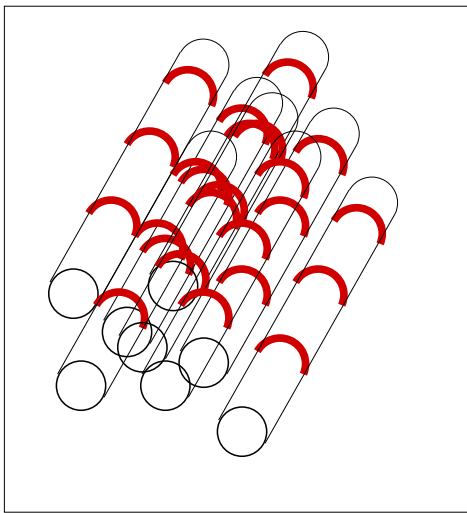
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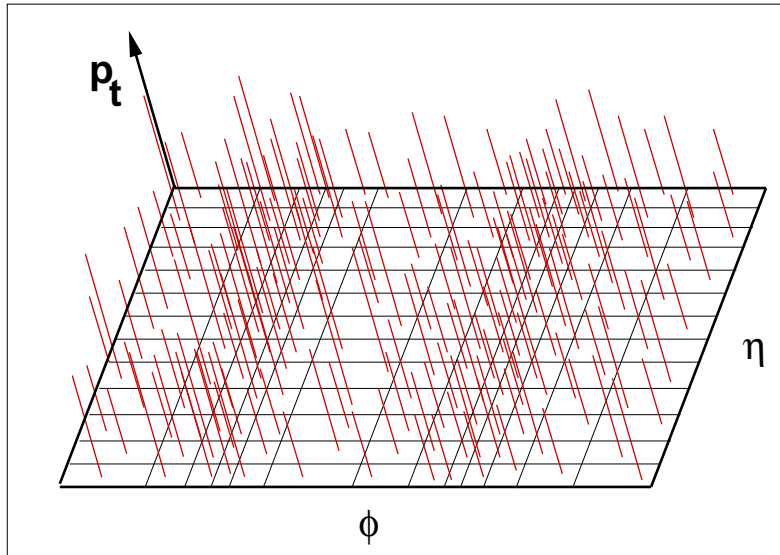


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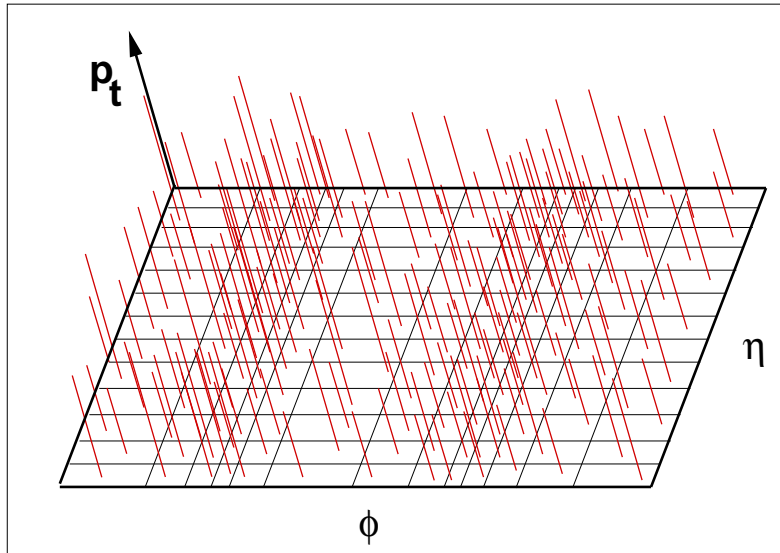
# 15 Number and $p_t$ correlations

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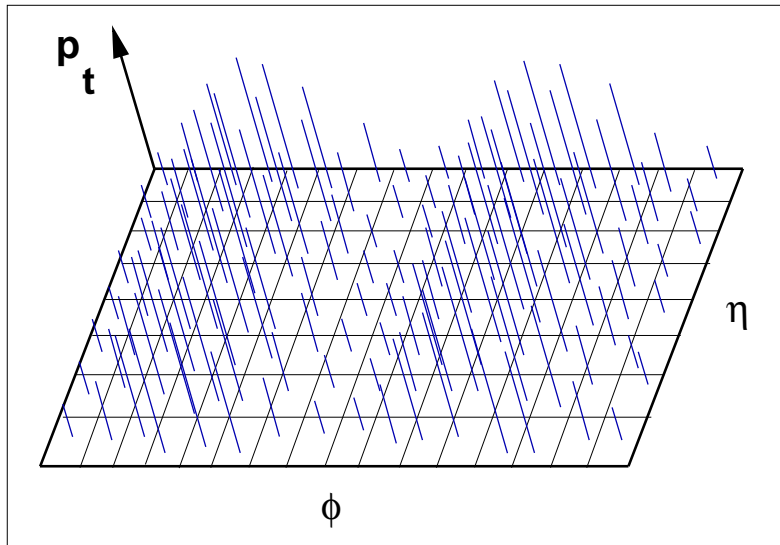


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Also elliptic... flow ( $p_t$  effect) !  
Pro: blast wave fits. Is there a **direct** measurement ?

## 16 Towards $p_t$ correlation/fluctuation analysis

Problem: need to tell apart  $p_{t,i}$  and number contributions to the

$p_t \equiv \sum_{i \in (\eta, \phi) \text{bin}} p_{t,i} \Rightarrow$  can extract the  $p_t$  correlation alone.

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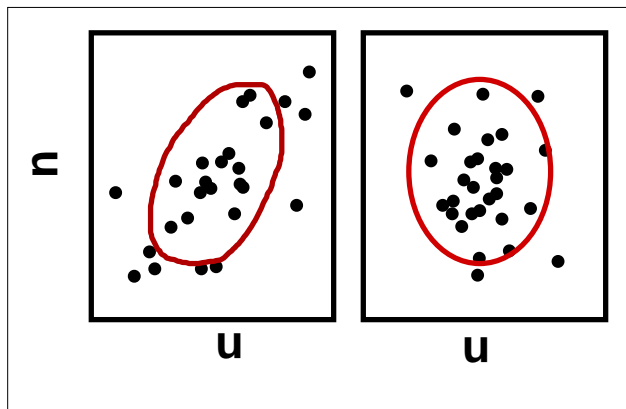
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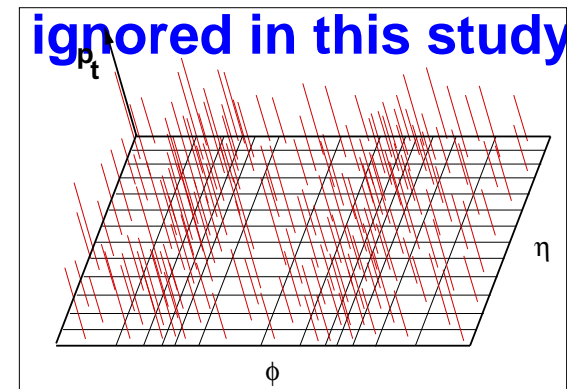
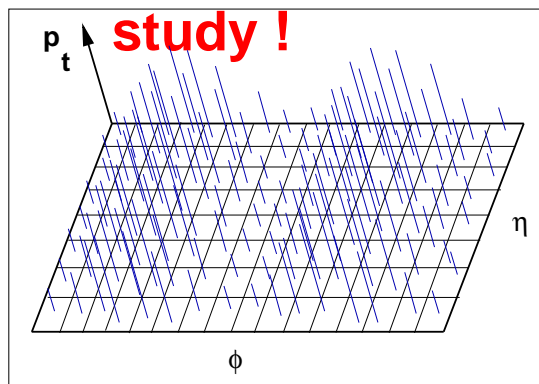
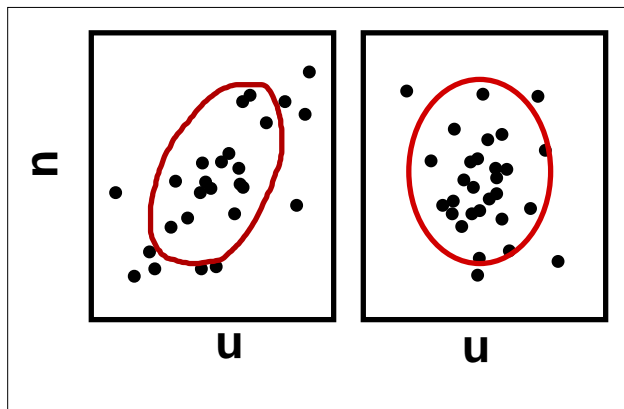
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# 17 Get correlations from fluctuations

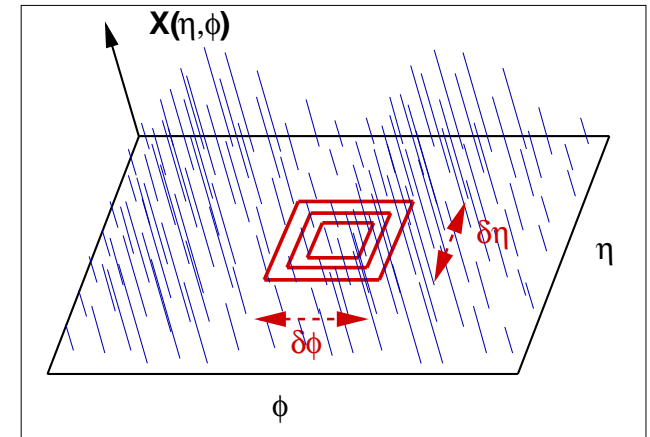


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Extract correlation structure of random field  $X$   
from the scale dependence of variance (van  
Marcke “Random Fields” MIT 1983;  
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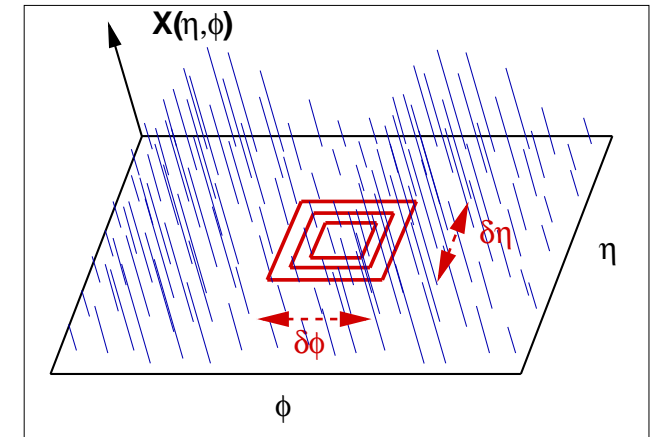
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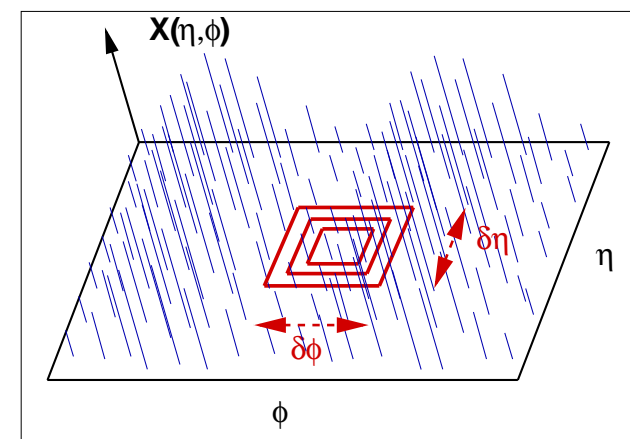
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$$\times [\overline{X(\eta_1, \phi_1)X(\eta_2, \phi_2)} - \overline{X(\eta_1, \phi_1)} \times \overline{X(\eta_2, \phi_2)}]$$

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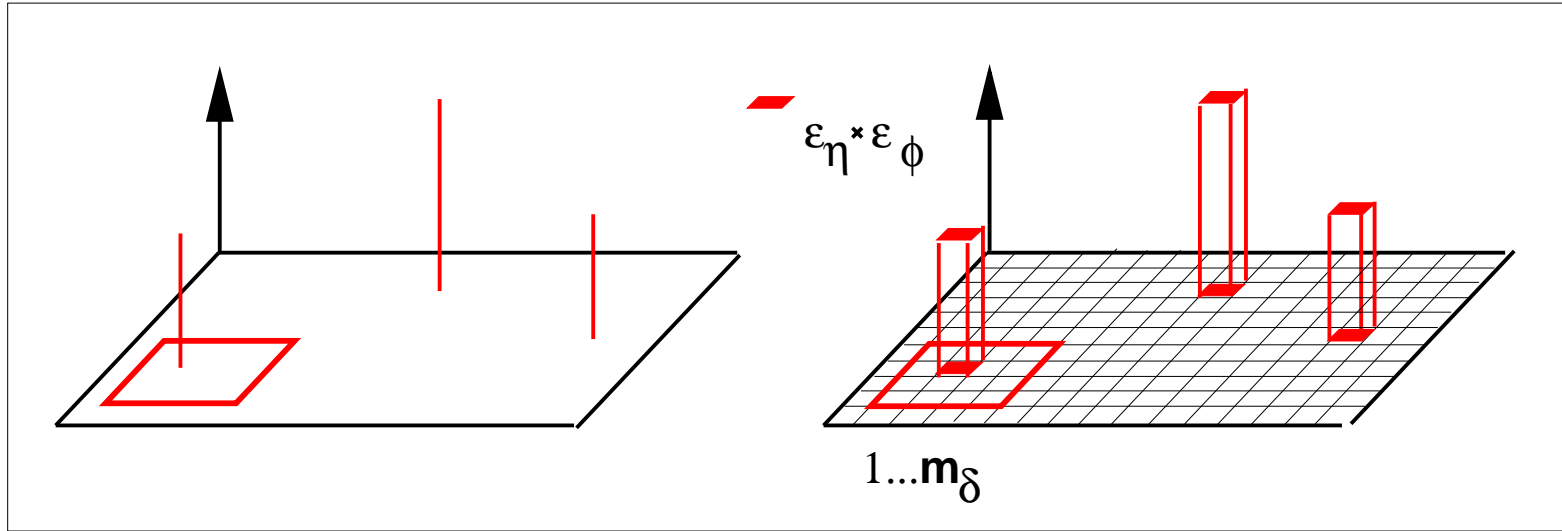
Compare with uncorrelated reference; recognize autocorrelation  $\rho(X, t_\Delta) \equiv \overline{X(t)X(t + t_\Delta)}$  ( $t$ -average).

$$\Delta\sigma^2(X, \delta\eta, \delta\phi) = \quad (9)$$

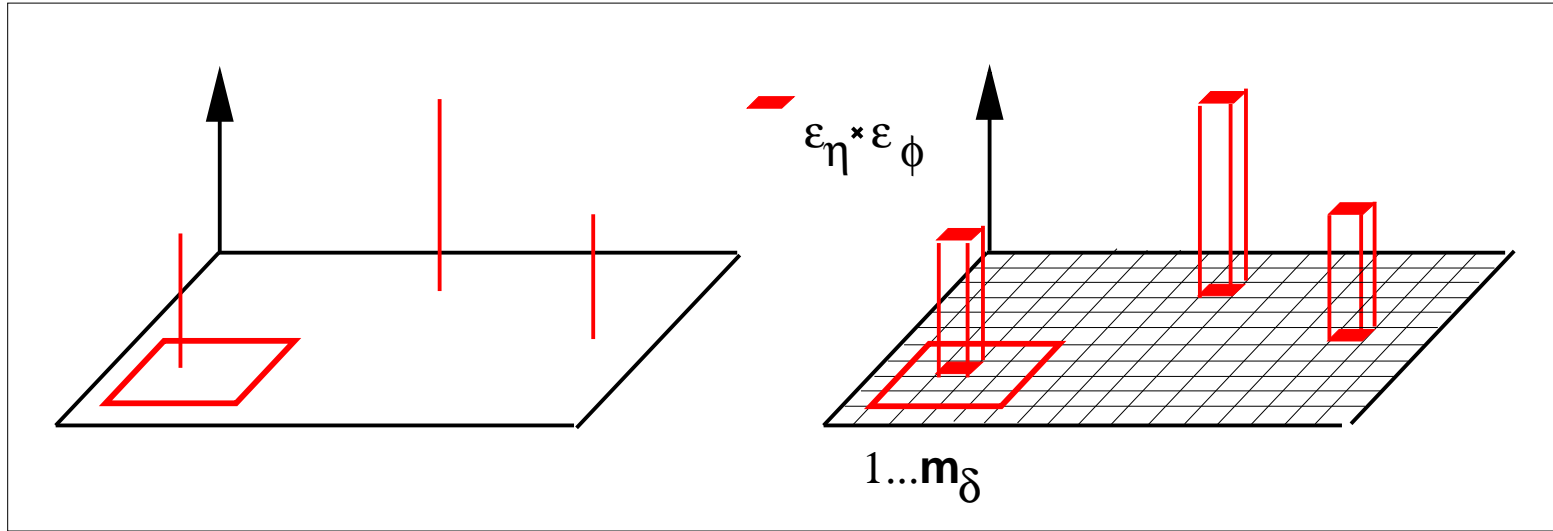
$$\int_{-\delta\eta/2}^{\delta\eta/2} d\eta_1 \int_{-\delta\phi/2}^{\delta\phi/2} d\phi_1 \int_{-\delta\eta/2}^{\delta\eta/2} d\eta_2 \int_{-\delta\phi/2}^{\delta\phi/2} d\phi_2 \Delta\rho(X, \eta_1 - \eta_2, \phi_1 - \phi_2) \quad (10)$$

$$= 2 \int_0^{\delta\eta} d\eta_\Delta 2 \int_0^{\delta\phi} d\phi_\Delta (\delta\eta - \eta_\Delta)(\delta\phi - \phi_\Delta) \Delta\rho(X, \eta_\Delta, \phi_\Delta) \quad (11)$$

# 18 The actual analysis is discrete: $\int \rightarrow \Sigma$



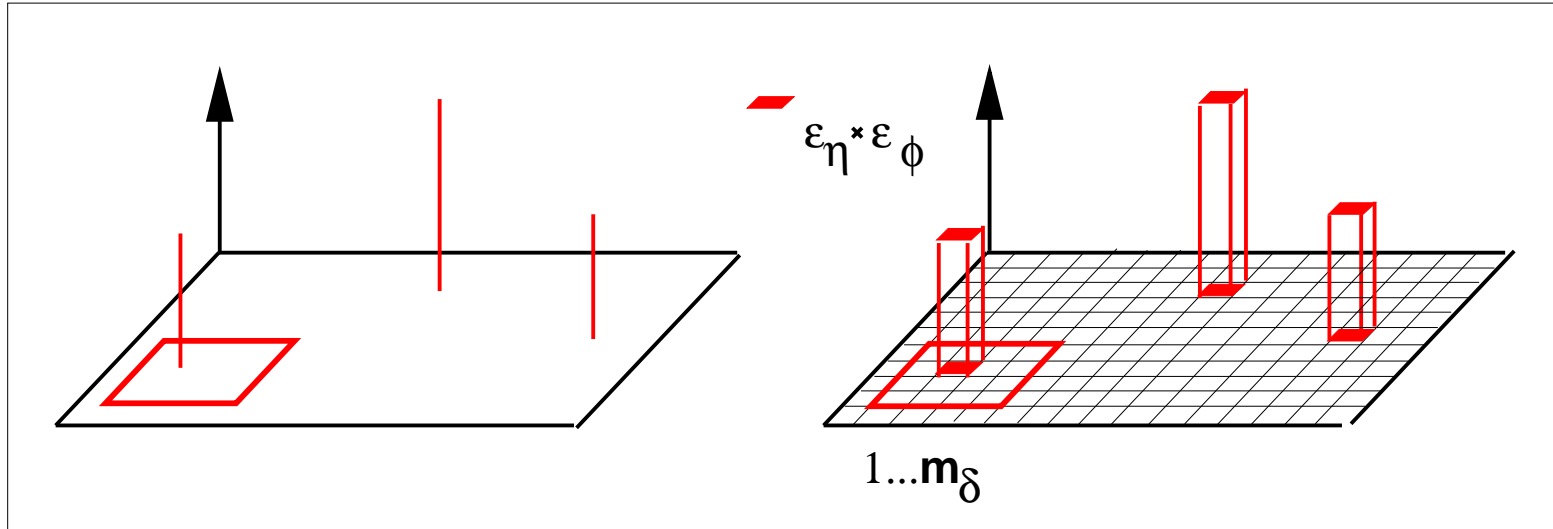
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kernel  $K$ :

$$(\delta\eta - \eta_{\Delta})(\delta\phi - \phi_{\Delta}) \rightarrow \varepsilon_{\eta}\varepsilon_{\phi}K_{m_{\delta}n_{\delta}:kl} \equiv \varepsilon_{\eta}\varepsilon_{\phi}(m_{\delta} - k + \frac{1}{2})(n_{\delta} - l + \frac{1}{2}) \quad (12)$$

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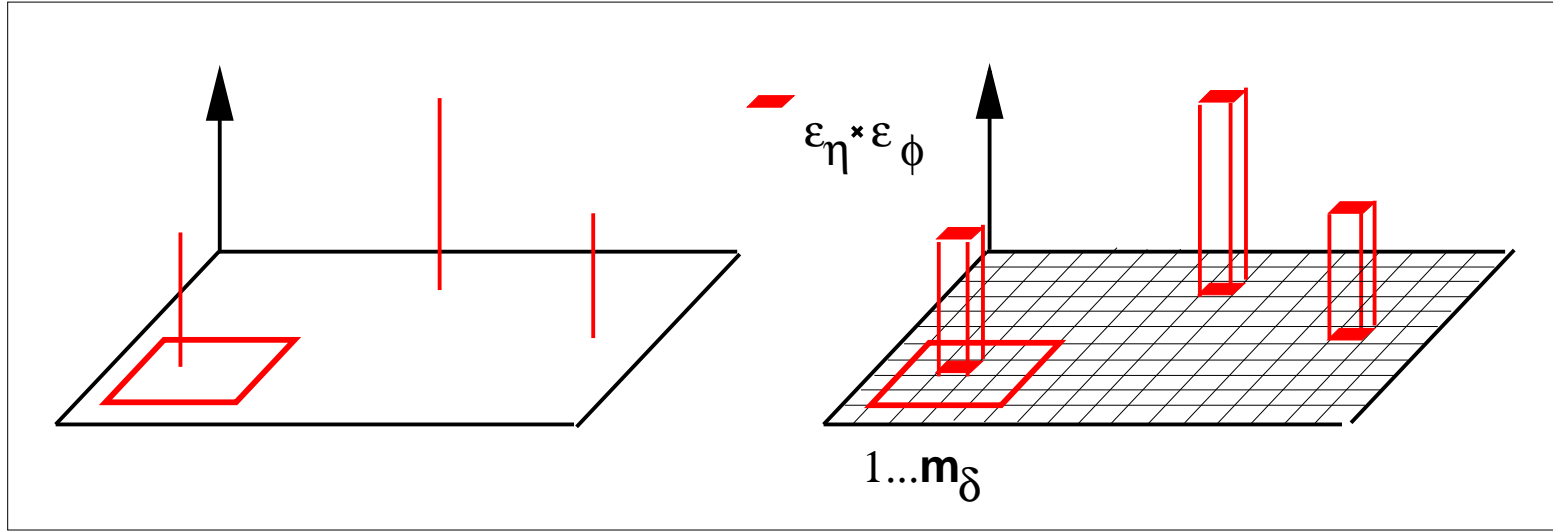
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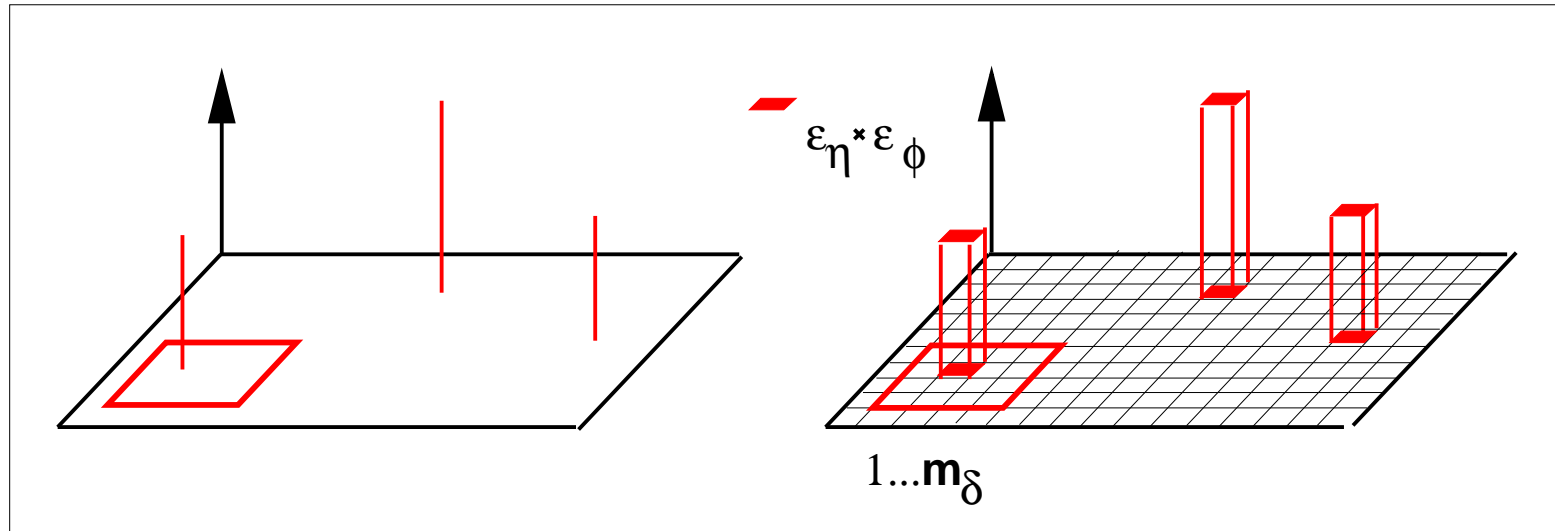
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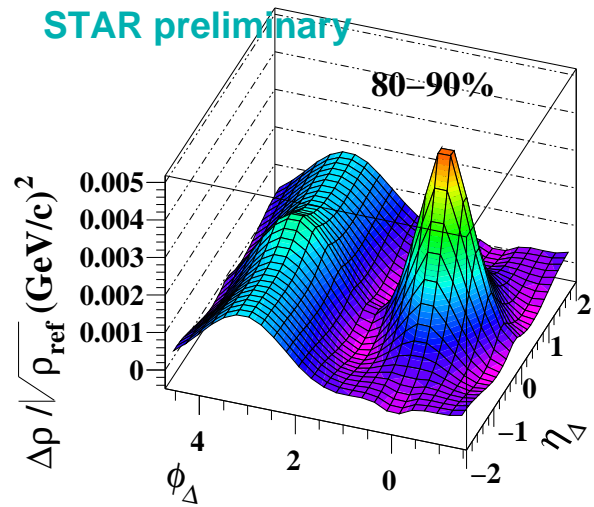
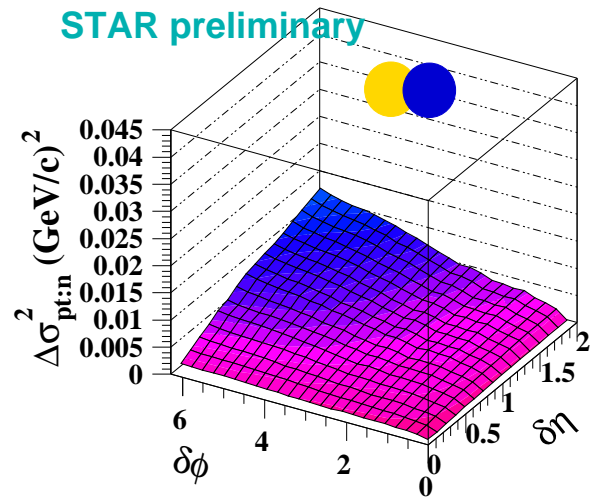
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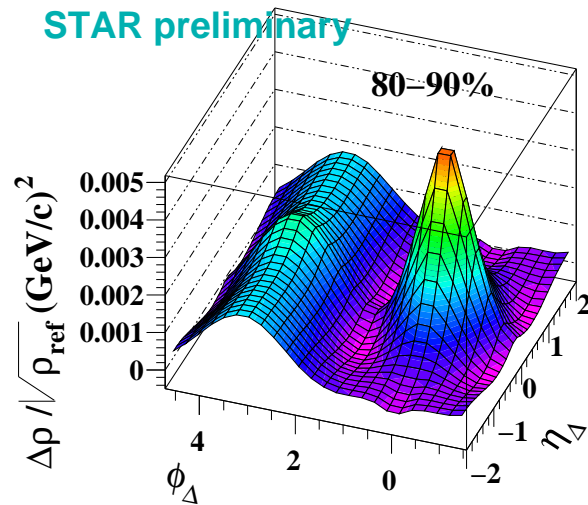
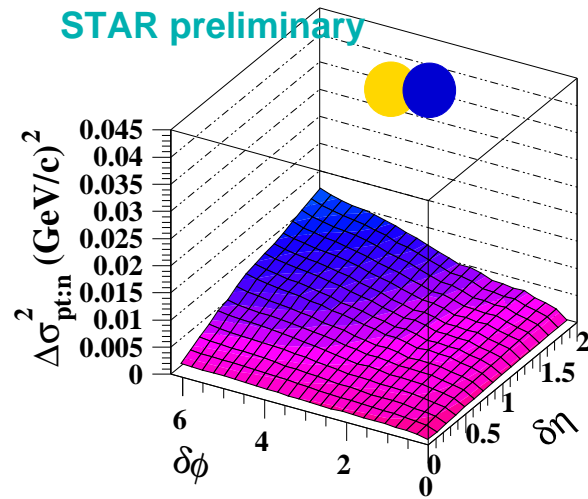
Inverse problem: knowing  $\Delta\sigma^2$ , solve for  $\Delta\rho/\sqrt{\rho_{\text{ref}}} \Rightarrow$  save  $O(N)$  in CPU time !

## 19 $p_t$ correlations from the inversion

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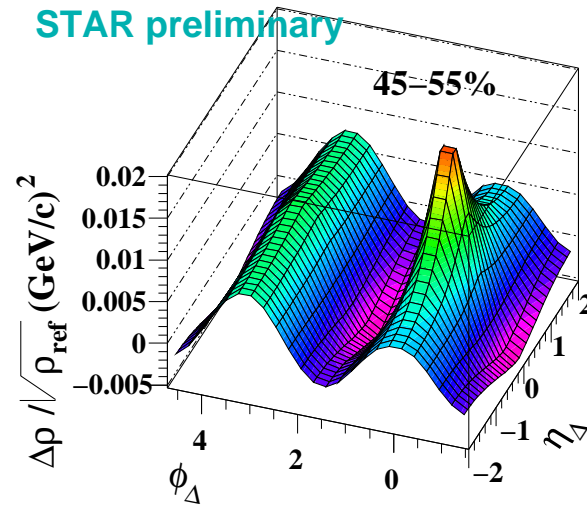
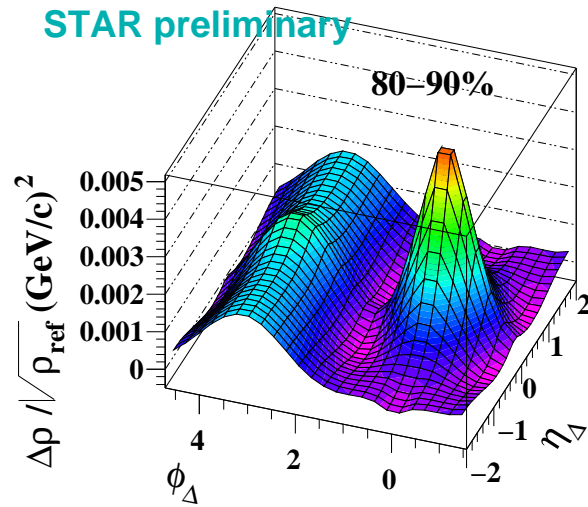
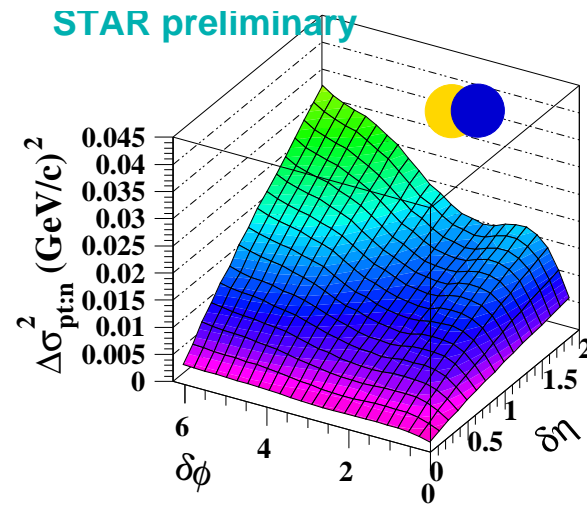
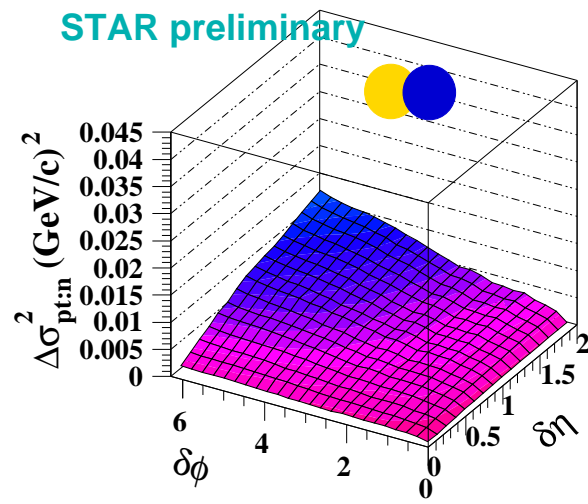


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**Top:**  
scale dependence of  
the “pure”  $p_t$  variance.  
**Bottom:**  
corresponding  
autocorrelation

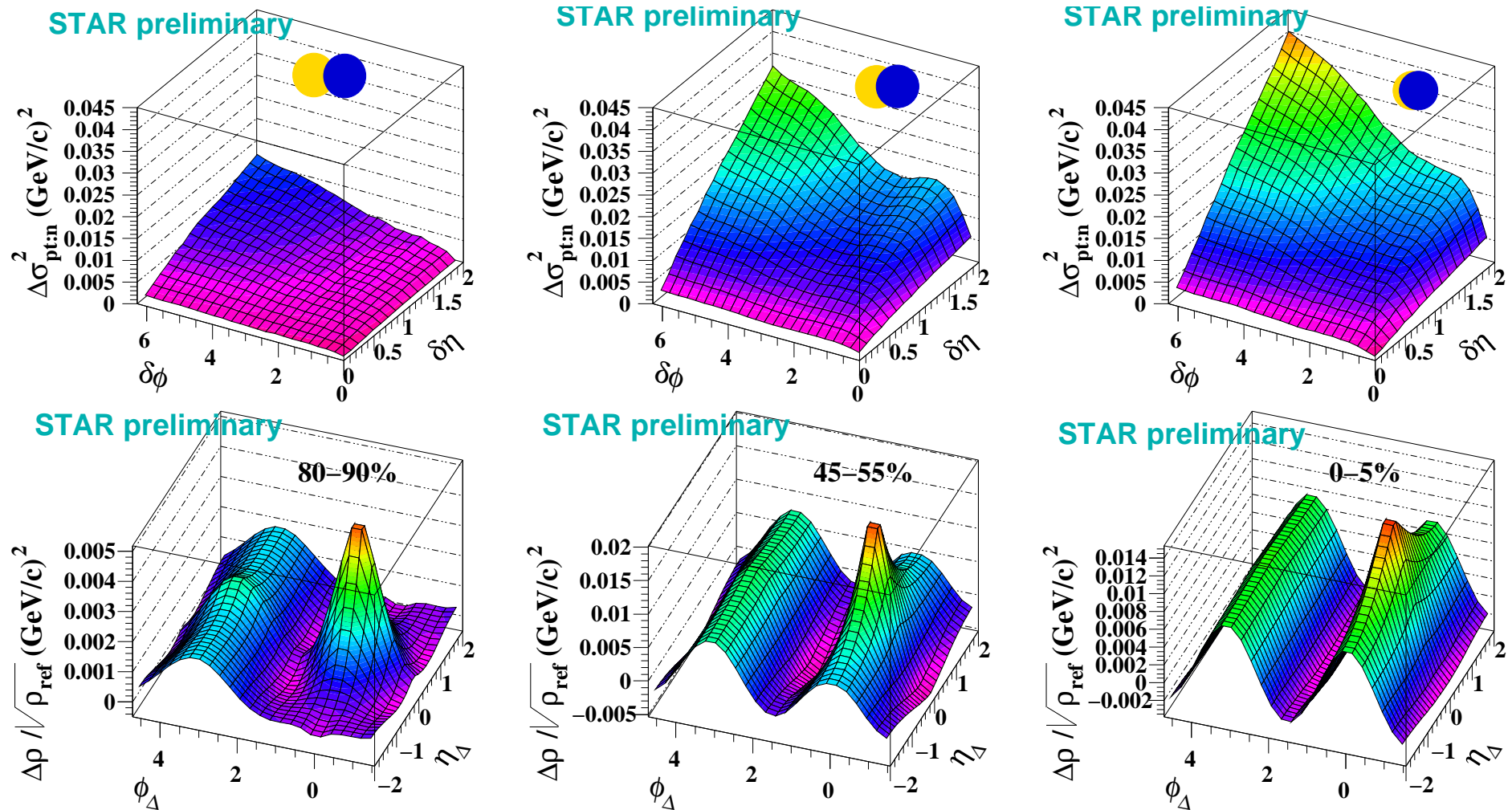
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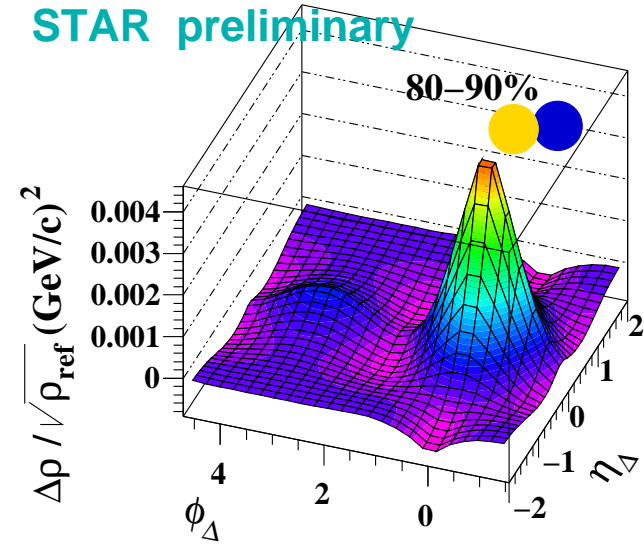
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First direct evidence of elliptic flow as a  $p_t$  blast.  
Next, subtract the flow contribution to look at  
minijets.

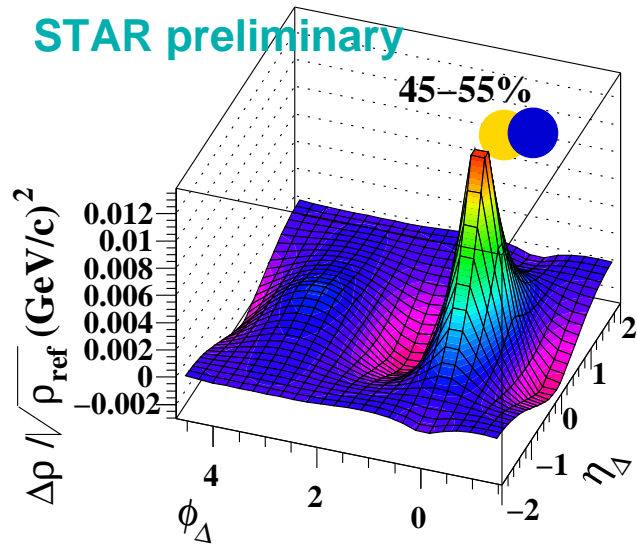
## 20 Localized $p_t$ correlations: minijets

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STAR preliminary



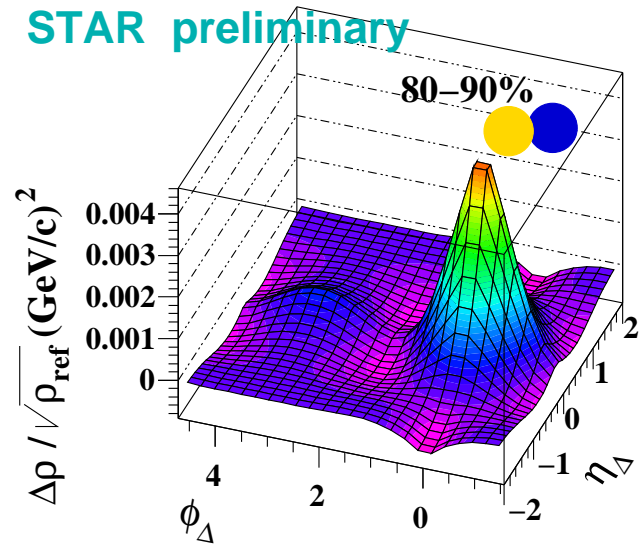
STAR preliminary



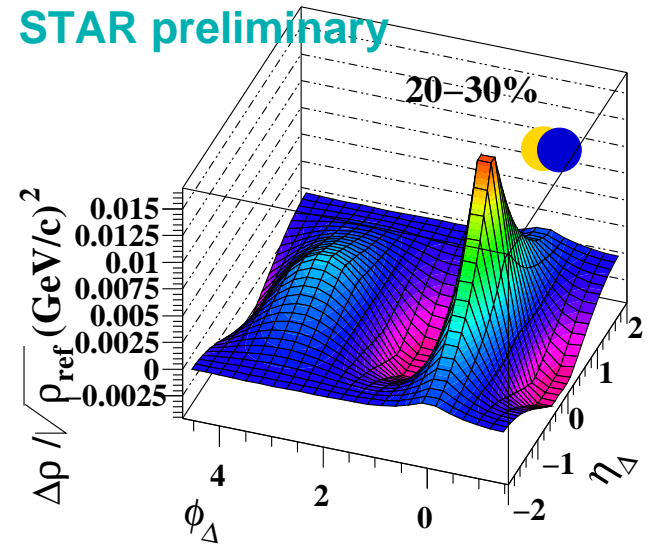


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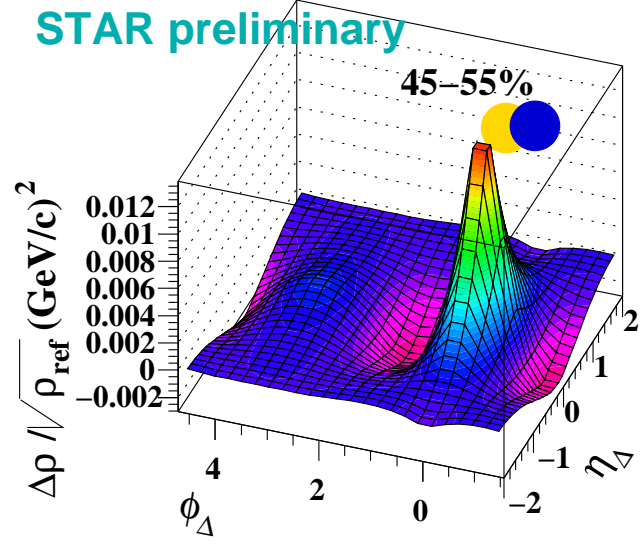
STAR preliminary



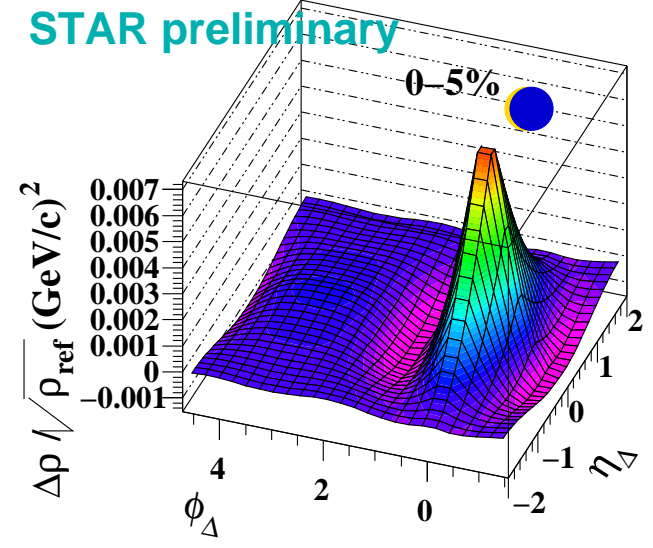
STAR preliminary



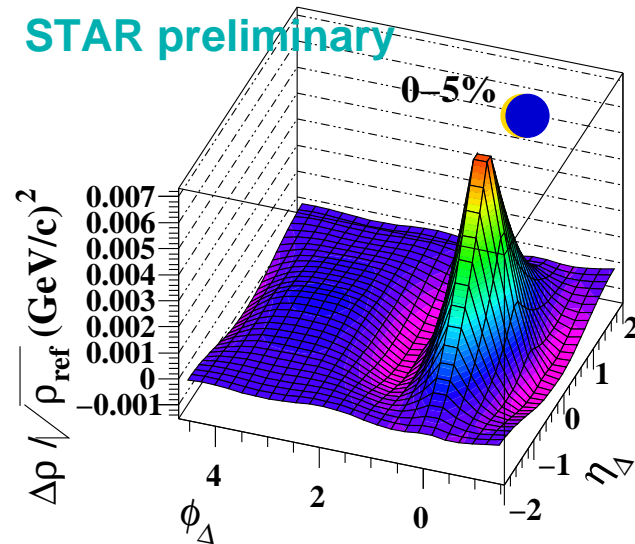
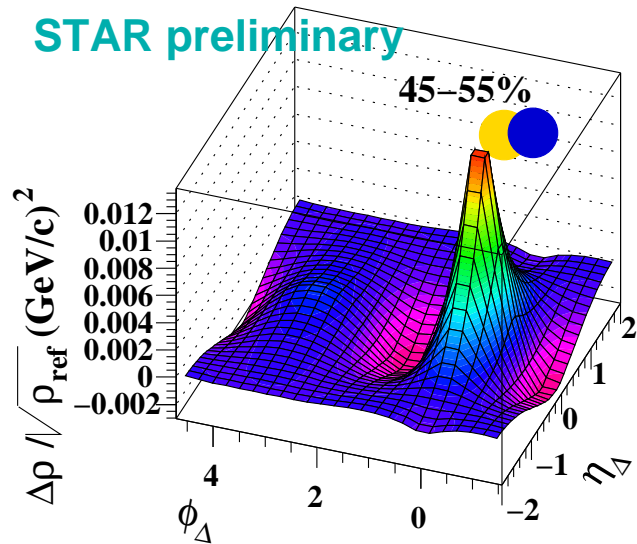
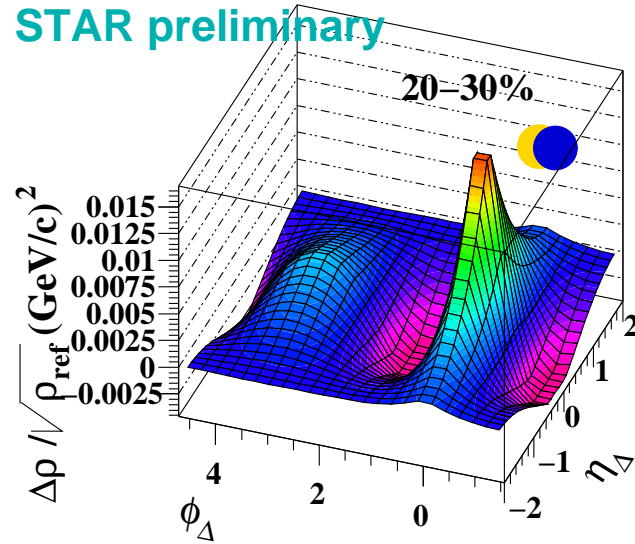
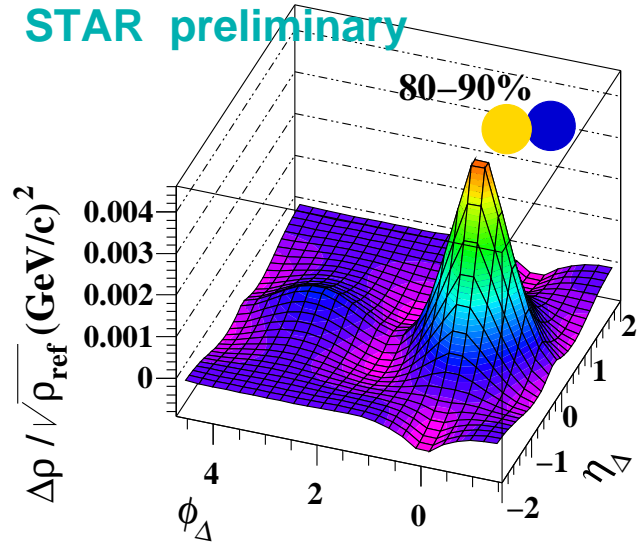
STAR preliminary



STAR preliminary



# 20 Localized $p_t$ correlations: minijets



AuAu 200 GeV. In  $\eta$ , correlation broadens with centrality; in  $\phi$  the trend is opposite. The surrounding background seems to recoil.

# 21 Conclusions

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- The minijet correlation structure is modified with centrality; the effect appears to “turn on” around  $\nu = (N_{\text{part}}/2)^{1/3} \approx 3$ . Broadening of the correlation in  $\eta$  and weakening of  $P_{\text{dyn}}^\eta$  on the coarse scale are consistent descriptions of the effect. How exactly does the coupling between longitudinal flow and mini-jets work ? What do we learn about the expanding fluid ?

## 21 Conclusions

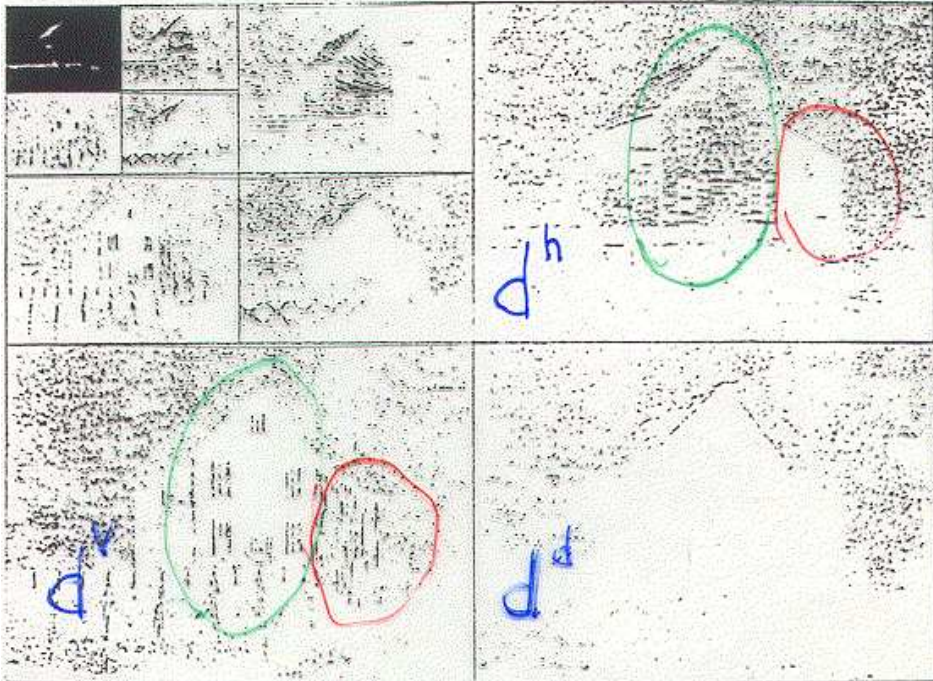
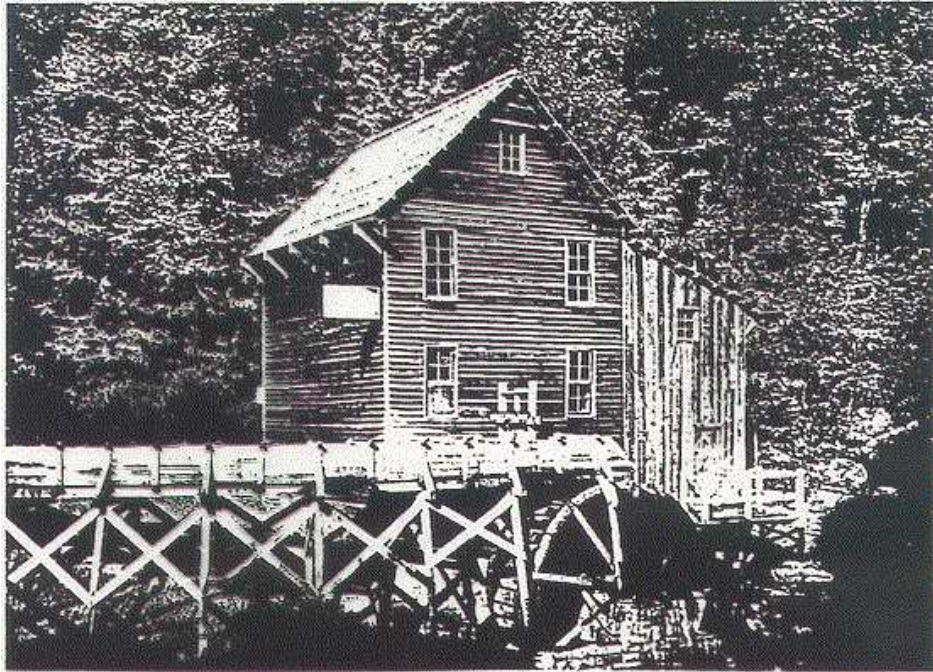
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- Increased symmetry of the charge-dependent correlation on  $(\eta, \phi)$  in the central collisions may point to a change in the hadronization geometry in the medium

## 22 Extra slides

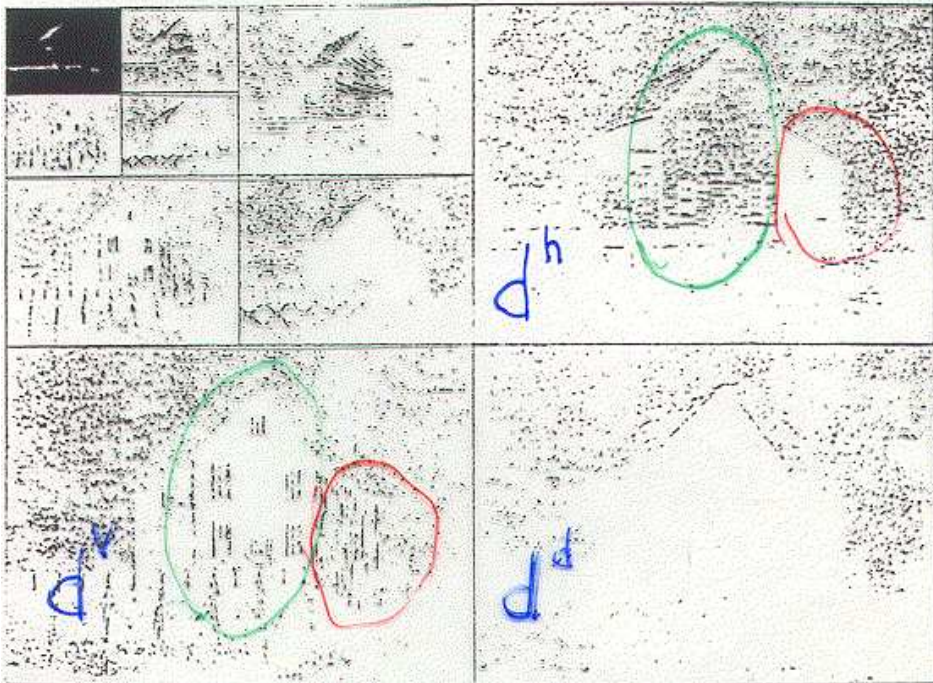
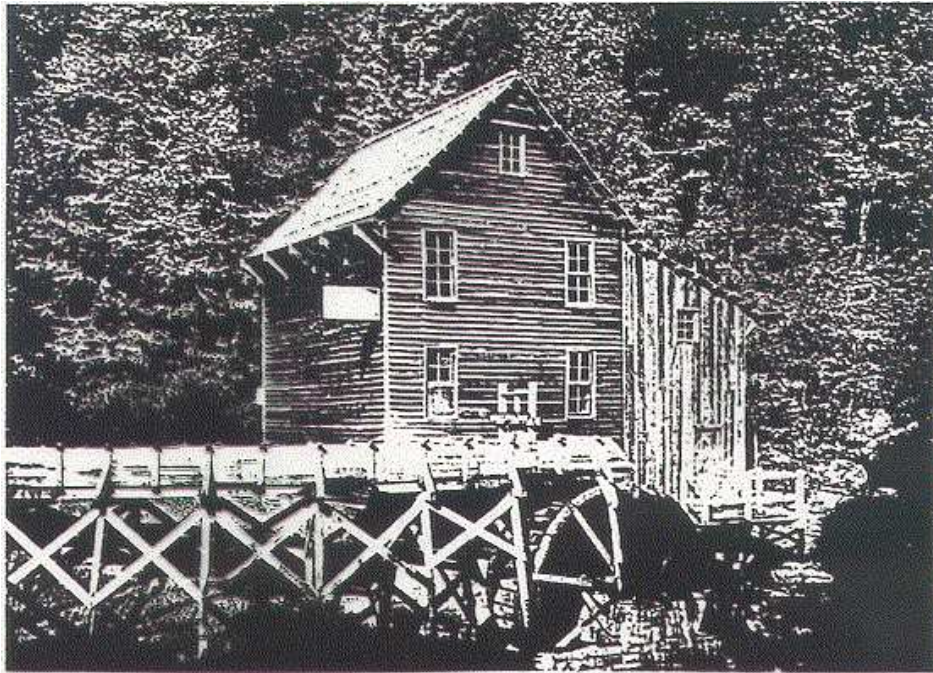


## 23 DWT of a photographic image

# 23 DWT of a photographic image



## 23 DWT of a photographic image



Reproduced from textbook:  
I. Daubechies, "Ten lectures on  
wavelets". The original caption: "A  
real image, and its wavelet  
decomposition into three  
multiresolution layers. On the  
wavelet components one clearly sees  
that the  $d^{j,v}$ ,  $d^{j,h}$ ,  $d^{j,d}$  emphasize,  
respectively, vertical, horizontal, and  
diagonal edges. In this figure, the  
bottom picture has been  
overexposed to make details in the  
 $d^{j,\lambda}$  more apparent. I would like to  
thank M. Barlaud for providing this  
figure." The colored marks are mine.