Basic symmetries in nuclear structure

- *Rotational and space inversion invariance:*
  - *result from the isotropy of space,*
  - *as a consequence spin and parity are good quantum numbers for nuclear states.*

- *Time reversal invariance:*
  - *result from the reversibility of motion in time.*

- *Notation:*

$R(\omega)$ –rotation,

$\boldsymbol{P}$ –space inversion,

$\boldsymbol{T}$ –time reversal.

Nuclear deformation

- *Occurs in the intrinsic, body fixed reference frame (non-inertial reference frame for rotating nuclei).*

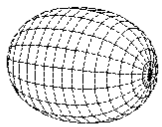
- *Violate basic symmetries in the intrinsic frame.*

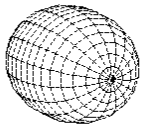
- *Broken symmetries are restored for the wave function in the laboratory reference frame by a transformation from the intrinsic reference frame.*

Spontaneous symmetry breaking or nuclear Jahn-Teller effect.

- *The violation of any symmetry in the intrinsic reference frame impacts the structure of the wave function in the laboratory reference frame and therefore is manifest as a distinct feature in excitation modes.*

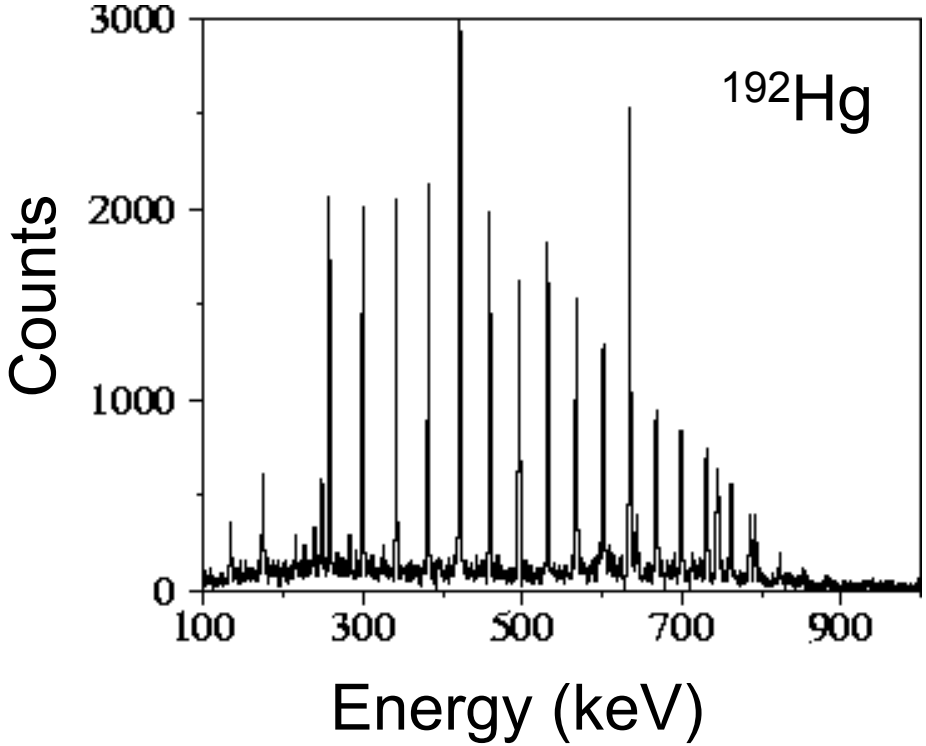
Rotational invariance:

$|\Psi\rangle =$ 

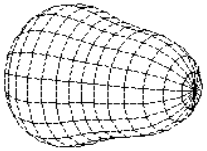
$R(\omega)|\Psi\rangle =$ 

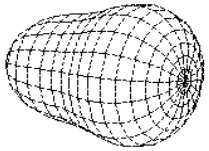
$R(\omega)|\Psi\rangle \neq |\Psi\rangle$

Gamma-Ray Spectrum

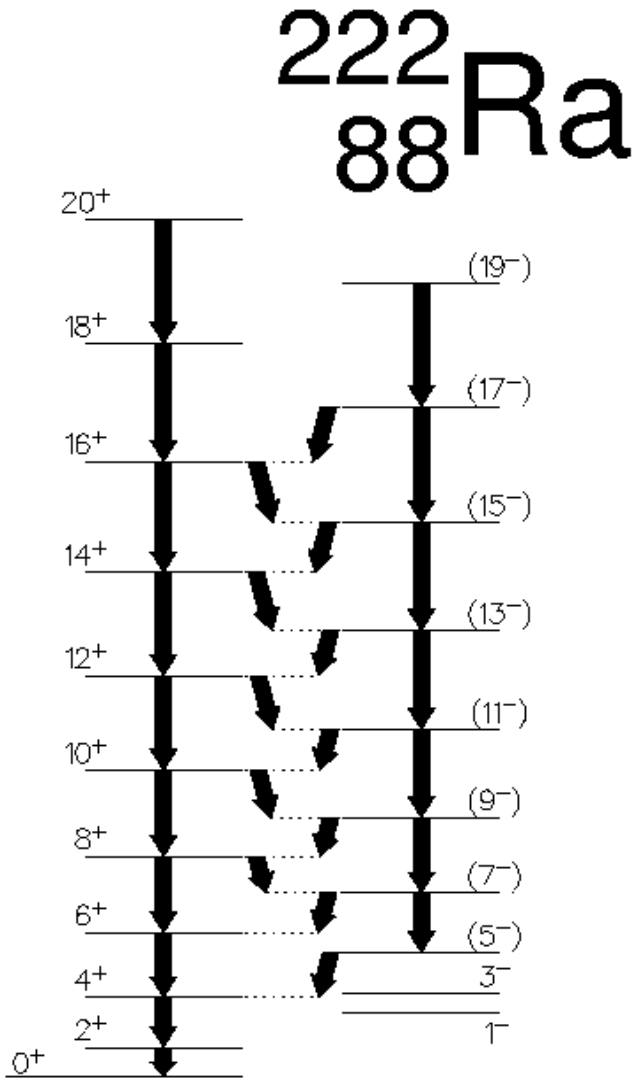


Space inversion invariance:

$$|\Psi\rangle = |$$  $$\rangle$$

$$P|\Psi\rangle = |$$  $$\rangle$$

$$P|\Psi\rangle \neq |\Psi\rangle$$

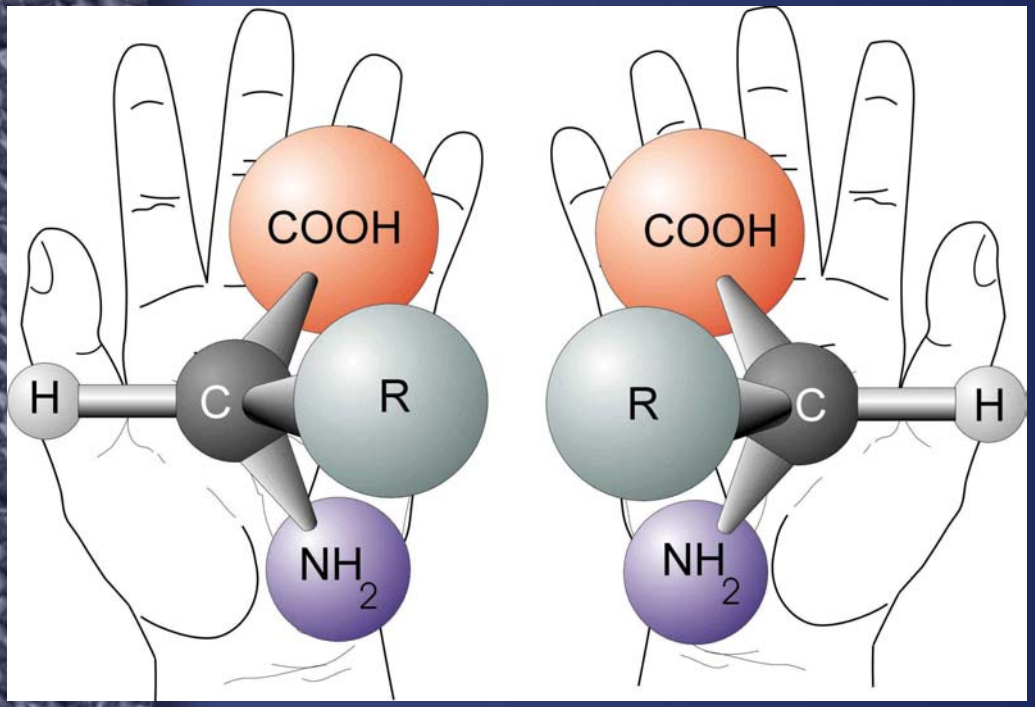


Chirality in Nuclear Structure

What is Chirality?

"I call any geometrical figure, or group of points, chiral, and say it has chirality, if its image in a plane mirror, ideally realized, cannot be brought to coincide with itself." - Lord Kelvin 1904

Examples of chiral systems are found throughout nature and in several disciplines of science



- Chemistry: molecules with opposite handedness react differently in similar environments

TENNESSEE

What is Chirality?

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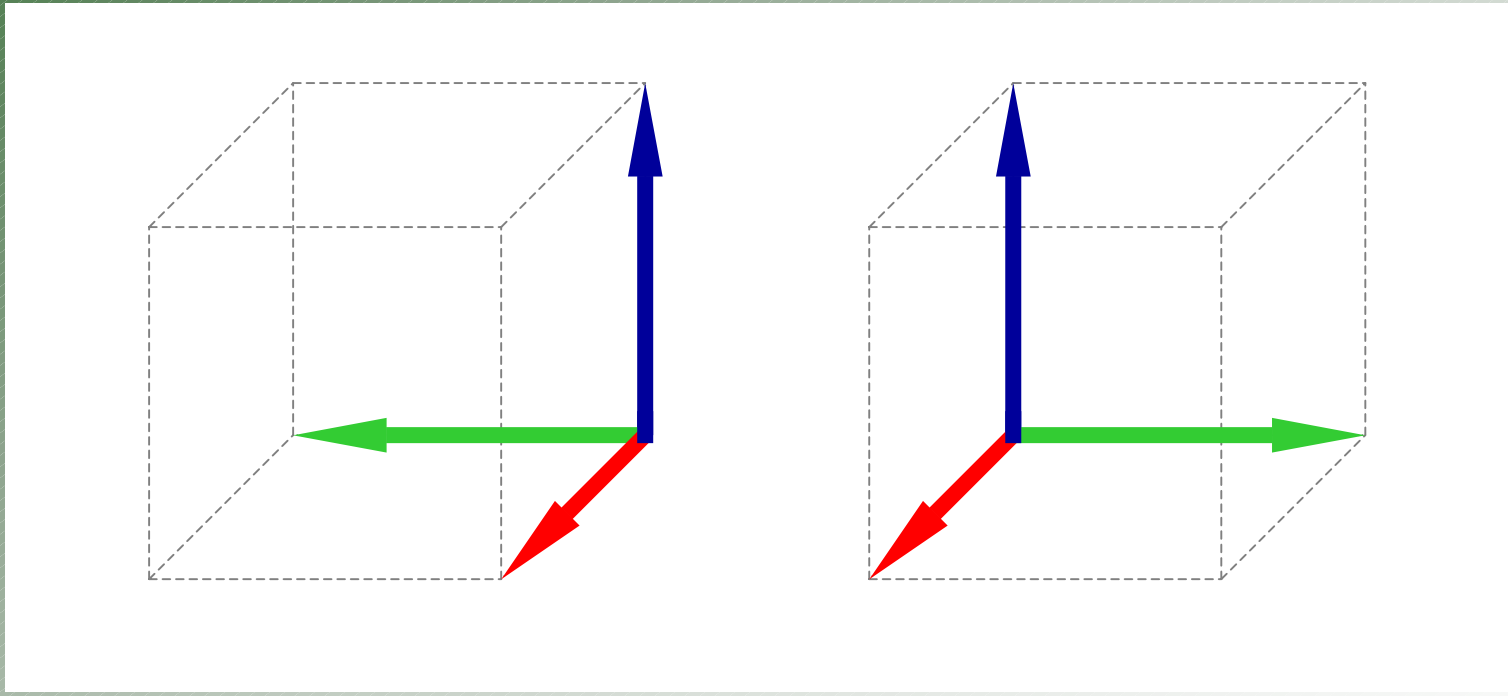
Examples of chiral systems are found throughout nature and in several disciplines of science

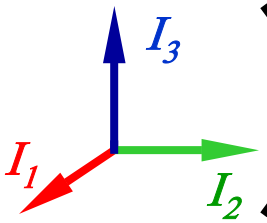
- Chemistry: molecules with opposite handedness react differently in similar environments
- Biology: DNA has right and left-handed "screws"
- Particle Physics

TENNESSEE

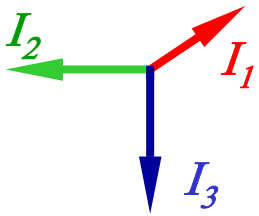
Three mutually orthogonal vectors can form two systems of opposite chirality.



For axial vectors of angular momenta systems of opposite chirality are related by time reversal.

$$|\Psi\rangle = |$$ 

$$\rangle$$

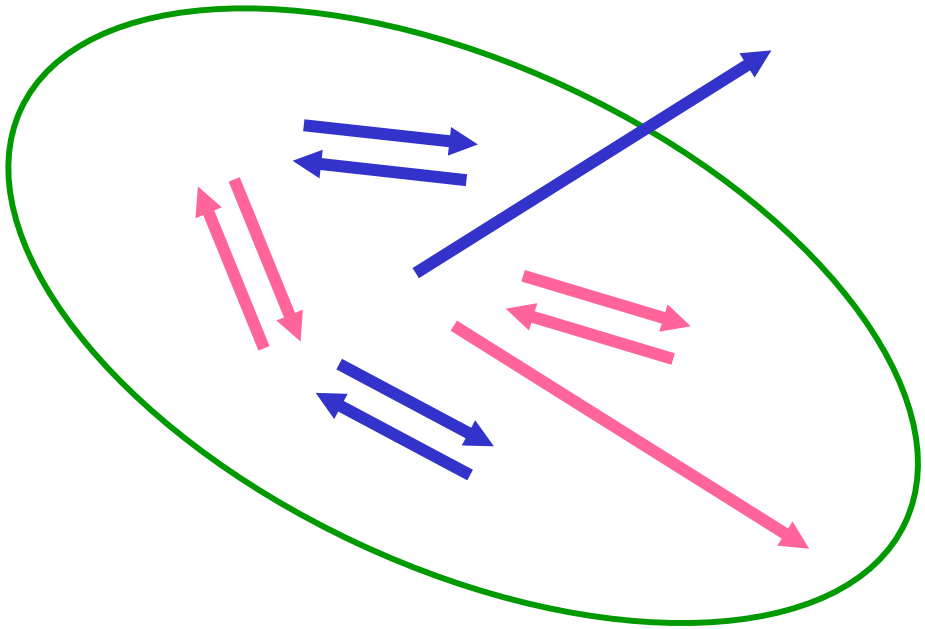
$$T|\Psi\rangle = |$$ 

$$\rangle$$

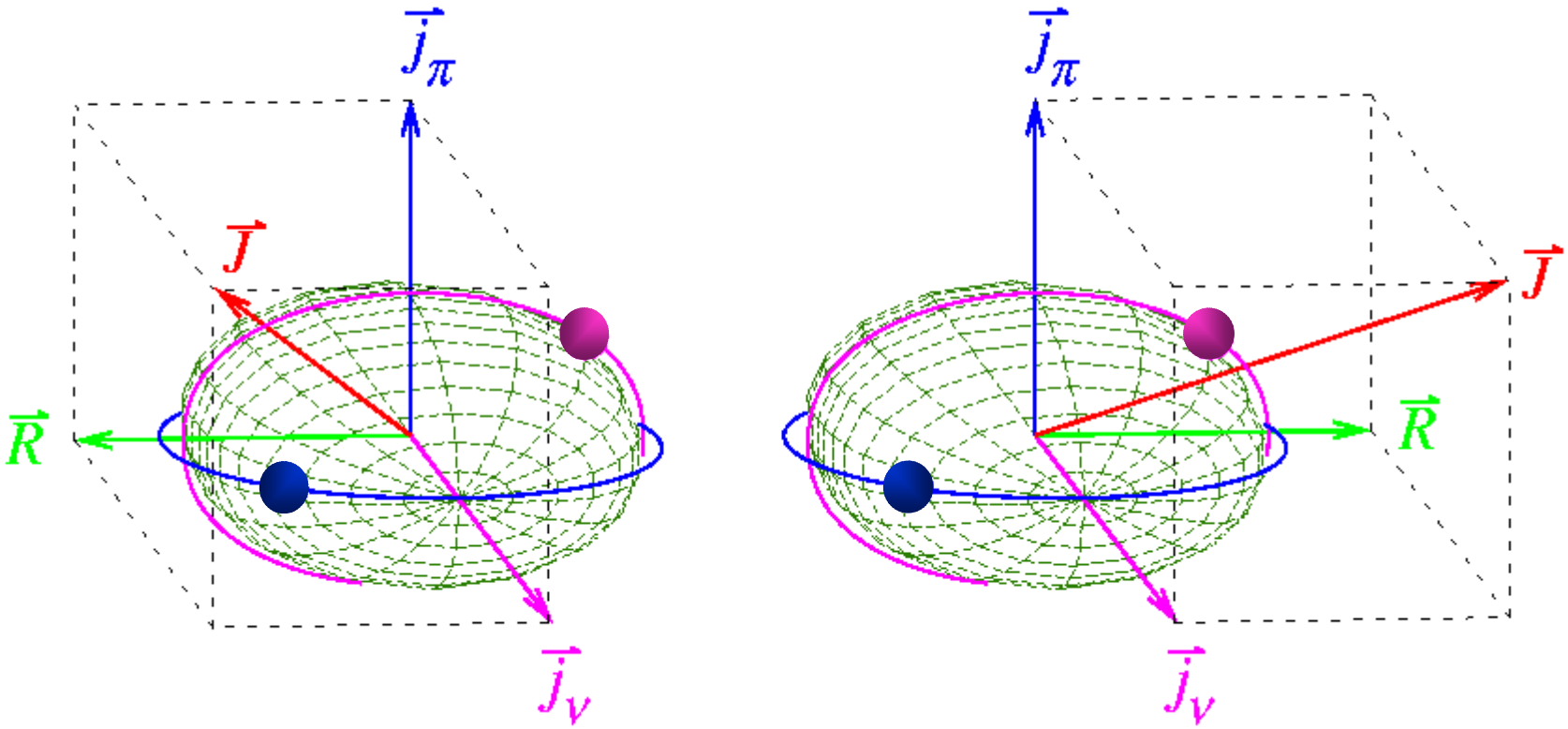
$$T|\Psi\rangle \neq |\Psi\rangle$$

Single-particle states in nuclei behave as gyroscopes.

- *Pairing interactions couple single particle states to Cooper pairs with no net angular momentum.*
- *Valence odd nucleons are unpaired.*



Triaxial odd-odd nuclei result in three perpendicular angular momenta for particle-hole configurations built on high-j orbitals .



Space inversion vs time reversal:

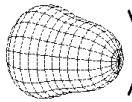
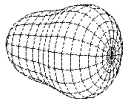
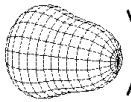
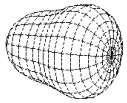
- Space inversion operation represented by $\boldsymbol{P}$ is linear:

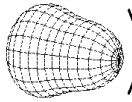
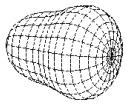
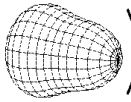
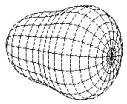
$$\boldsymbol{P}\,(\boldsymbol{a}|\psi_1\rangle+\boldsymbol{b}|\psi_2\rangle)=\boldsymbol{a}\,\boldsymbol{P}\,|\psi_1\rangle+\boldsymbol{b}\,\boldsymbol{P}\,|\psi_2\rangle.$$

- For the unitary operator $\boldsymbol{P}$:

$$\boldsymbol{P}^2=1,\quad \boldsymbol{P}^{-1}=\boldsymbol{P}=\boldsymbol{P}^{\dagger}.$$

- Eigenstates of $\boldsymbol{P}$ with eigenvalues $\pi=\pm 1$ can be formed:

$\boldsymbol{P}\,1/\sqrt{2}\,(|$  $\rangle+|$  $\rangle)=1/\sqrt{2}\,(|$  $\rangle+|$  $\rangle)$,

$\boldsymbol{P}\,1/\sqrt{2}\,(|$  $\rangle-|$  $\rangle)=-1/\sqrt{2}\,(|$  $\rangle-|$  $\rangle)$.

- If $[\,\boldsymbol{P}\,,\boldsymbol{H}\,]=0$, parity is a good quantum number for nuclear states.

Space inversion vs time reversal:

- *Time reversal operation represented by* $\boldsymbol{T}$ *is antilinear:*

$$\boldsymbol{T}(\boldsymbol{a}|\psi_1\rangle+\boldsymbol{b}|\psi_2\rangle)=\boldsymbol{a}^*\,\boldsymbol{T}\,|\psi_1\rangle+\boldsymbol{b}^*\,\boldsymbol{T}|\psi_2\rangle.$$

- *For the antiunitary operator* $\boldsymbol{T}$ *:*

$$\boldsymbol{T}^2=(-1)^{2I}=(-1)^A,\quad \boldsymbol{I}\text{-spin},\quad \boldsymbol{A}\text{-number of fermions.}$$

- *Despite* $[\,\boldsymbol{T}\,,\boldsymbol{H}\,]=0$, *<u>there is no</u> quantum number associated with the time reversal symmetry.*

- *Eigenstates of T <u>can not</u> be defined.*

- For the nuclear hamiltonian which is invariant under time reversal the wave functions for physical states are required to be invariant under the operator $O = T\, R_y(\pi)$.

- T denotes time reversal .

- $R_y(\pi)$ denotes rotation by 180° around the axis perpendicular to the quantization axis.

- With these definitions the O operator is a complex conjugation of expansion coefficients for wave functions in $|IM\rangle$ basis.

Since:

$$O\,|IM\rangle = |IM\rangle,$$

$$O\,\alpha|IM\rangle = \alpha^*|IM\rangle,$$

$$O^2\alpha|IM\rangle = \alpha|IM\rangle,$$

for an odd-odd nucleus there are two classes of states:

1) *planar states invariant under* O:

$$O\,|IP\rangle = |IP\rangle,$$

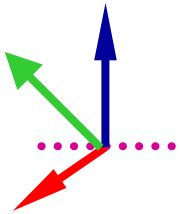
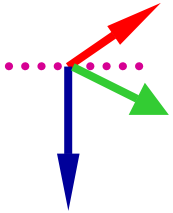
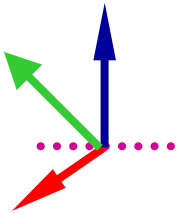
2) *right- and left- handed chiral states related by* O:

$$O\,|IR\rangle = |IL\rangle,$$

$$O\,|IL\rangle = |IR\rangle.$$

For the planar states of three angular momenta:

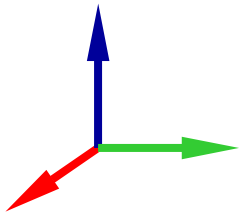
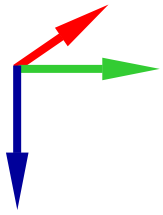
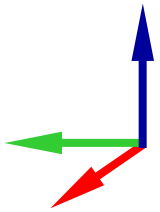
$$O\,|I\,P\rangle = T\,R_y(\pi)\,|I\,P\rangle = |I\,P\rangle\,.$$

$T\,R_y(\pi)\Big|$  $\Big\rangle = T\Big|$  $\Big\rangle = \Big|$  $\Big\rangle$

For the right- and left-handed states of three mutually orthogonal angular momenta:

$$O\,|I\,R\rangle = T\,R_y(\pi)\,|I\,R\rangle = |I\,L\rangle\,,$$

$$O\,|I\,L\rangle = T\,R_y(\pi)\,|I\,L\rangle = |I\,R\rangle\,.$$

$T\,R_y(\pi)\Big|$  $\Big\rangle = T\Big|$  $\Big\rangle = \Big|$  $\Big\rangle$

- *For $|I\,R\rangle$ and $|I\,L\rangle$ quantum mechanical analysis for a two level system directly applies.*

*(See for example **Feynman lectures on Physics**.)*

- *Physical states invariant under O are:*

$$|I\;+\rangle = \frac{1}{\sqrt{2}}(|IR\;\rangle + |IL\;\rangle),$$

$$|I\;-\rangle = \frac{i}{\sqrt{2}}(|IR\;\rangle - |IL\;\rangle),$$

- *For* $\langle IR | H | IL \rangle = 0,$

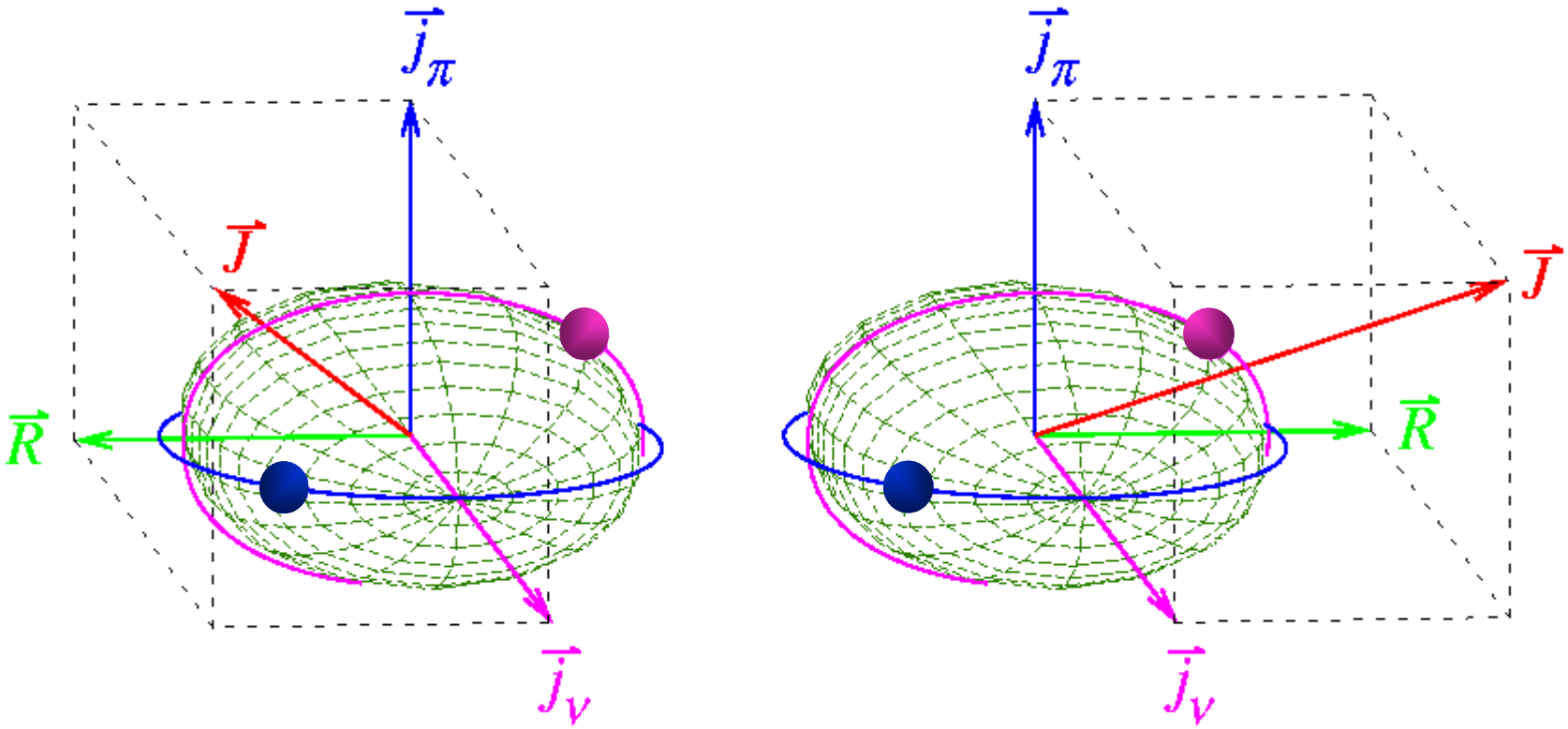
if amplitude of the planar component is zero, then

$$\langle I+ | H | I+ \rangle = \langle I- | H | I- \rangle,$$

and physical states $|I+\rangle$, $|I-\rangle$ *are degenerate.*

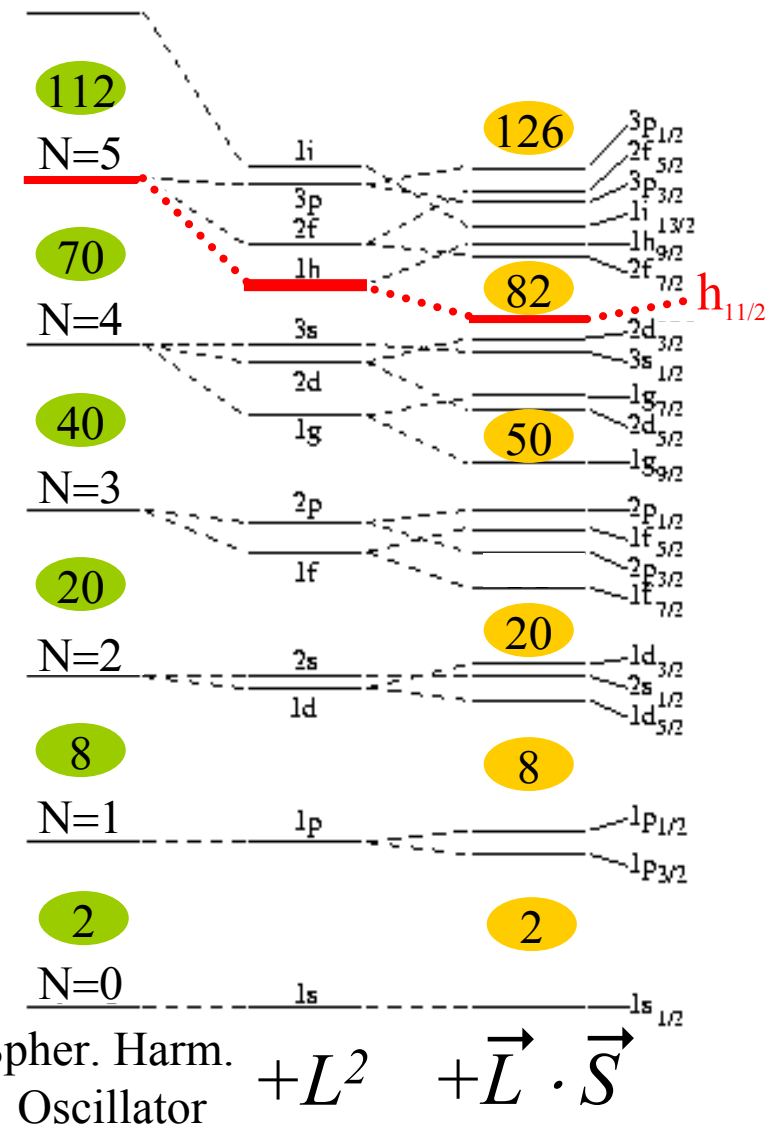
- *If the planar component contributes significantly the degeneracy is lifted, the separation energy for the physical states* $|I+\rangle$, $|I-\rangle$ *depends on the planar component amplitude.*

Triaxial odd-odd nuclei result in three perpendicular angular momenta for particle-hole configurations built on high-j orbitals .



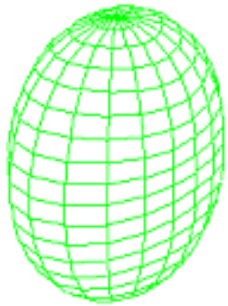
Nuclear single-particle shell model states.

$$H_{SM} = V(r) + V_{LS}(r)\,\vec{L}\cdot\vec{S}$$



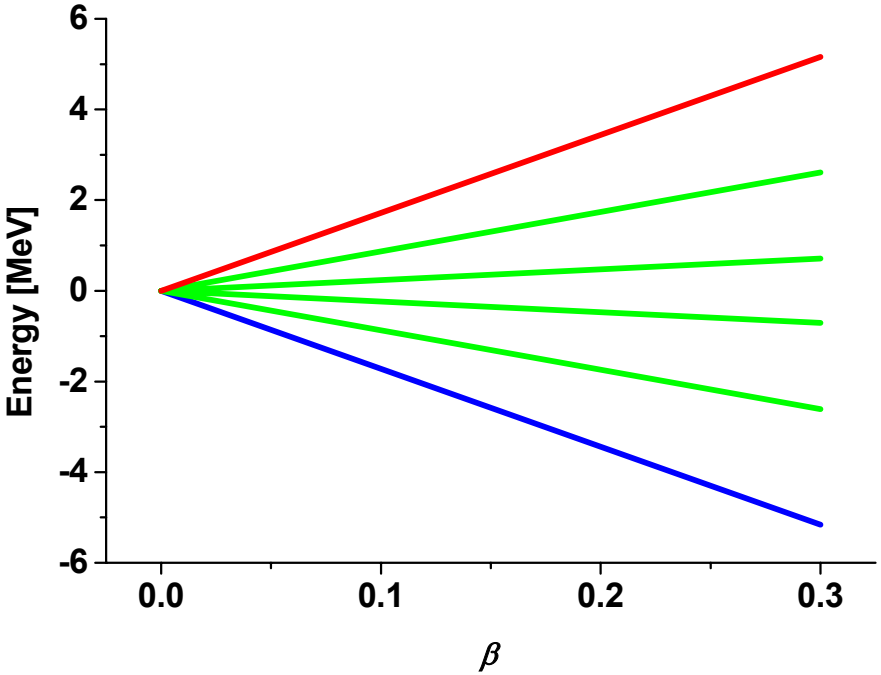
Unique parity $h_{11/2}$ state in quadrupole-deformed triaxial potential.

$$H = H_{SM} + H_{def}$$



Triaxial shape for $\beta = 0.3$, $\gamma = 30°$.

$$H_{def} = k\beta\,[\,\cos(\gamma)Y_{20}(\theta,\phi) + 1/\sqrt{2}\sin(\gamma)\{Y_{22}(\theta,\phi) + Y_{2-2}(\theta,\phi)\}]$$



$\langle j_s \rangle = 0.00 \quad \langle \sigma_s \rangle = 1.36$

$\langle j_i \rangle = 0.00 \quad \langle \sigma_i \rangle = 2.01$

$\langle j_l \rangle = \pm 5.46 \quad \langle \sigma_l \rangle = 0.30$

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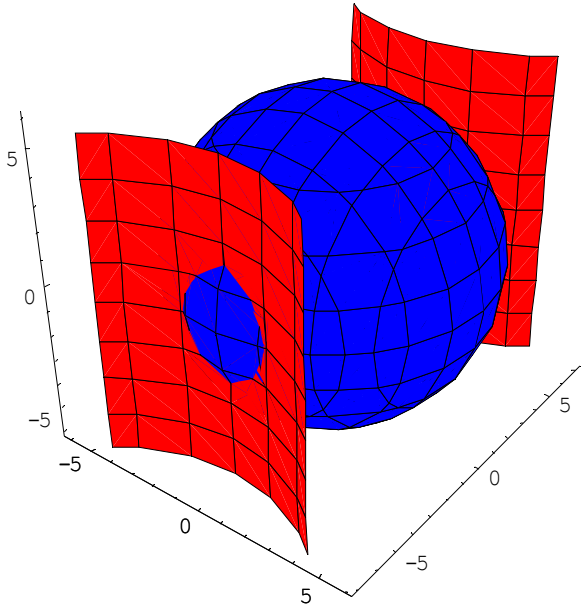
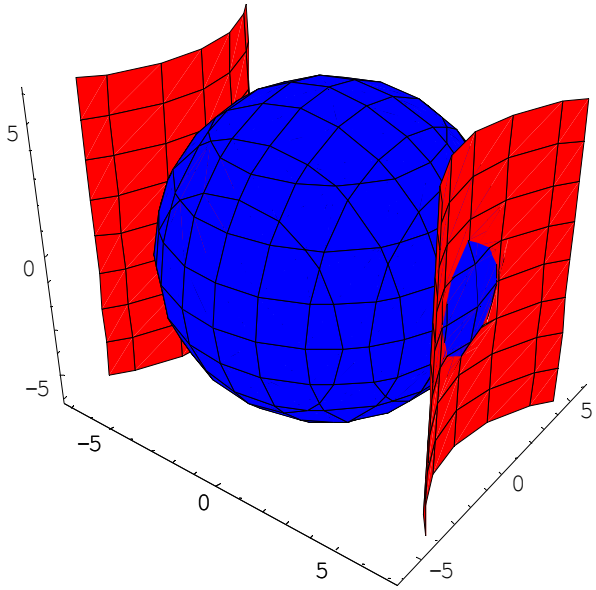
Semi classical analysis for single-particle hamiltonian in a triaxial nucleus.

$$j^2 = j_x^2 + j_y^2 + j_z^2 \qquad E - E_F = \kappa\,(j_x^2 - j_y^2)$$

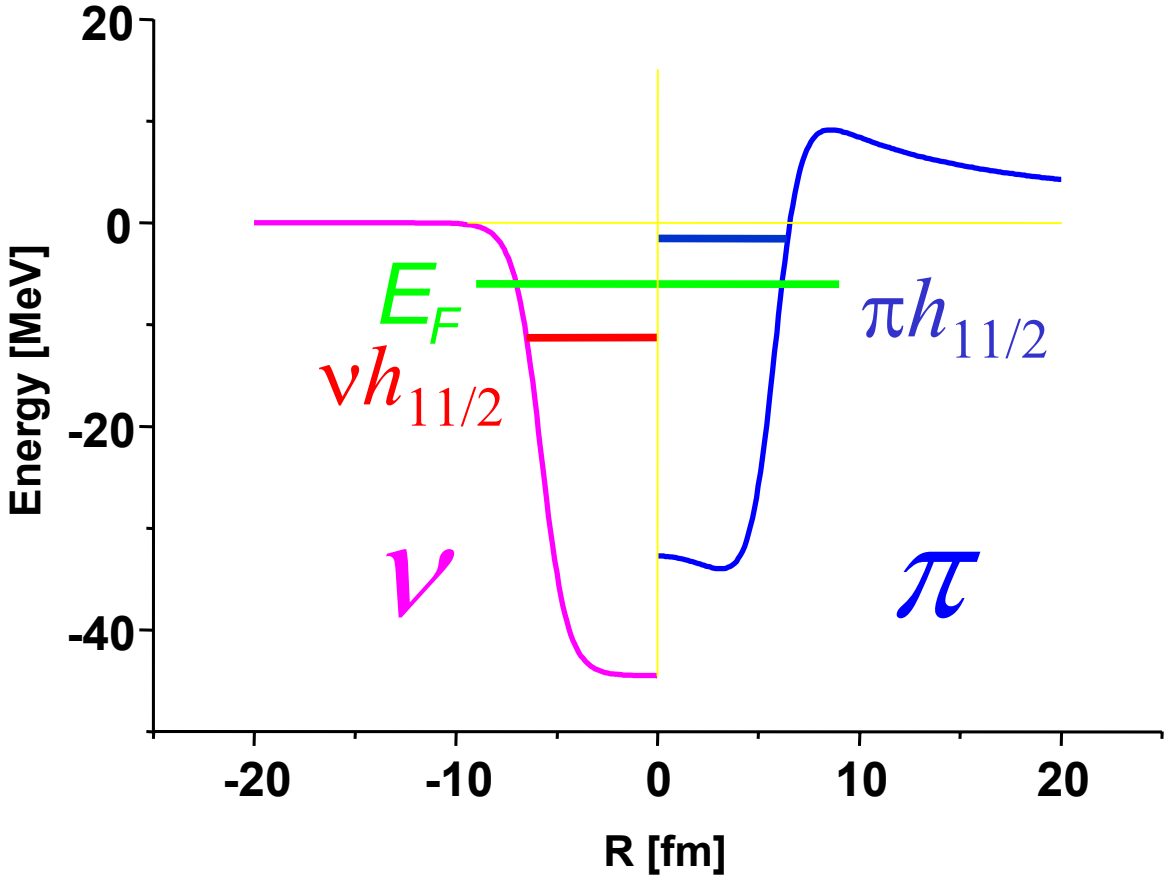
$E > E_F$



$E < E_F$

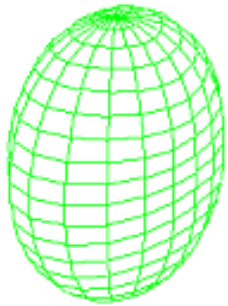


Position of the Fermi level relative to the $h_{11/2}$ states in the mass A~130 region (Z~59,N~75).



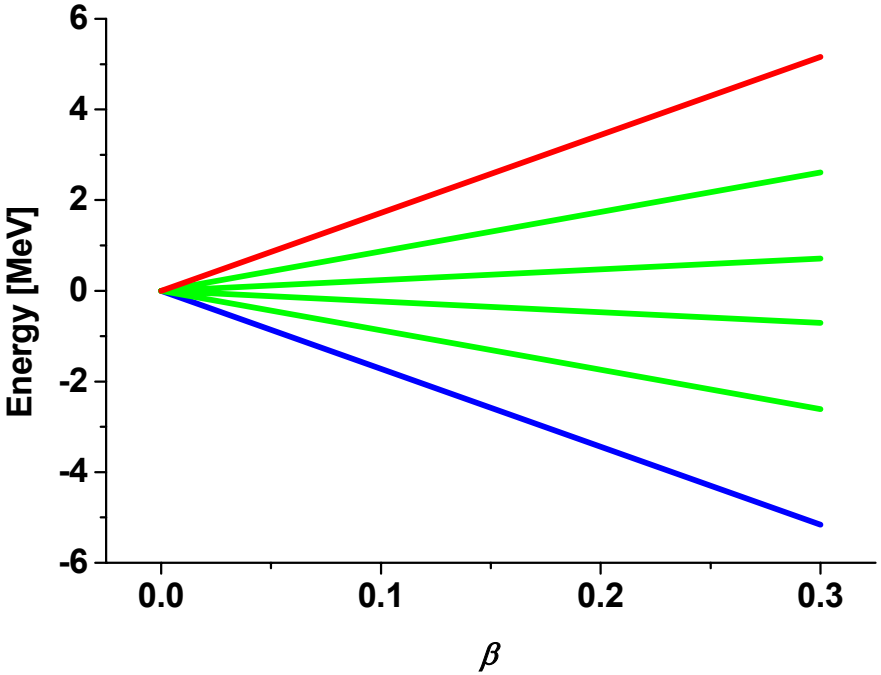
Unique parity $h_{11/2}$ state in quadrupole-deformed triaxial potential.

$$H = H_{SM} + H_{def}$$



Triaxial shape for $\beta = 0.3$, $\gamma = 30°$.

$$H_{def} = k\beta\,[\,\cos(\gamma)Y_{20}(\theta,\phi) + 1/\sqrt{2}\sin(\gamma)\{Y_{22}(\theta,\phi) + Y_{2-2}(\theta,\phi)\}]$$



$\langle j_s \rangle = 0.00 \quad \langle \sigma_s \rangle = 1.36$

$\langle j_i \rangle = 0.00 \quad \langle \sigma_i \rangle = 2.01$

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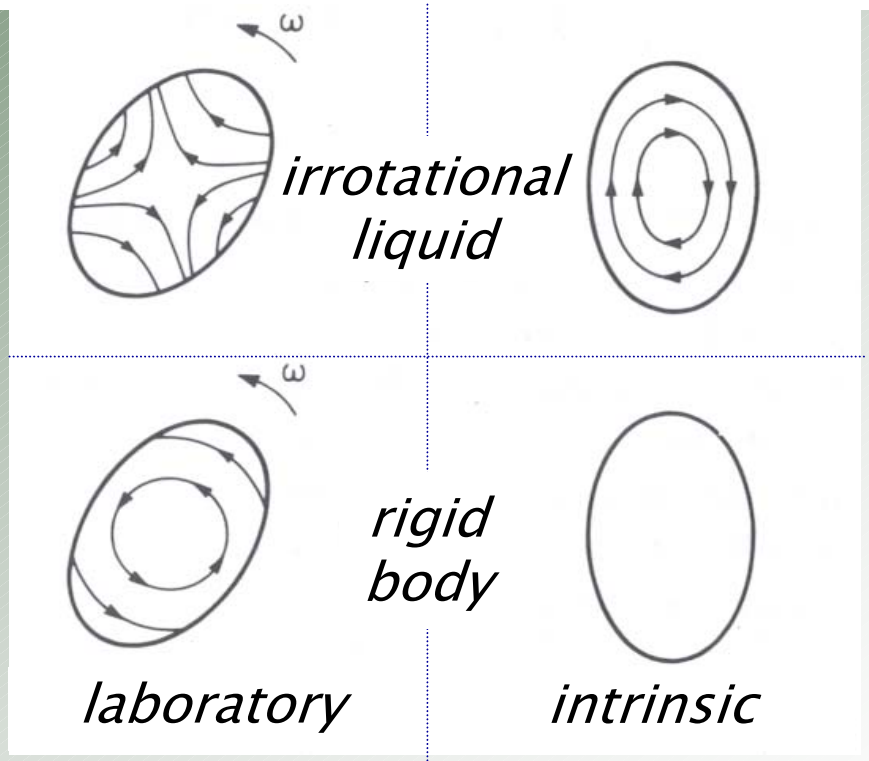
$\langle j_l \rangle = 0.00 \quad \langle \sigma_l \rangle = 1.36$

Quantum mechanics of collective nuclear rotation.

Parameterization of nuclear radius:

$$r(t)=r_0\,[\,1+\Sigma_{\lambda\mu}\,a(t)\,Y_{\lambda\mu}(\theta,\varphi)\,].$$

Rotation resembles that of irrotational liquid but is different than that of a rigid body. In particular moments of inertia for both cases differ significantly.



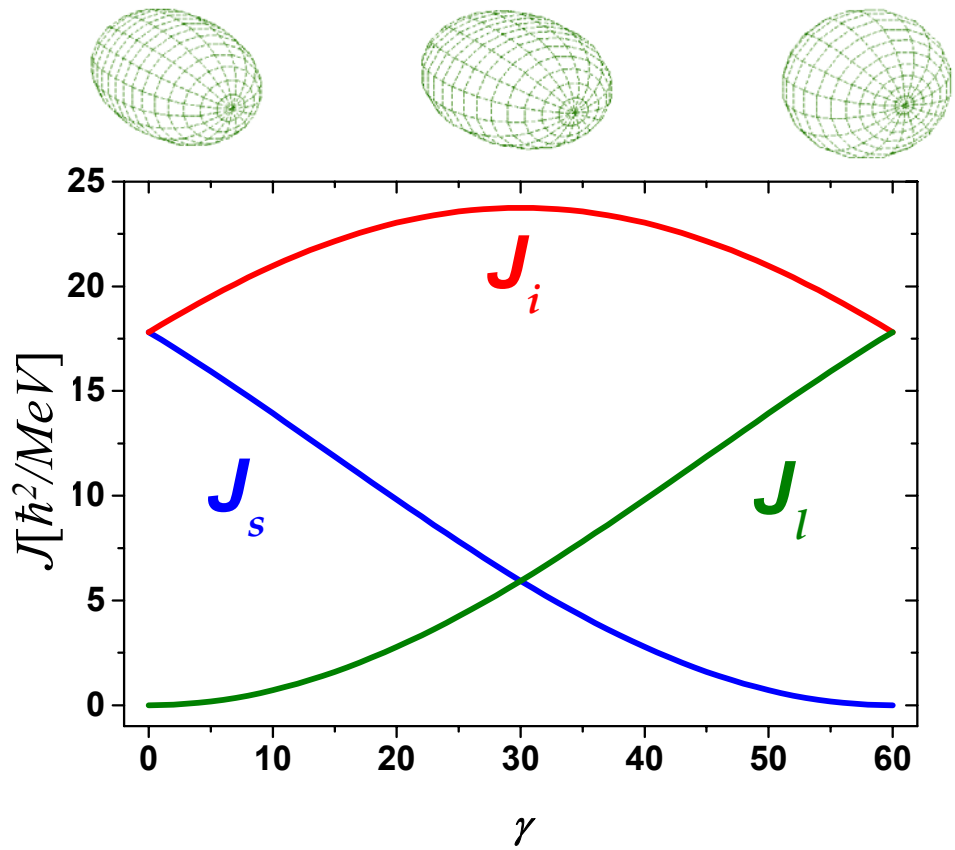
Angular momentum for rotating triaxial body with irrotational flow moment of inertia aligns along intermediate axis.

$$H = \frac{R_s^2}{2J_s} + \frac{R_i^2}{2J_i} + \frac{R_l^2}{2J_l}$$

$$J_s = 4B\beta^2 \sin^2(\gamma - 120°)$$

$$J_i = 4B\beta^2 \sin^2(\gamma - 240°)$$

$$J_l = 4B\beta^2 \sin^2(\gamma)$$



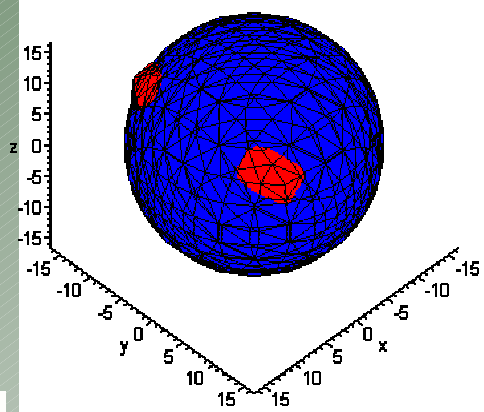
Semi classical analysis for a triaxial rotor.

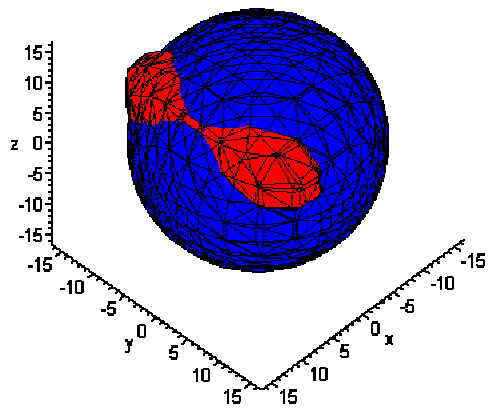
$$J_i = 4J_s = 4J_l = J_0$$

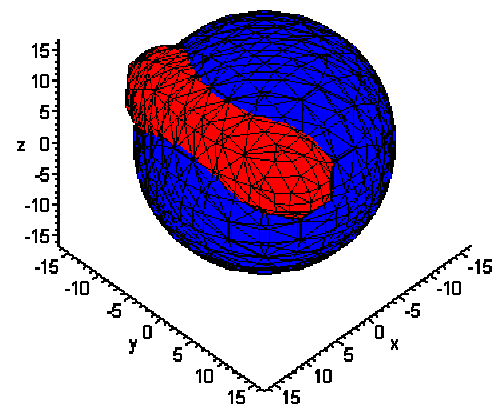
$$I^2 = I_s^2 + I_i^2 + I_l^2 \qquad E = \frac{\hbar^2}{2J_0}\left[4(I_s - j_s)^2 + I_i^2 + 4(I_l - j_l)^2\right]$$

$I = 15[\hbar]$ $\qquad J_0 = 27.7[\hbar^2/MeV]$ $\qquad j_l = j_s = 5.5[\hbar]$

$E = 3.3[MeV]$ $\qquad E = 3.8[MeV]$ $\qquad E = 4.3[MeV]$



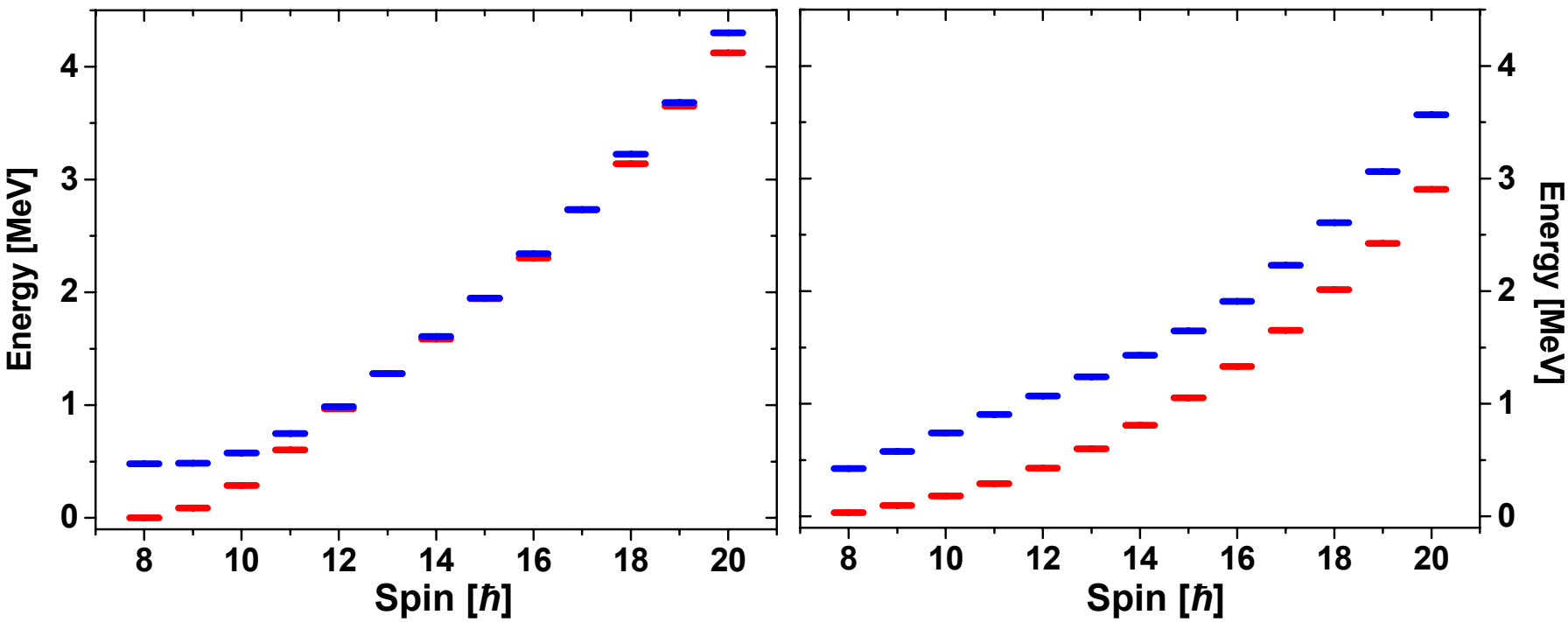




$\pi h_{11/2} \nu h_{11/2}$ particle-hole coupling at β=0.3, γ=30°

$$J_k^{irr.} = J_0 \frac{4}{3} \sin^2(\gamma - \frac{2}{3}\pi k)$$

$$J_k^{rig.} = J_0 (1 + \sqrt{\frac{5}{4\pi}} \beta \cos(\gamma - \frac{2}{3}\pi k))$$



Experimental evidence for doublet bands:

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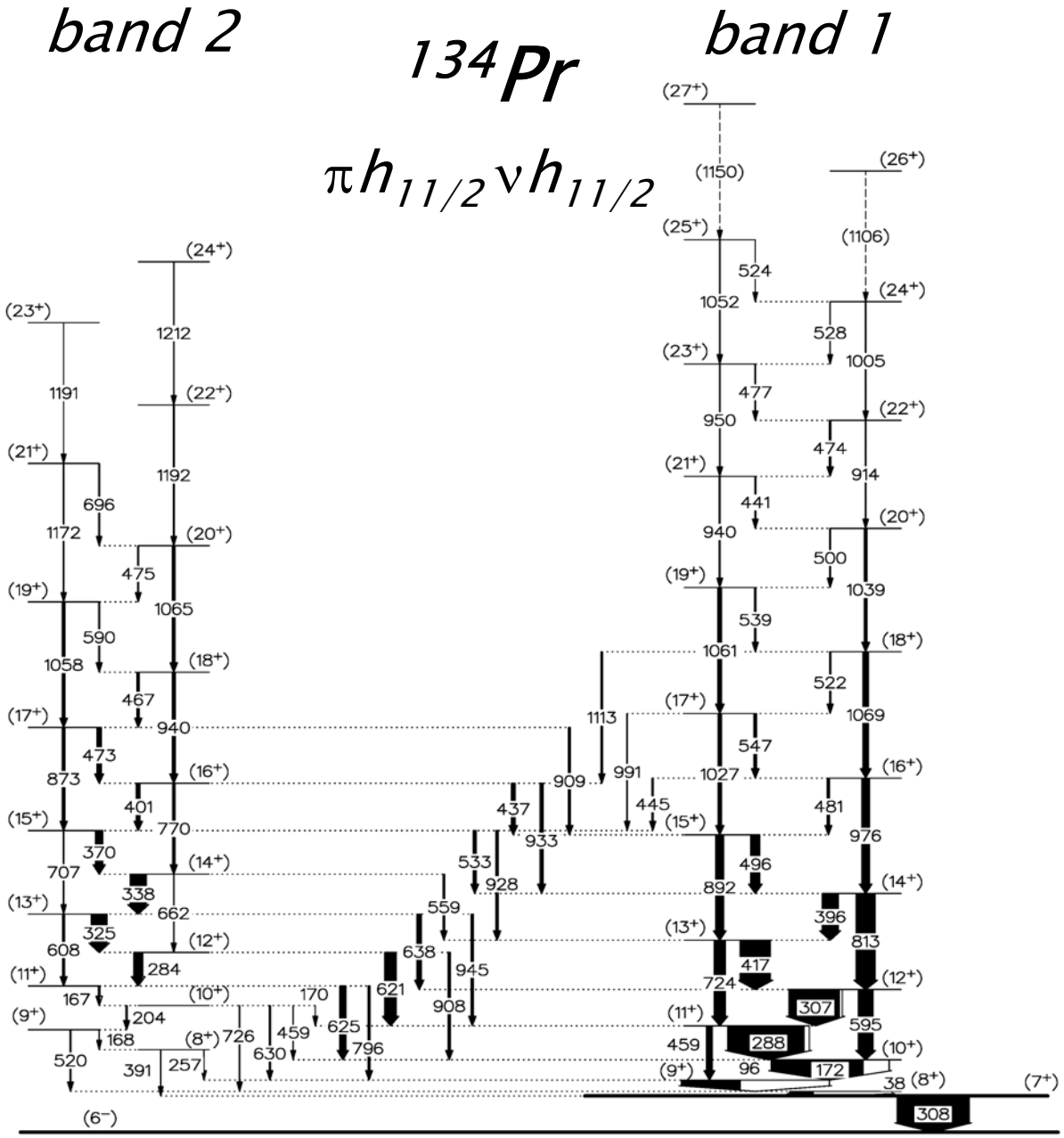
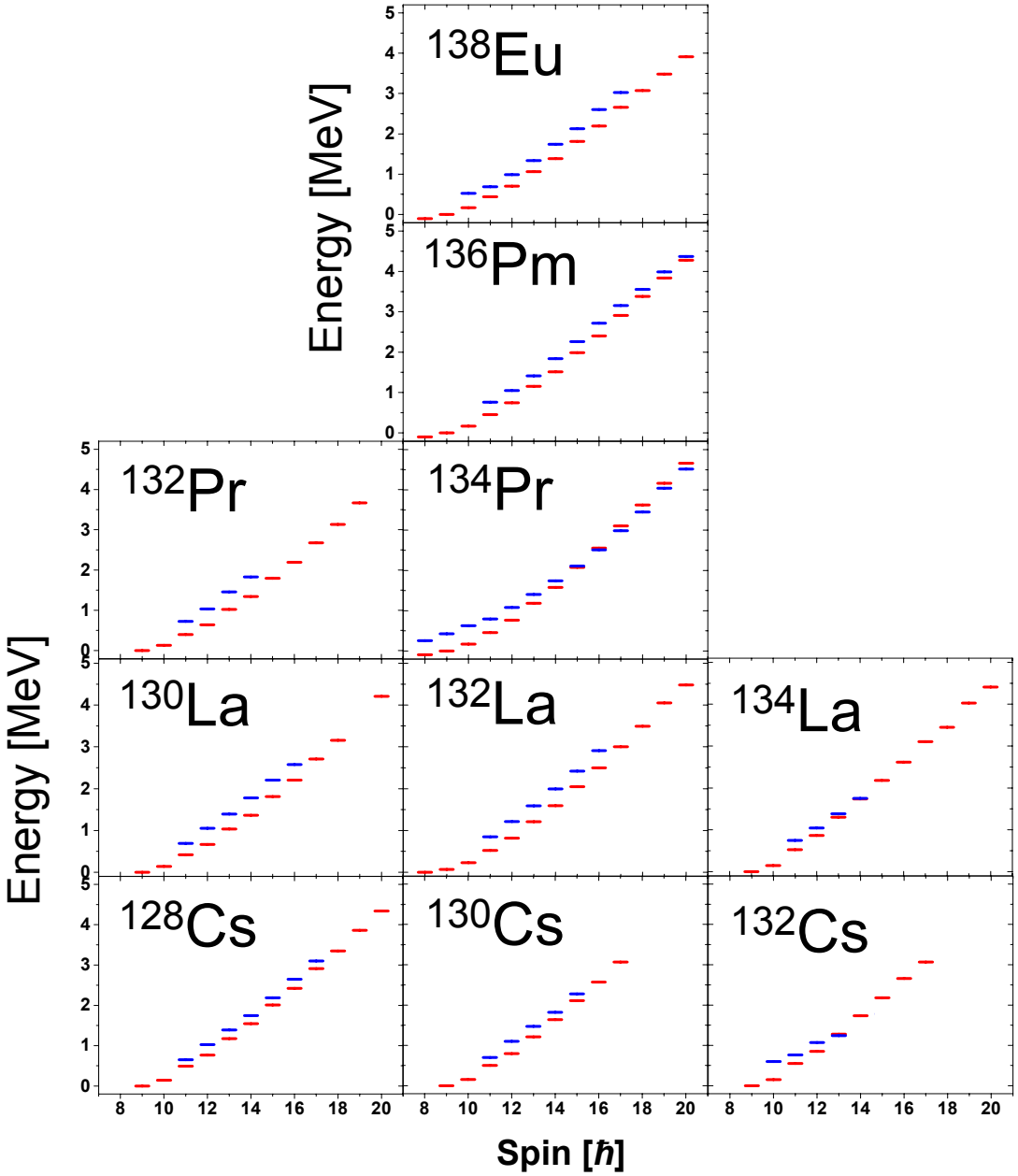
Results of the Gammasphere GS2K009 experiment.

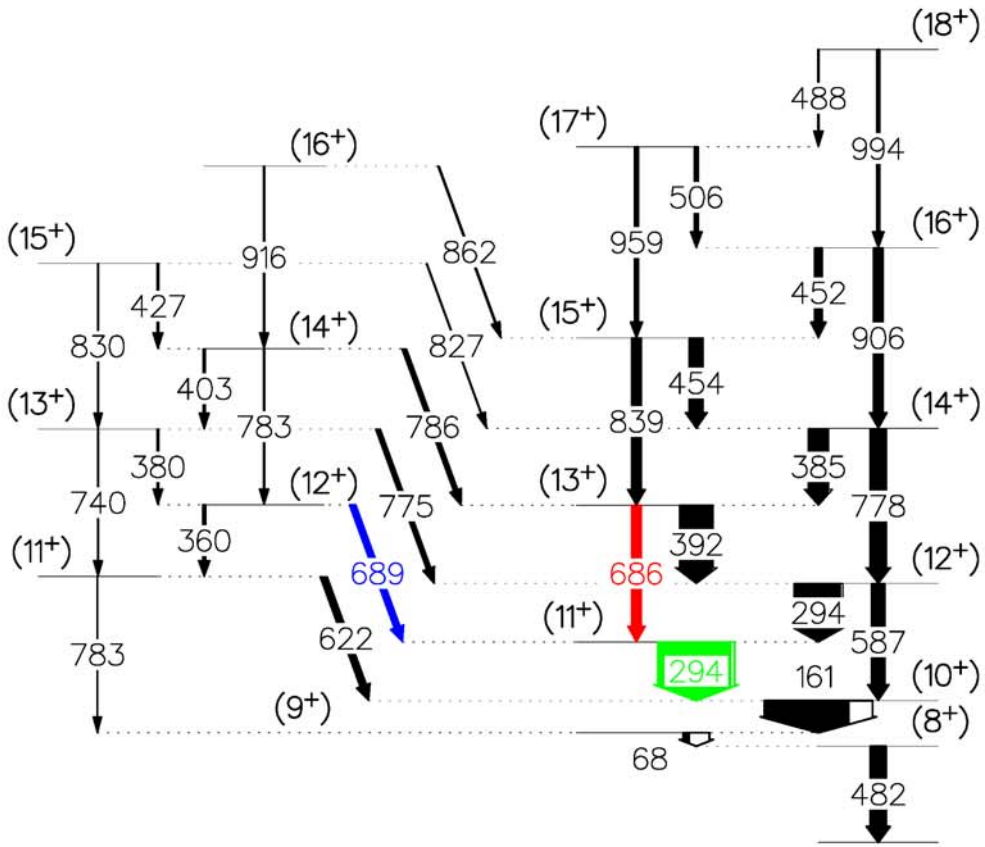


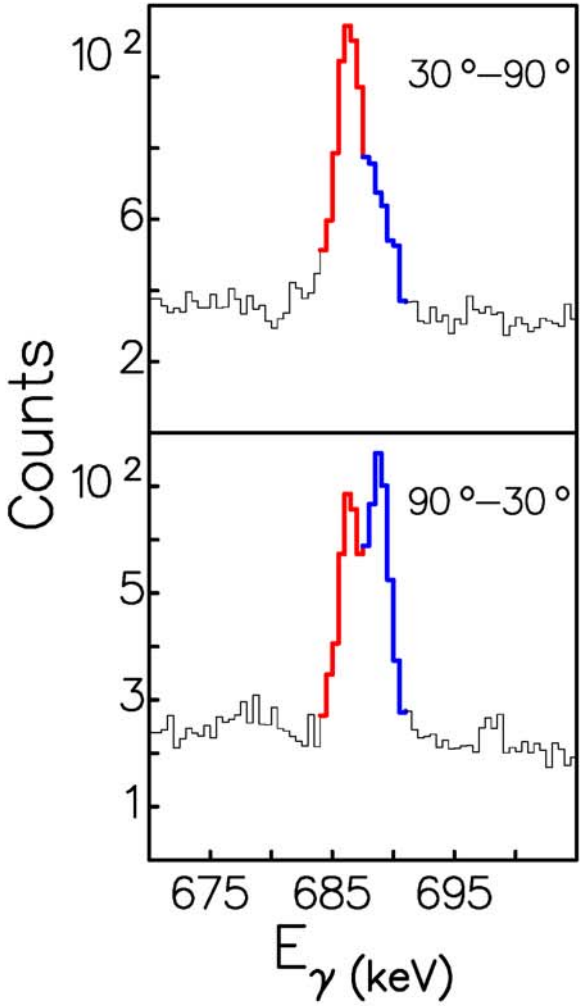
Systematics of partner bands in odd-odd A~130 nuclei.



Angular correlation (DCO) measurement for ^{132}La.

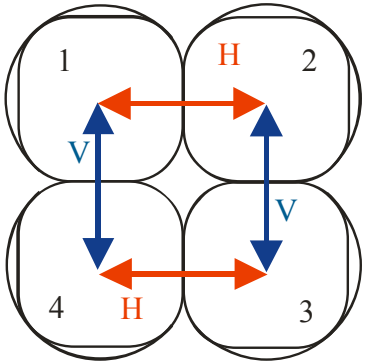
R_{DCO}(294 *keV M1/E2*-686 *keV E2*) = 0.46(6)
R_{DCO}(294 *keV M1/E2*-689 *keV ?*) = 1.4(2)





Polarization measurements in ^{132}Cs

[G Rainovski et al., PRC 68, (2003) 024318].

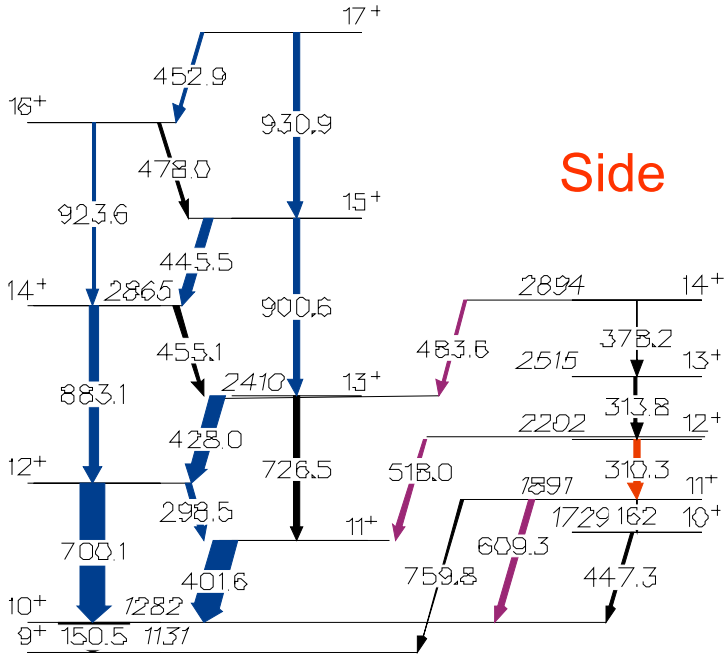


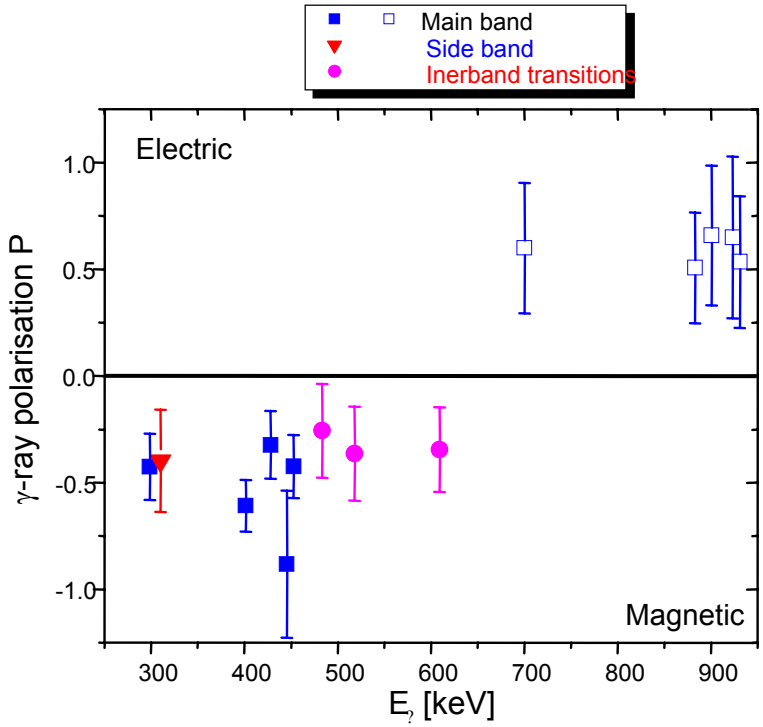
$$P = \frac{1}{Q(E_\gamma)} \frac{N_v - N_h}{N_v + N_h}$$

$Q(E_\gamma)$ – P.M. Jones *et al.*, NIM A 362 (1995) 556

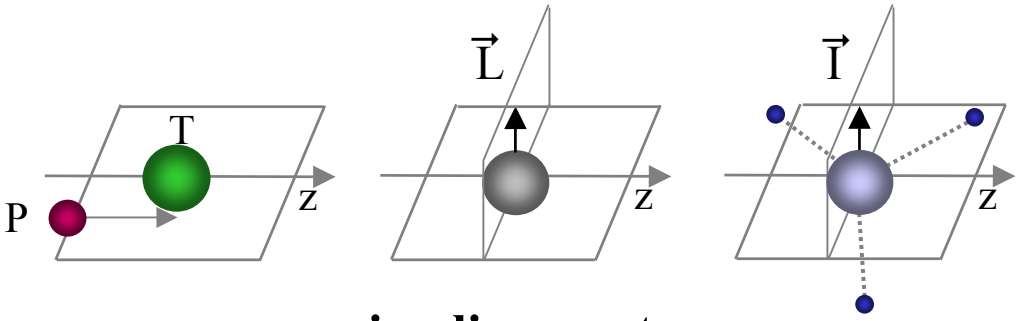
Main

Side

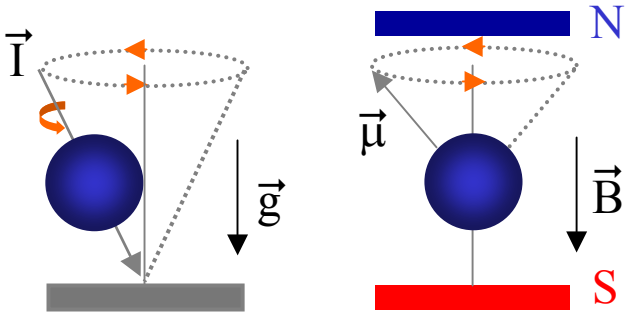




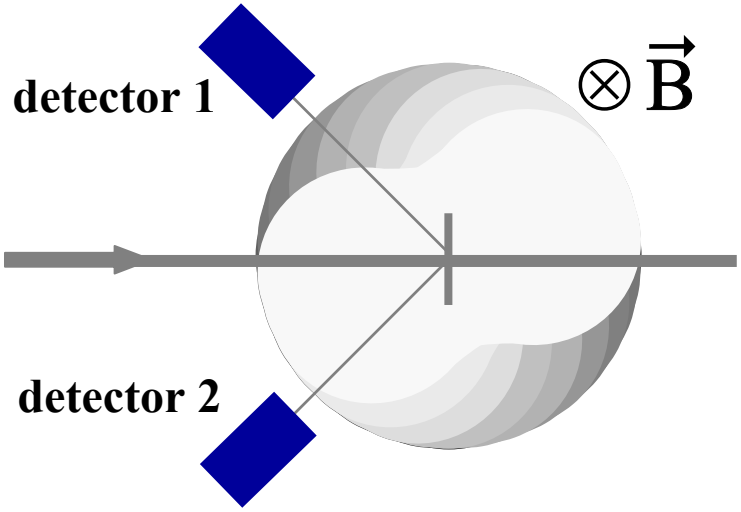
Unique parity configuration assignments for the main band from magnetic moment measurements.



spin alignment

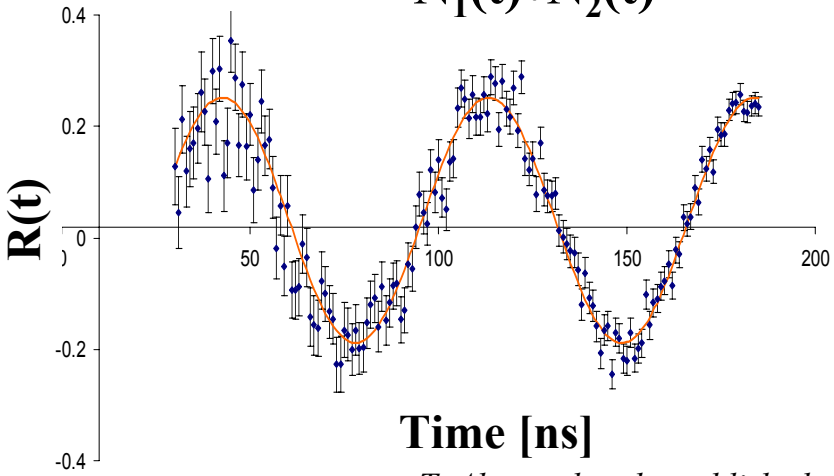


Larmor precession



precession of γ-ray angular distribution

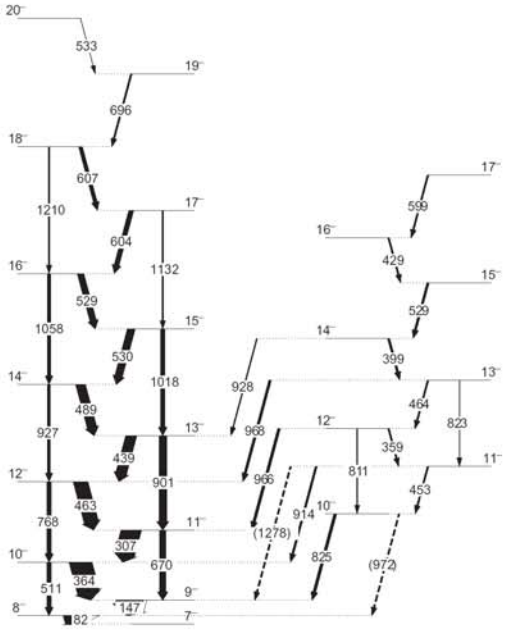
^{128}Cs $R(t) = \dfrac{N_1(t) - N_2(t)}{N_1(t) + N_2(t)}$



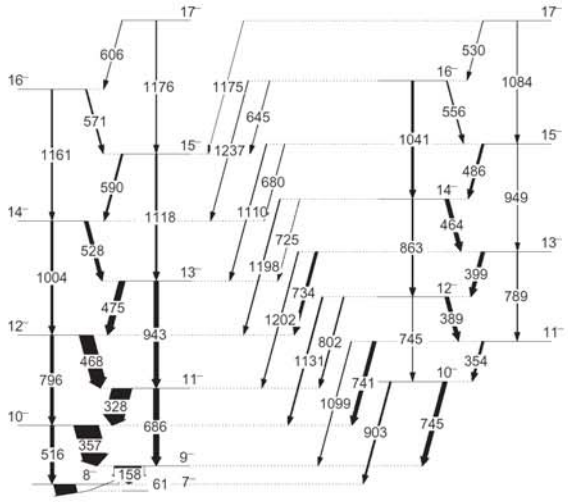
T. Ahn et al. to be published.

Systematics of odd-odd Rh isotopes near A~104

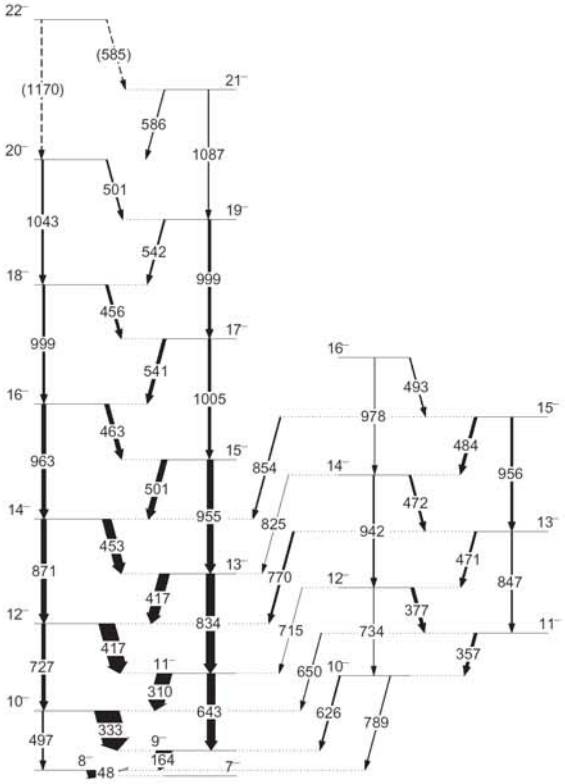
C. Vaman PRL92 (2004) 032501, P. Joshi et al., PLB in print.



^{102}Rh



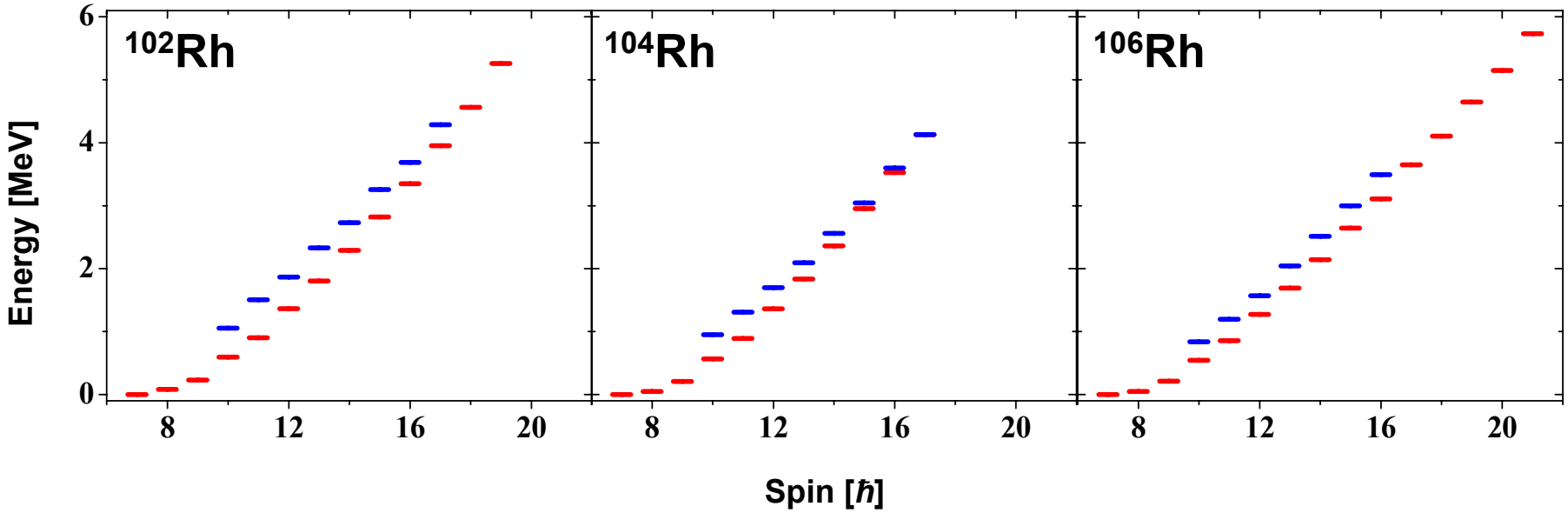
^{104}Rh



^{106}Rh

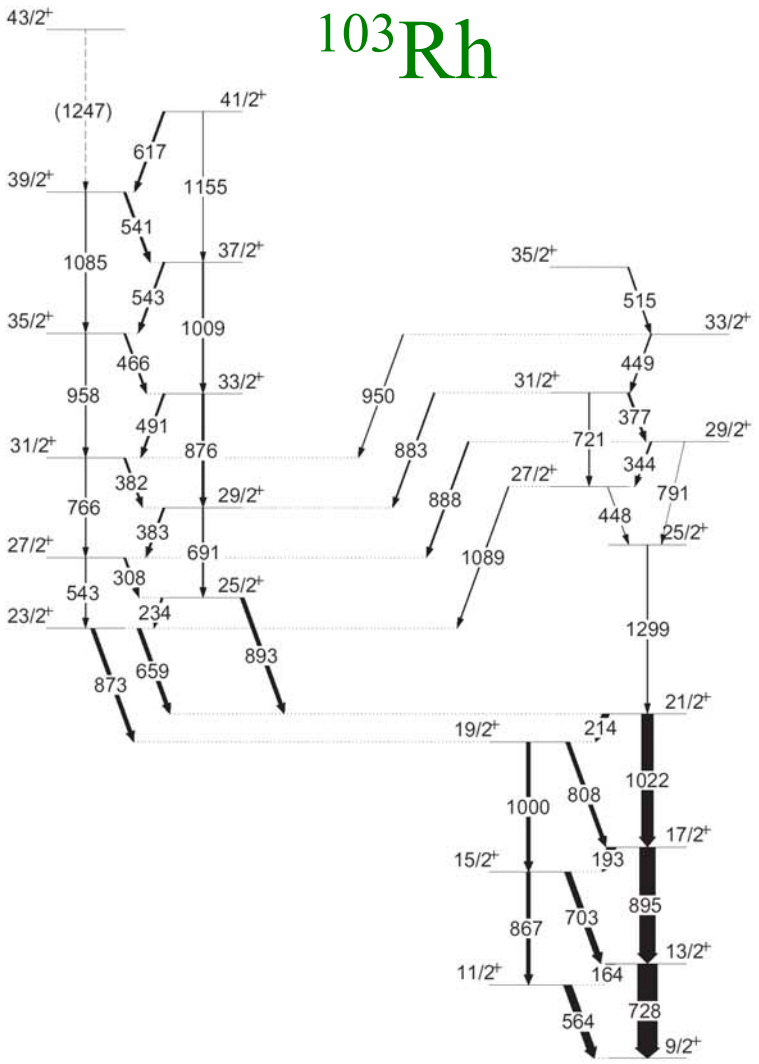
Systematics of odd-odd Rh isotopes near A~104

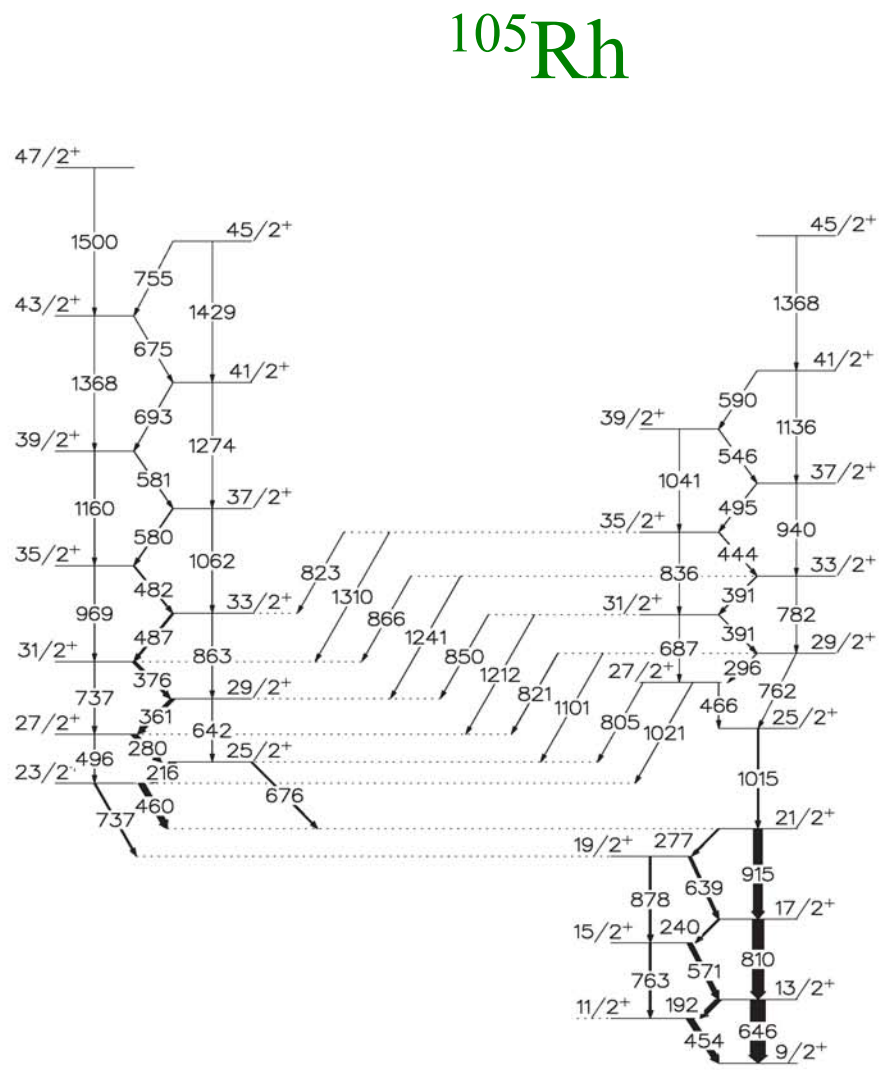
C. Vaman PRL92 (2004) 032501, P. Joshi et al., PLB in print.



Doublet bands in odd-even ^{103}Rh and ^{105}Rh near ^{104}Rh

C. Vaman et al., J. Timar et al., submitted to PLB.





What have we learned from chirality so far?

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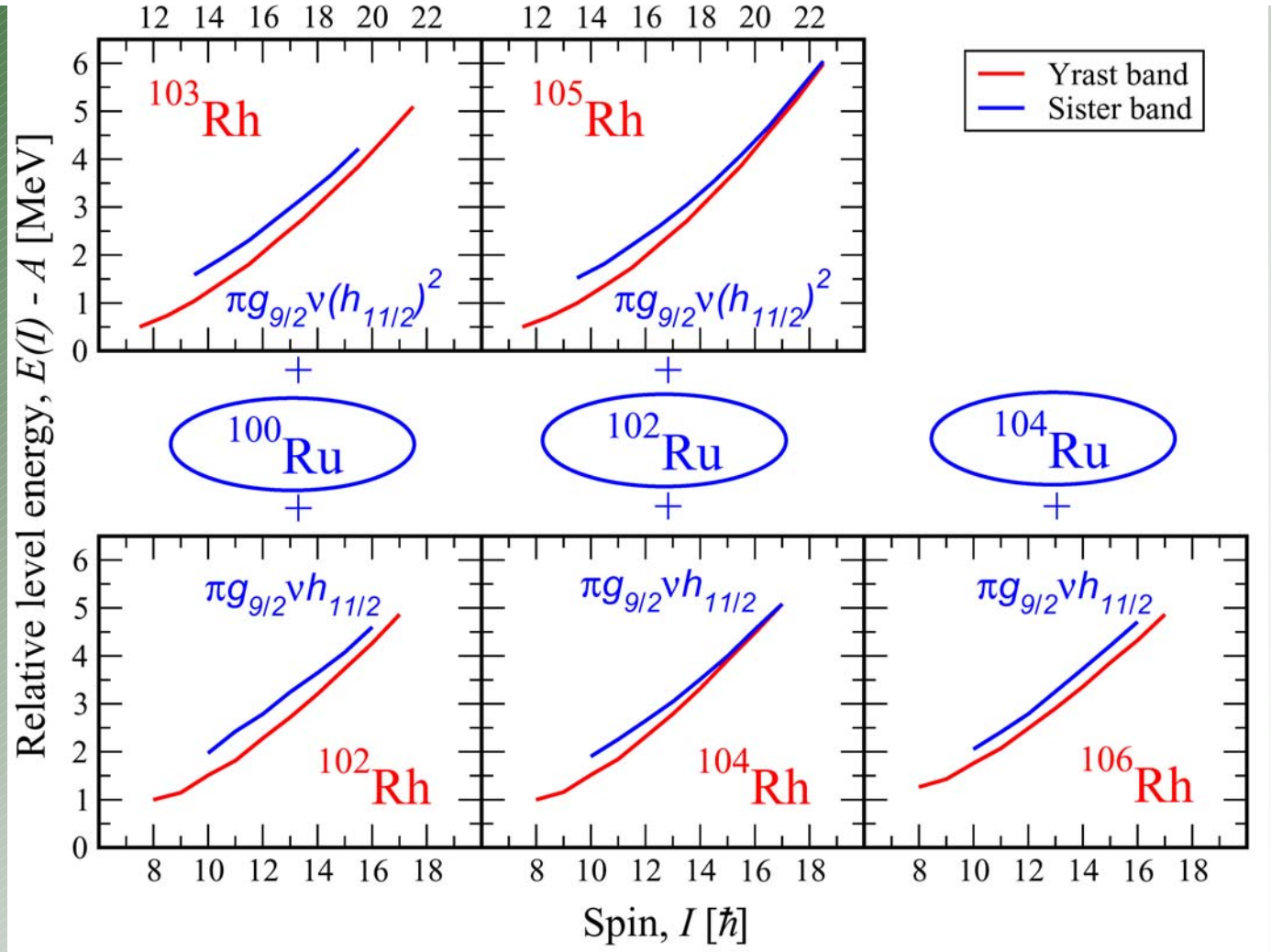
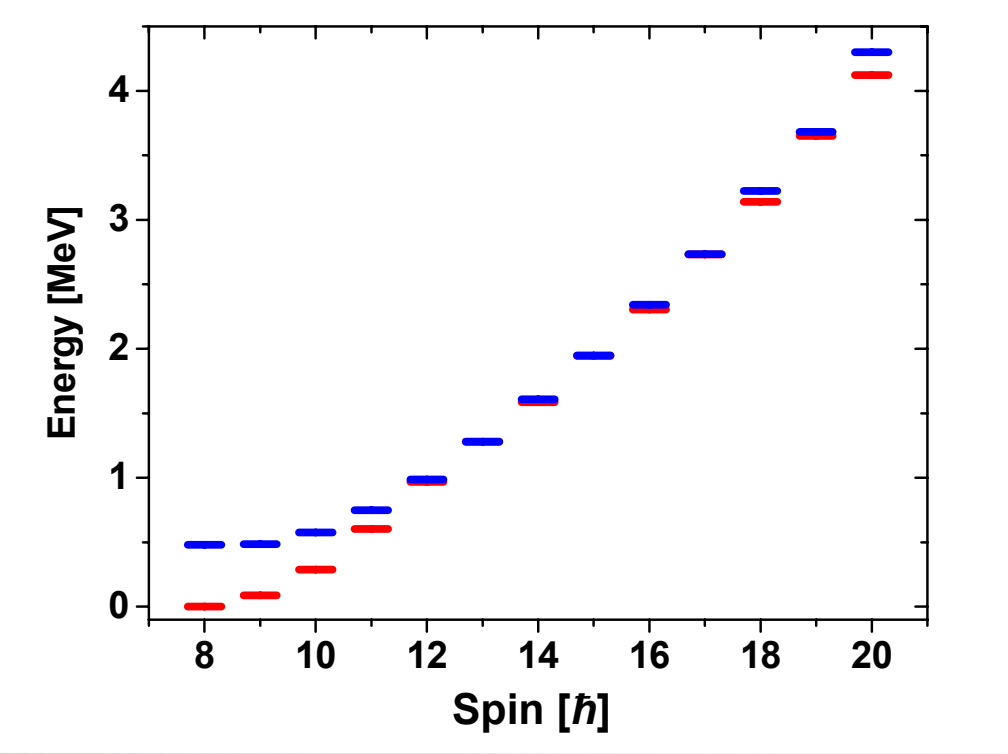
Chirality is a general phenomenon in triaxial nuclei:

- *two mass regions identified up to date,*
- *partner bands in odd-odd and odd-A nuclei.*



Energy separation vs. spin trend understood:

- *planar components at the low spin,*
- *degenerate levels at the medium spin,*
- *Coriolis alignment at the high spin.*

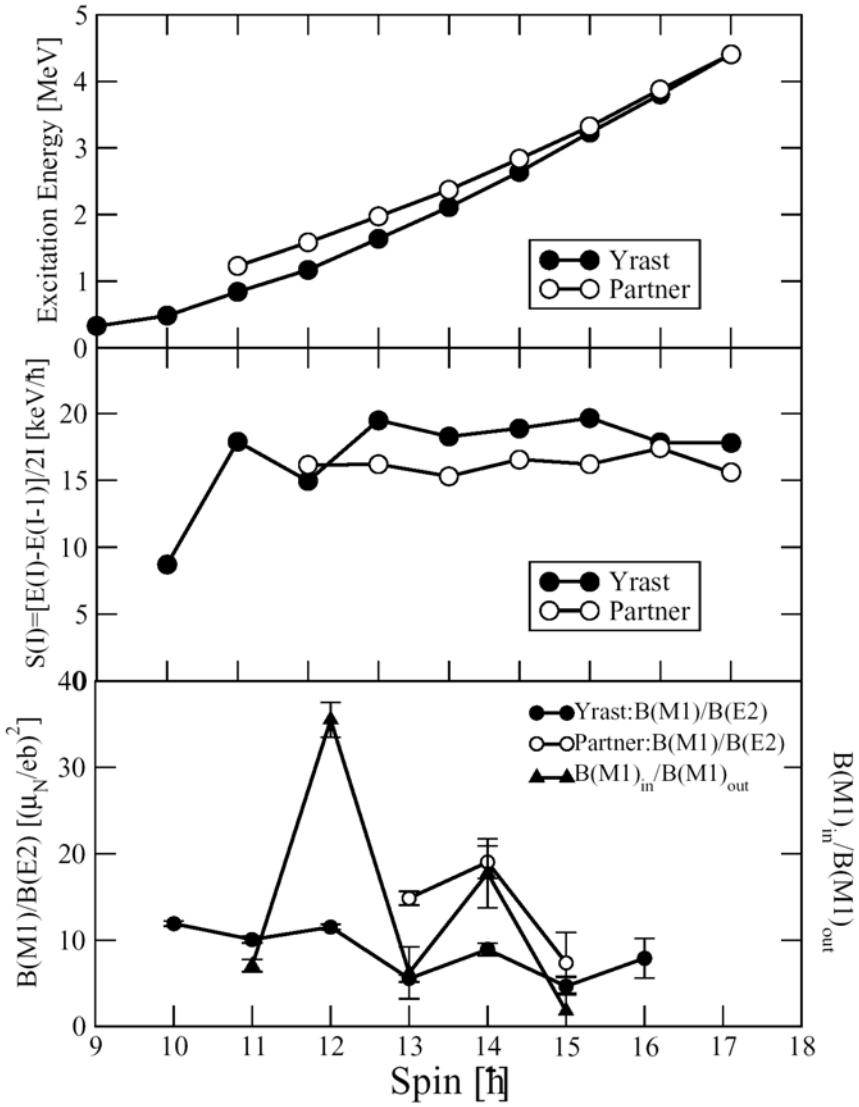


The smoking gun has been identified:

three fingerprints currently established for chirality in odd-odd triaxial nuclei:

- *near degenerate doublet $\Delta I=1$ bands for a range of spin I ;*
- $S(I)=[E(I)-E(I-1)]/2I$ *independent of spin I;*
- *chiral symmetry restoration selection rules for M1 and E2 transitions vs. spin resulting in staggering of the absolute and relative transition strengths.*

Based on the above fingerprints ^{104}Rh provides the best example of chiral bands observed up to date.



- ✓ doubling of states
- ✓ $S(I)$ independent of I
- ✓ $B(M1)$, $B(E2)$ staggering

C. Vaman et al. PRL 92(2004)032501

A new limit of particle rotor model for triaxial nuclei has been identified:

For irrotational flow moment of inertia there are two special cases for which two out of three moments are equal:

axial symmetry

for $\gamma=0^o$ (prolate shapes)

$J_s=J_i=J_0 \quad J_l=0$

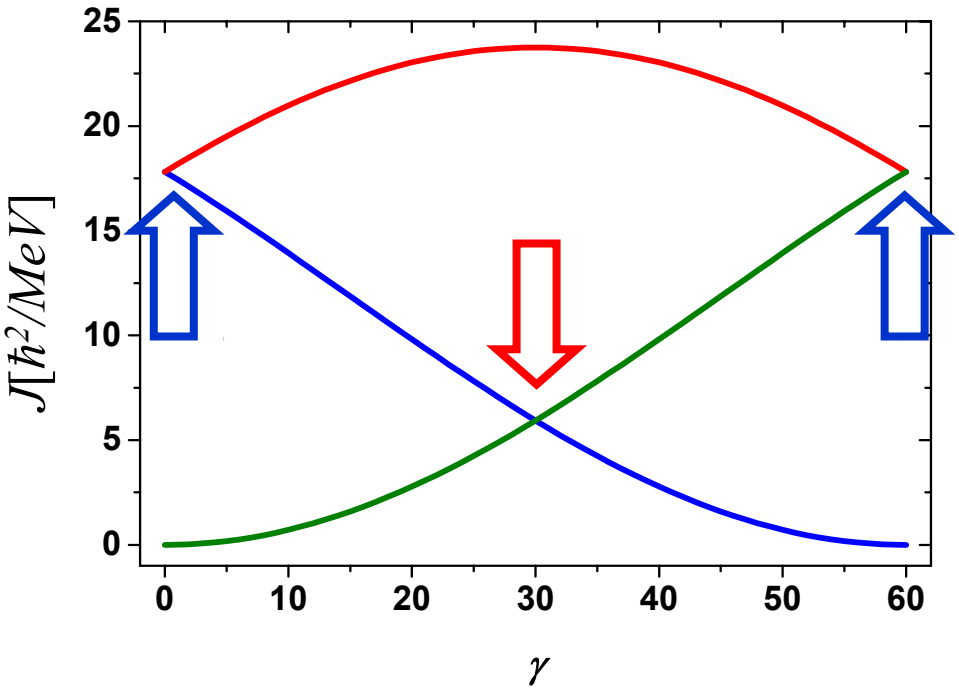
for $\gamma=60^o$ (oblate shapes)

$J_l=J_i=J_0 \quad J_s=0$

triaxiality

for $\gamma=30^o$ (triaxial shapes)

$J_l=J_s=J_0 \quad J_i=4J_0.$

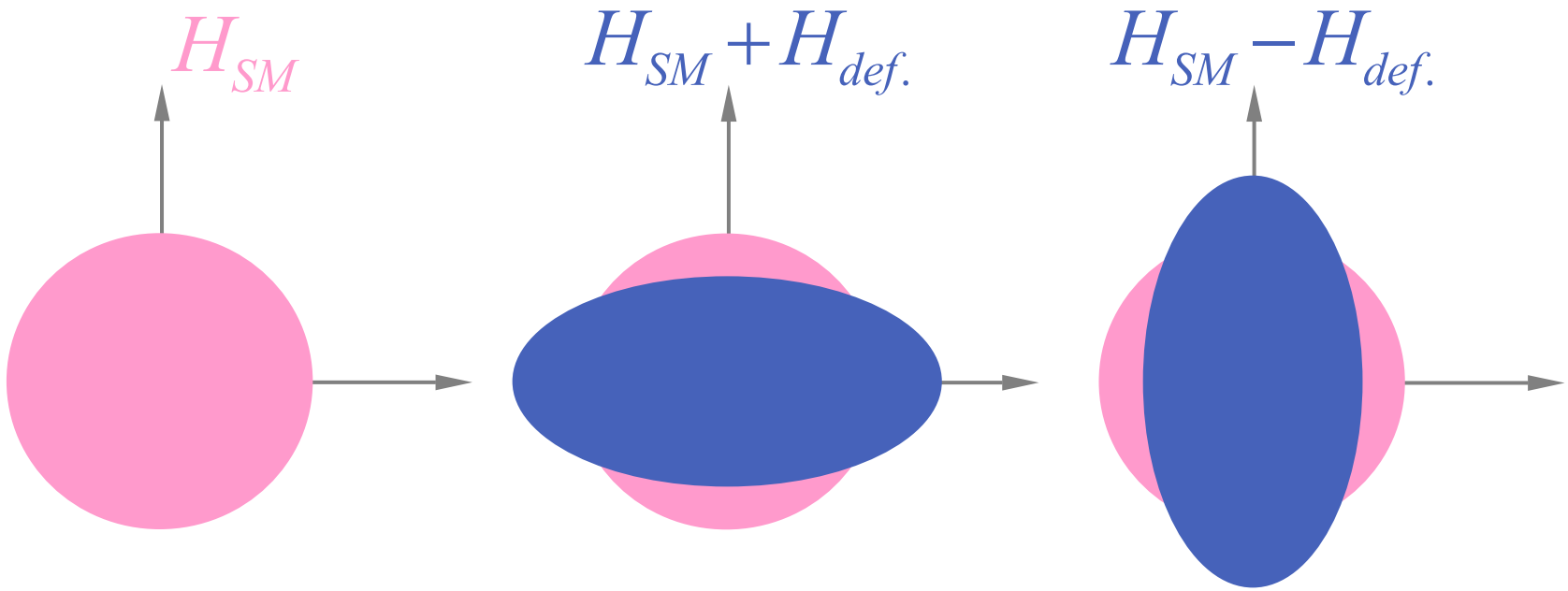


For triaxial shapes at $\gamma=30^o$ the rotor hamiltonian has a very similar structure to the hamiltonian for axially symmetric shapes.

The intermediate axis becomes an effective symmetry axis.

New symmetry of particle rotor model for odd-odd chiral nuclei:

a) rotation by 90 or 270 deg. about the intermediate axis

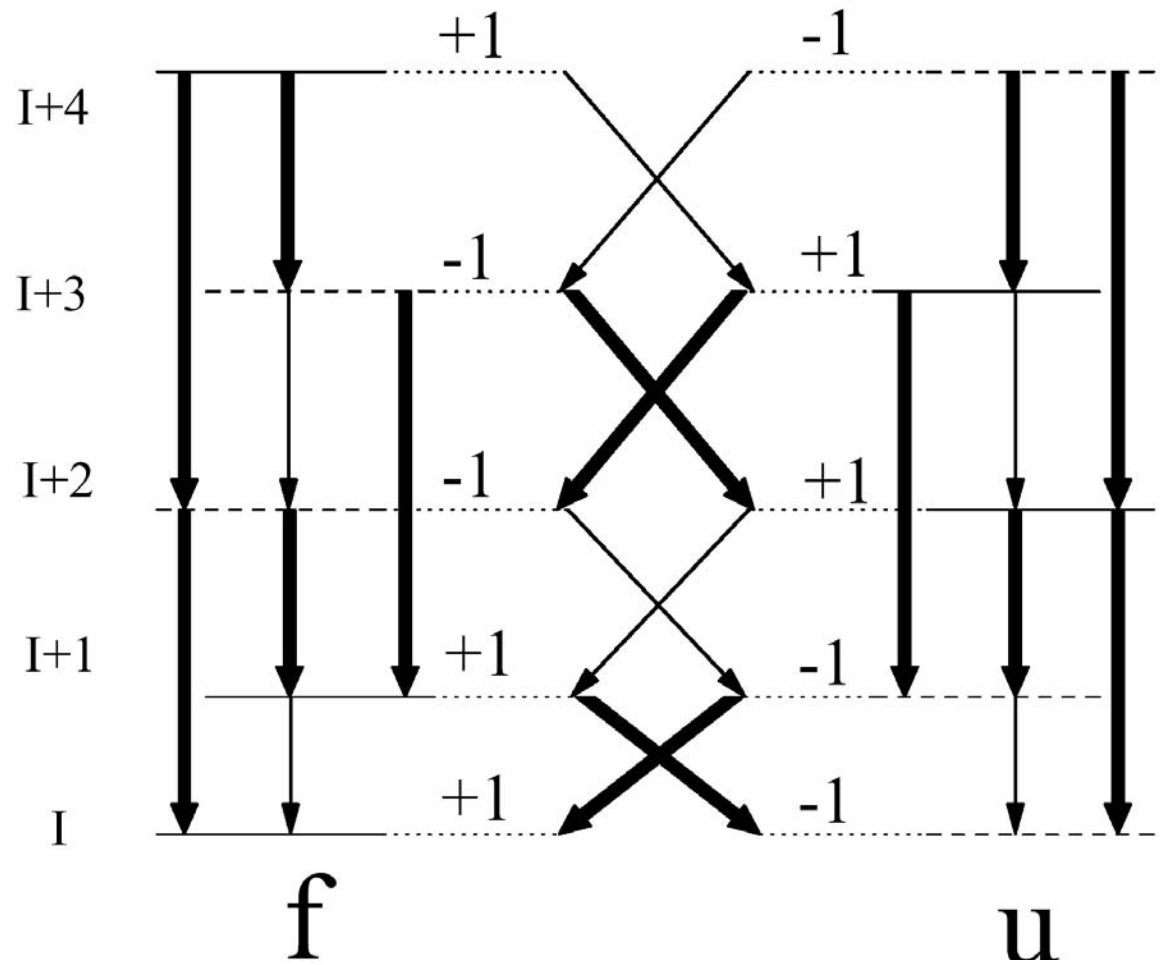


$$R_i(\pi/2)H_{def.}R_i^{-1}(\pi/2) = -H_{def.}$$

$$|\Psi'\rangle = R_i(\pi/2)|\Psi\rangle$$

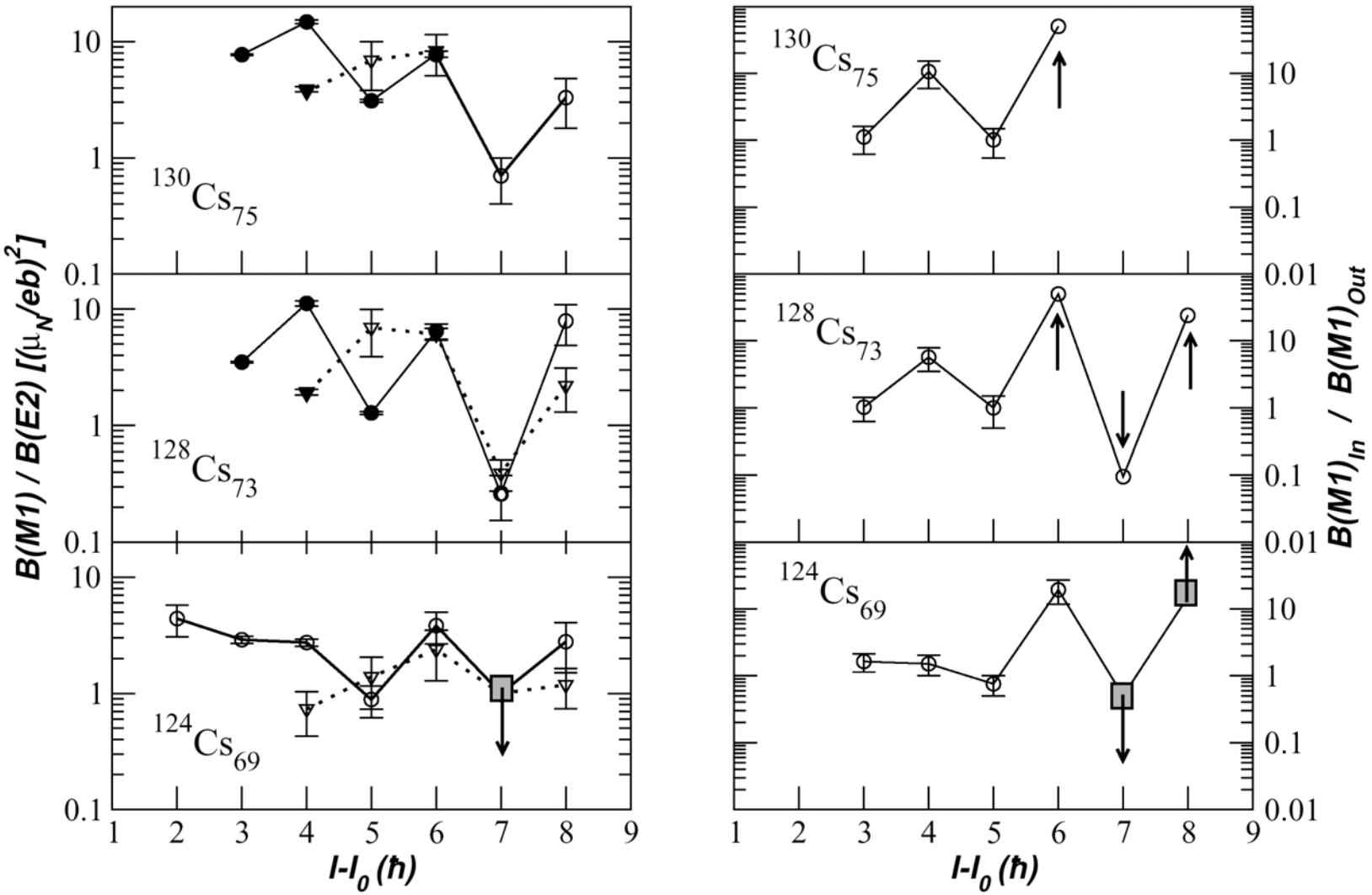
$$H_{def.}|\Psi\rangle = \varepsilon|\Psi\rangle \Rightarrow H_{def.}|\Psi'\rangle = -\varepsilon|\Psi'\rangle$$

New symmetry of particle rotor model for odd-odd chiral nuclei: results in selection rules and unique predictions for electromagnetic transition rates without a need for calculation.

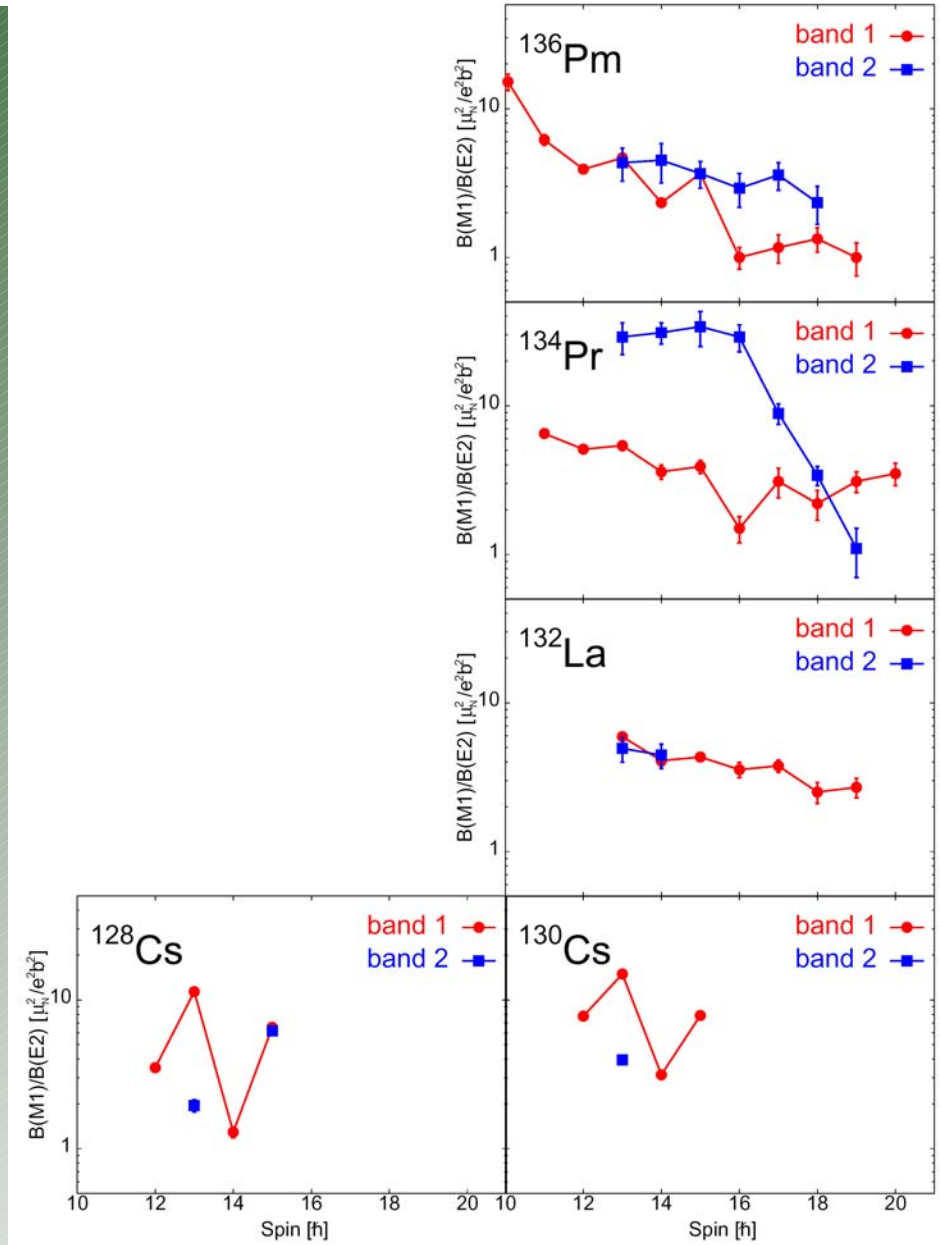


T. Koike et al. , submitted to PRL

Electromagnetic properties – pronounced staggering in experimental B(M1)/B(E2) and $B(M1)_{in}$ / $B(M1)_{out}$ ratios as a function of spin [T.Koike et al. PRC 67 (2003) 044319].



Electromagnetic properties – unexpected B(M1)/B(E2) behavior for ^{134}Pr and heavier N=75 isotones.



I like to recall his [von Laue's] question as to which results derived in the present volume I considered most important. My answer was that the explanation of the concept of parity and the quantum theory of vector addition model appeared to me most significant. Since that time, I have come to agree with his answer that the recognition that almost all rules of spectroscopy follow from the symmetry of the problem is the most remarkable result.

Eugene P. Wigner
Group Theory,
Author's Preface