To Mix or Not to Mix: Vector and Axial Vector Spectral Densities at Finite Temperature

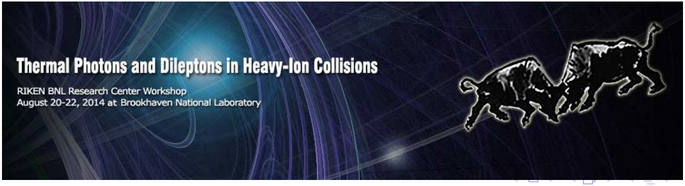
Alejandro Ayala*, C. A. Dominguez, M. Loewe, Y. Zhang

arXiv:1405.2228 [hep-ph], *to appear in PRD*

(*) Instituto de Ciencias Nucleares, UNAM

ayala@nucleares.unam.mx

Workshop on Thermal Photons and Dileptons in Heavy-Ion Collisions, BNL, August 2014

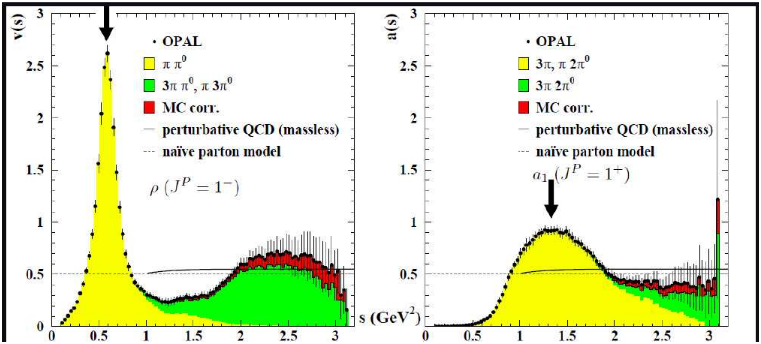


Spontaneous symmetry breaking

- **Nambu & Goldstone** (late 50's) discovered a way through which a symmetry of a system can be realized: **spontaneous breaking/restauration of the symmetry**
- The symmetry is not realized in the particle mass spectrum. **Parity partners are non-degenerate in masses**

$$N(\tfrac{1}{2}^{+}, 938) = N(\tfrac{1}{2}^{-}, 1535),$$
$$\pi(0^{-}, 140) = \sigma(0^{+}, 600),$$
$$\rho(1^{-}, 770) = a_1(1^{+}, 1260)$$

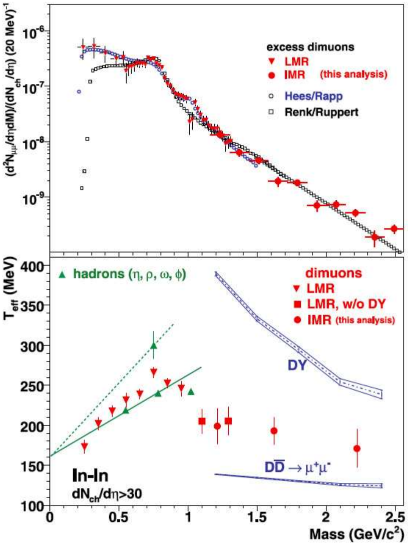
$\tau \longrightarrow$ hadrons



OPAL Collaboration

Information on vector spectral density at finite T and μ_B from low mass dileptons

NA60, Eur. Phys. J. C **59**, 607 (2009)

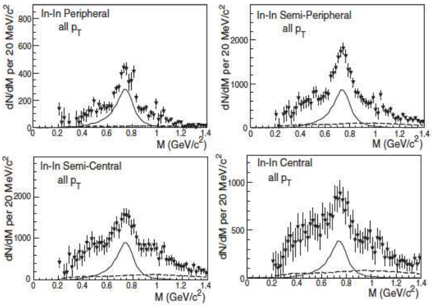


- High-quality NA60 data

Clear change in the ρ peak at SPS: Width grows, mass remains

- ✓ Spectral function shows a clear peak at the nominal ρ mass
- ✓ Peak broadens for the most central collisions
- ✓ Total dilepton yield also increases with centrality

NA60, Phys. Rev. Lett. **96**, 162302 (2006)



What about a_1 at finite T and μ_B?

There is no equivalent experimental information on the thermal properties of a_1

Need theoretical link between QCD and hadron properties

Weinberg Sum Rules: Finite Temperature

- Relate **vector** and **axial** current correlators

$$\begin{aligned}\Pi_{\mu\nu}(q_0^2,\mathbf{q}^2) &= i\int d^4x e^{iq\cdot x}\langle\mathcal{T}[J_{\mu A,V}(x)J^\dagger_{\nu A,V}(0)]\rangle \\ &= -q^2\left[\Pi^T_{A,V}(q_0^2,\mathbf{q}^2)P^T_{\mu\nu}+\Pi^L_{A,V}(q_0^2,\mathbf{q}^2)P^L_{\mu\nu}\right]\end{aligned}$$

$$W_1=\int_0^\infty ds\frac{1}{\pi}\left(\mathrm{Im}\Pi^T_V-\mathrm{Im}\Pi^T_A\right)=2f_\pi^2$$

$$W_2=\int_0^\infty ds\ s\frac{1}{\pi}\left(\mathrm{Im}\Pi^T_V-\mathrm{Im}\Pi^T_A\right)=0$$

To relate QCD properties to hadron properties

Gell-Mann–Oakes–Renner formula,

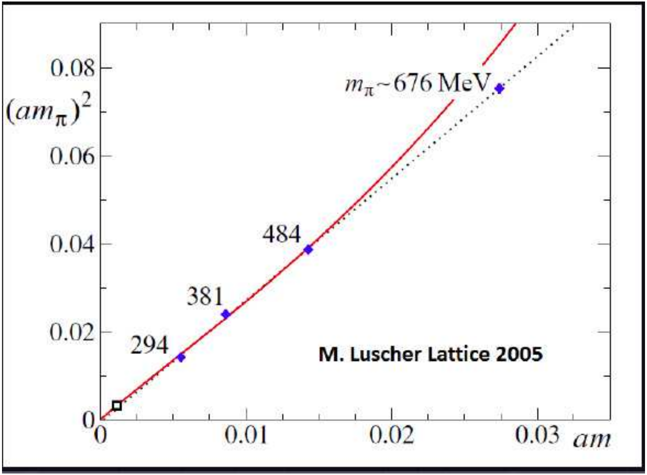
$$f_\pi^2 m_\pi^2 = -\frac{1}{2}(m_u + m_d) \langle 0 | \bar{\psi}_u \psi_u + \bar{\psi}_d \psi_d | 0 \rangle$$

$$f_\pi = 93 MeV,\ m_\pi = 139 MeV,\ m_u + m_d \approx 14 MeV$$

$$\langle 0 | \bar{\psi}_u \psi_u | 0 \rangle = \langle 0 | \bar{\psi}_d \psi_d | 0 \rangle = \langle 0 | \bar{\psi}_s \psi_s | 0 \rangle$$

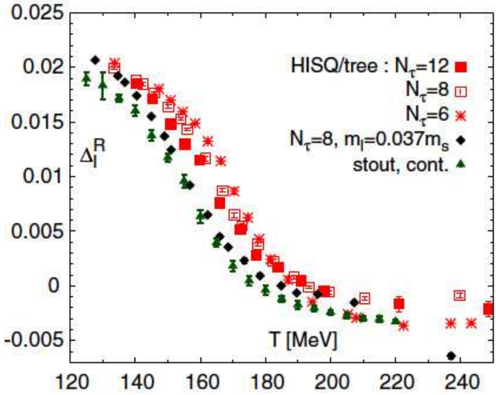
$$= -(225 MeV)^3 = -1.5 fm^{-3}$$

GMOR on the lattice



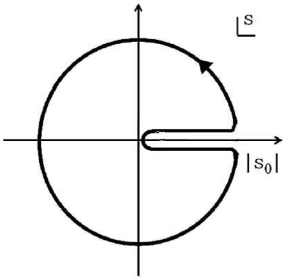
Quark-antiquark condensate

HotQCD Collaboration, Phys. Rev. D **85**, 0545031 (2012)



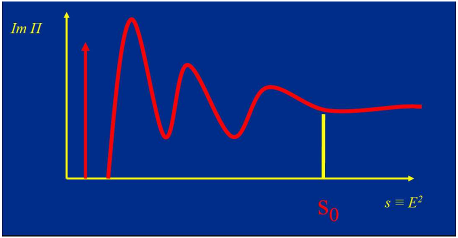
Finite Energy QCD Sum Rules

- ✓ Quantum field theory based on OPE of current-current correlators and Cauchy's theorem on complex energy squared-plane
- ✓ Relates hadron spectral function to QCD condensates and fundamental degrees of freedom (quark-hadron duality)
- ✓ Finite Energy refers to finite radius of integration s_0 called the energy squared-threshold for the continuum



Melting of resonances

- ✓ Hadron spectral function made out of resonances plus a continuum
- ✓ At finite temperature/density, s_0 decreases. Resonances melt
- ✓ FESR allow exploring how the resonance parameters change with temperature/density



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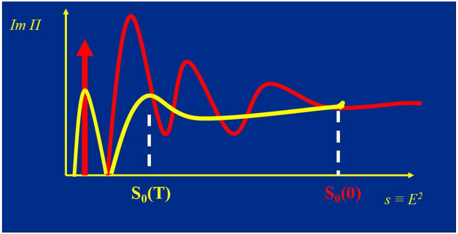


Finite Energy QCD Sum Rules

- Current correlator at finite temperature

$$\begin{aligned}\Pi_{\mu\nu}(q_0^2, \mathbf{q}^2) &= i \int d^4x e^{iq\cdot x} \langle \mathcal{T}[J_\mu(x) J_\nu^\dagger(0)] \rangle \\ &= -q^2 \left[\Pi^T(q_0^2, \mathbf{q}^2) P_{\mu\nu}^T + \Pi^L(q_0^2, \mathbf{q}^2) P_{\mu\nu}^L \right]\end{aligned}$$

- Work in the limit $\mathbf{q} \to 0$ where $\Pi_{\mu\nu}$ contains only spatial components
- Integrating the function $\frac{s^N}{\pi} \Pi^T(s \equiv q_0^2)$ in the complex s–plane along a contour with a fixed radius $|s| = s_0$

$$\frac{1}{2\pi i} \oint_{C(|s_0|)} ds \; s^N \Pi^T(s) = -\frac{1}{\pi} \int_0^{s_0} ds \; s^N \mathrm{Im} \Pi^T(s).$$

Finite Energy QCD Sum Rules

- The integrand on the right-hand side can be written entirely in terms of hadronic degrees of freedom.
- The integrand on the left-hand side can be written entirely in terms of QCD degrees of freedom, using the OPE, as

$$\Pi^{\text{QCD}}(s) = \sum_{M=0} \frac{C_{2M}\langle O_{2M}\rangle}{(-s)^M}.$$

- The term with $M = 0$ corresponds to the perturbative (pQCD) contribution. The FESR are

$$\begin{aligned}(-1)^{N+1} C_{2N}\langle O_{2N}\rangle &= 8\pi^2 \left[\frac{1}{\pi}\int_0^{s_0} ds s^{N-1} \text{Im}\Pi_0^{\text{had}}(s) \right. \\ &- \left. \frac{1}{\pi}\int_0^{s_0} ds s^{N-1} \text{Im}\Pi_0^{\text{pQCD}}(s) \right]\end{aligned}$$

Weinberg Sum Rules: Finite Temperature

$$W_1 = \int_0^\infty ds \frac{1}{\pi} \left(\mathrm{Im}\Pi_V - \mathrm{Im}\Pi_A \right) = 2 f_\pi^2$$

$$W_2 = \int_0^\infty ds\ s \frac{1}{\pi} \left(\mathrm{Im}\Pi_V - \mathrm{Im}\Pi_A \right) = 0$$

- These become FESR

$$W_1 = \int_0^{s_0} ds \frac{1}{\pi} \left(\mathrm{Im}\Pi_V - \mathrm{Im}\Pi_A \right) = 2 f_\pi^2$$

$$W_2 = \int_0^{s_0} ds\ s \frac{1}{\pi} \left(\mathrm{Im}\Pi_V - \mathrm{Im}\Pi_A \right) = 0$$

Use Finite Energy QCD Sum Rules to describe **Vector** spectral density

- ρ-saturation and BW form

$$\frac{1}{\pi}\mathrm{Im}\Pi_0^{\mathrm{had}}(s) = \frac{1}{\pi}\frac{1}{f_\rho^2}\frac{M_\rho^3\Gamma_\rho}{(s-M_\rho^2)^2+M_\rho^2\Gamma_\rho^2},$$

- **Three** leading FESR ($N = 1, 2, 3$)

$$\begin{aligned}(-1)^{N+1}C_{2N}\langle O_{2N}\rangle &= 8\pi^2\left[\frac{1}{\pi}\int_0^{s_0} ds s^{N-1}\mathrm{Im}\Pi_0^{\mathrm{had}}(s)\right.\\ &\quad - \left.\frac{1}{\pi}\int_0^{s_0} ds s^{N-1}\mathrm{Im}\Pi_0^{\mathrm{pQCD}}(s)\right]\end{aligned}$$

Finite Energy QCD Sum Rules: Finite Temperature

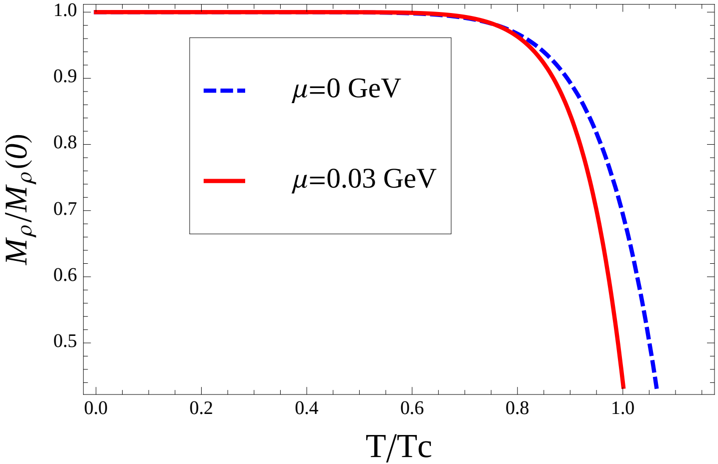
- **Three** leading FESR, **six** unknowns
- Strategy: provide expected behavior of three unknowns based on experience from other channels
- Choose $\Gamma_\rho(T)$, $M_\rho(T)$ and $C_6\langle O_6\rangle(T)$ as inputs

$$\begin{aligned} \Gamma_\rho(T) &= \Gamma_\rho(0)\left[1-(T/T_c)^3\right]^{-1}, \\ C_6\langle O_6\rangle(T) &= C_6\langle O_6\rangle(0)\left[1-(T/T_q^*)^8\right], \\ M_\rho(T) &= M_\rho(0)\left[1-(T/T_M^*)^{10}\right], \end{aligned}$$

$\Gamma_\rho(0) = 0.145$ MeV, $C_6\langle O_6\rangle(0) = -0.951667$ GeV^6 and $M_\rho(0) = 0.776$ GeV, $T_c = 0.197$ GeV, $T_q^* = 0.187$ GeV and $T_M^* = 0.222$ GeV

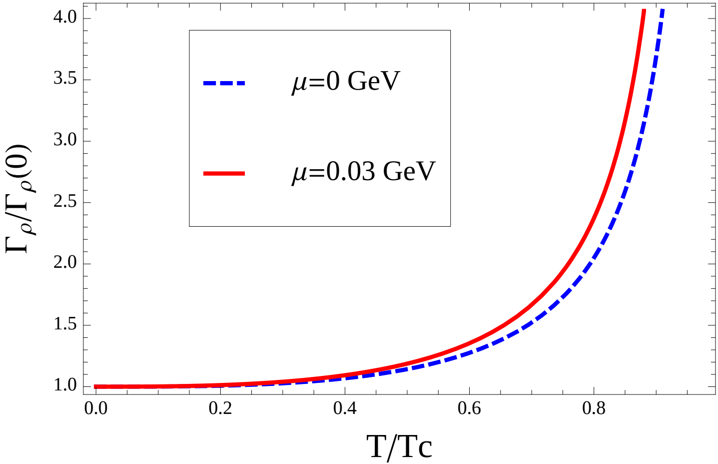
- Solve for $f\rho(T)$, $s_0(T)$ and $C_4\langle O_4\rangle(T)$ [A.A., C.A. Dominguez, M. Loewe, Y. Zhang, Phys. Rev. D **86**, 114036 (2012)]

$M_\rho(T,\mu)$

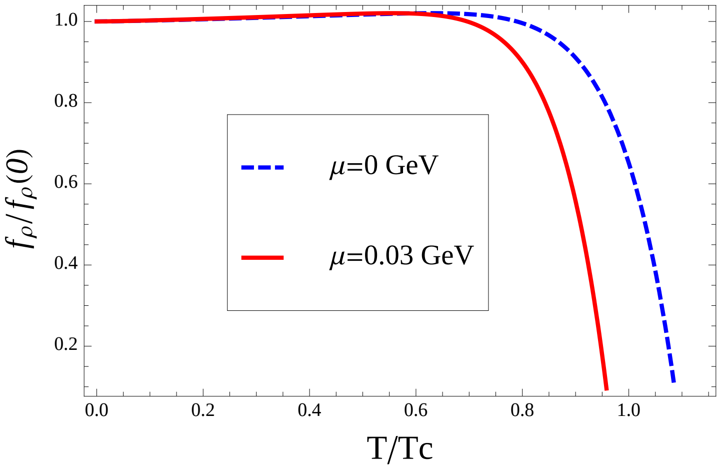


1.0
0.9
0.8
0.7
0.6
0.5
0.0
0.2
0.4
0.6
0.8
1.0
μ=0 GeV
μ=0.03 GeV
$M_\rho/M_\rho(0)$
T/Tc

$\Gamma_\rho(T, \mu)$

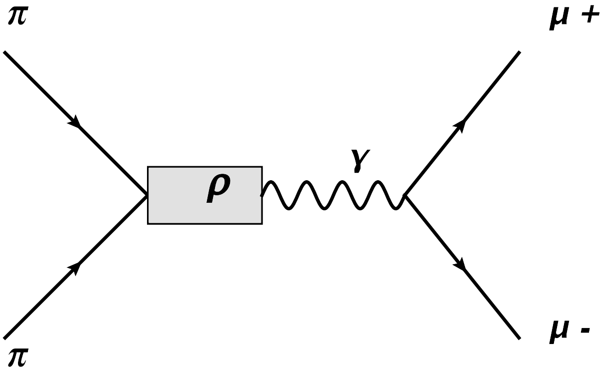


$f_\rho(T, \mu)$

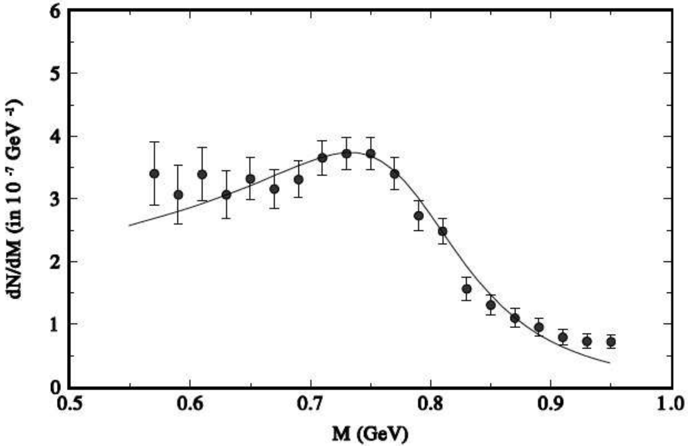


Test: computing dilepton rate at the ρ peak

- Consider processes where pions anhilate into ρ's which in turn decay into dimuons by vector dominance



Comparison with NA60 data



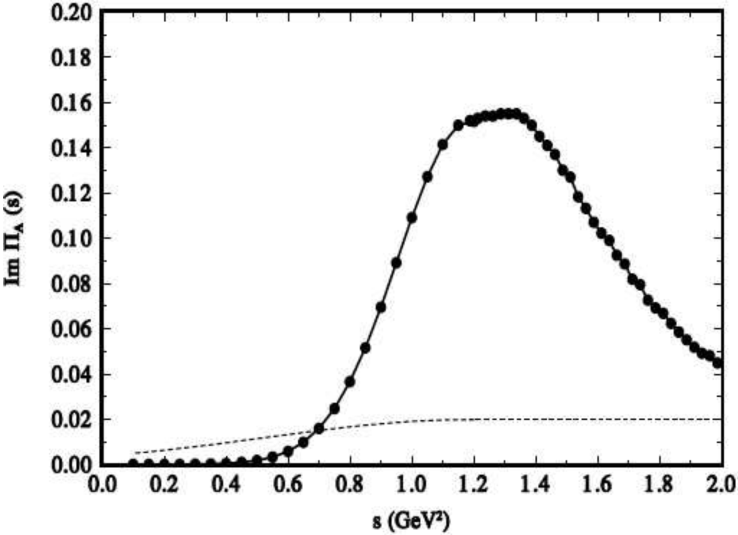
Use Finite Energy QCD Sum Rules to describe **Axial** spectral density

- a_1-saturation and Gaussian piece-wise form

$$
\begin{aligned}
\frac{1}{\pi}\mathrm{Im}\Pi_A(s) &= Cf_{a_1}\exp\left[-\left(\frac{s-M_{a_1}^2}{\Gamma_{a_1}^2}\right)^2\right] \\
(0 \leq\ & s \leq 1.2\ \mathrm{GeV}^2) \\
\frac{1}{\pi}\mathrm{Im}\Pi_A(s) &= Cf_{a_1}\exp\left[-\left(\frac{1.2\ \mathrm{GeV}^2-M_{a_1}^2}{\Gamma_{a_1}^2}\right)^2\right] \\
(1.2\ \mathrm{GeV}^2 \leq\ & s \leq 1.45\ \mathrm{GeV}^2) \\
\frac{1}{\pi}\mathrm{Im}\Pi_A(s) &= Cf_{a_1}\exp\left[-\left(\frac{s-M_{a_1}^2}{\Gamma_{a_1}^2}\right)^2\right] \\
(1.45\ \mathrm{GeV}^2 \leq\ & s \leq m_\tau^2)
\end{aligned}
$$

$$M_{a_1} = 1.230\mathrm{GeV} \quad \Gamma_{a_1} = 0.560\mathrm{GeV} \quad C = 0.662 \quad f_{a_1} = 0.073$$

Fit to ALEPH data



Finite temperature a_1 parameters

- Solving the FESR's s_0 **turns out** to be identical to the one in the vector channel

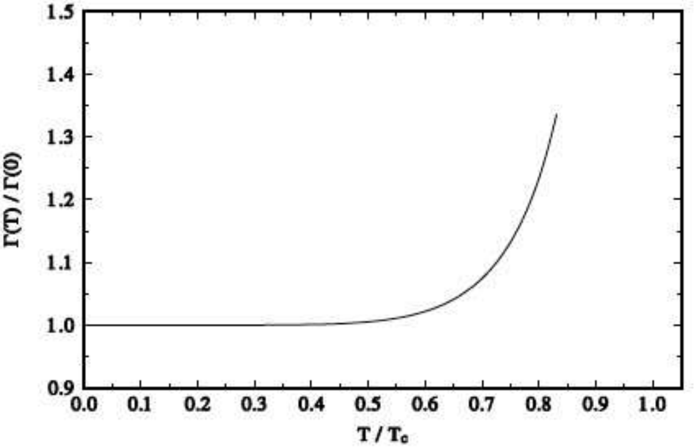
$$s_0 = 1.44 \text{ GeV}^2$$

- The **finite temperature** results for $s_0(T)$, $f_\pi(T)$, $f_{a_1}(T)$ and $\Gamma_{a_1}(T)$ are written generically as

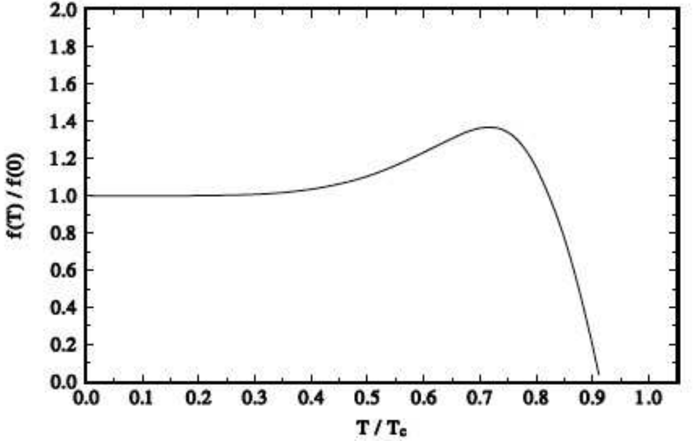
$$Y(T) = Y(0)\left(1 + a_1(T/T_c)^{b_1} + a_2(T/T_c)^{b_2}\right)$$

Parameter	a_1	a_2	b_1	b_2
$s_0(T)$	- 28.5	-0.6689	35.60	3.93
$f_\pi(T)$	- 0.2924	- 0.7557	73.43	11.08
$f_{a_1}(T)$	- 19.34	14.27	7.716	6.153
$\Gamma_{a_1}(T)$	2.323	1.207	20.24	7.869

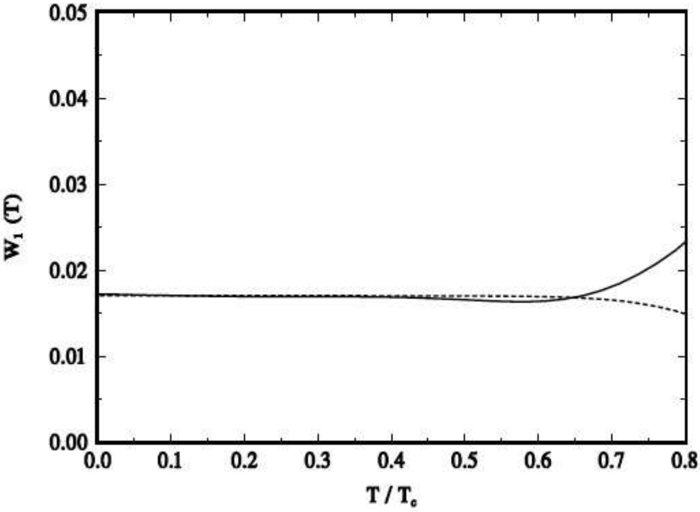
a_1 width as a function of temperature



a_1 weak coupling as a function of temperature

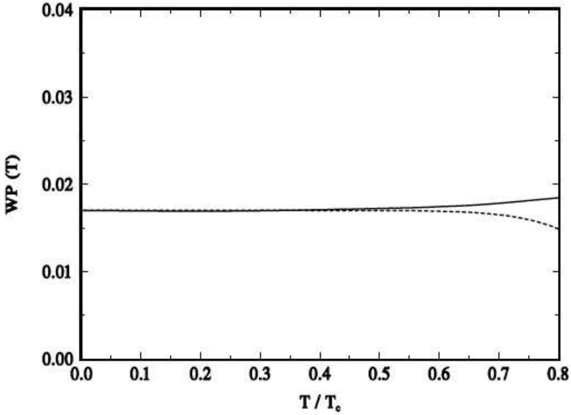


Weinberg Sum Rule 1



Combining WSR1 and WSR2 obtain WSR with **pinched kernel**

$$WP = \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right) \frac{1}{\pi} \left(\mathrm{Im}\Pi_V - \mathrm{Im}\Pi_A\right) = 2f_\pi^2$$



DISCUSION

- Is there any sign of **mixing of vacuum spectral densities?**

$$\begin{aligned}\Pi_V(q,T) &= (1-\epsilon(T))\Pi_V(q,0)+\epsilon(T)\Pi_A(q,0)\\ \Pi_A(q,T) &= (1-\epsilon(T))\Pi_A(q,0)+\epsilon(T)\Pi_V(q,0)\\ \epsilon(T) &= \frac{T^2}{6f_\pi^2}\ \text{ from } \chi\text{PT}\end{aligned}$$

[M. Dey, V. L. Eletsky, and B. L. Ioffe, Phys. Lett. B 252, 620 (1990); J. I. Kapusta and E.V. Shuryak, Phys. Rev. D 49, 4694 (1994); N. P. M. Holt, P. M. Hohler and R. Rapp, Phys. Rev. D 87, 076010 (2013); P. M. Hohler, R. Rapp, Nucl. Phys. A 892 (2012) 58.]

- Our findings show that the **parameters** describing the vector and axial spectral densities **evolve independently** from each other at finite T
- What seems to matter is general features such as **diverging widths** and vanishing s_0 at $T_c$as well as **constant mass** up to T close to T_c
- Important to further elaborate on these issues to clarify properties of spectral densities and thus for the detailed understanding of the approach to chiral symmetry restoration