Direct-photon spectra and flow in Pb–Pb collisions at the LHC measured with the ALICE experiment

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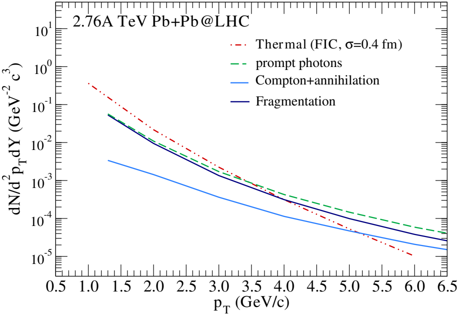
Outline

1. Introduction to Direct Photon Measurements
2. Measurement of Direct Photon Spectrum
3. Direct Photon v_2 Measurement
4. Alternative Representation of Direct Photon Flow
5. Inclusive Photon v_3 Measurement

ideal hydro th. photon v_2 for different QGP formation times τ_0
arXiv:0809.0548 [nucl-th]



Additional sources Pb–Pb collisions

Thermal Photons

- Scattering of thermalized particles
- Exponentially decreasing but dominant at low p_T

Jet-Medium Interactions

- Scattering of hard partons with thermalized partons
- In-medium (photon) bremsstrahlung emitted by quarks

pp & Pb–Pb collisions

Prompt Photons

- Calculable within NLO pQCD
- Dominant at high p_T
- γ leaves medium unaffected $\Rightarrow$ ideal probe
- Test of binary scaling in Pb–Pb

Direct Photon Flow

Initial azimuthal asymmetry in coordinate space in non-central A+A
$\Rightarrow$ asymmetry in momentum space

$$\frac{\mathrm{d}N}{\mathrm{d}\varphi} = \frac{1}{2\pi}\left(1 + 2\sum_{n\geq 1} v_n \cos(n(\varphi - \Psi_n^{RP}))\right)$$

- v_2: elliptic flow, collective expansion at low p_{T}
- v_3: triangular flow

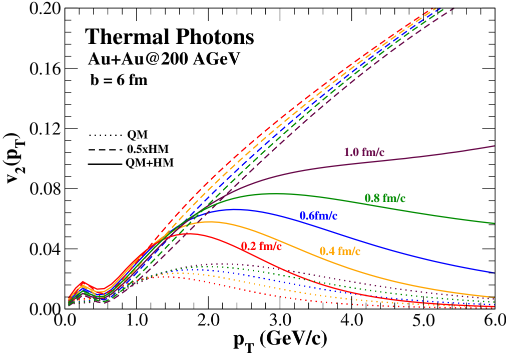
Thermal Photon v_2

- Constrains onset of direct photon production
  - Early production $\rightarrow$ small v_2
  - Late production $\rightarrow$ hadron-like v_2

Thermal Photon v_3

- Allows to distinguish different initial conditions & exotic models

arXiv:0809.0548 [nucl-th]



Direct Photon Transverse Momentum Spectrum

Direct Photon Extraction

Subtraction Method:

$$\gamma_{\text{direct}} = \gamma_{\text{inc}} - \gamma_{\text{decay}} = (1 - \frac{\gamma_{\text{decay}}}{\gamma_{\text{inc}}}) \cdot \gamma_{\text{inc}}$$
$$= (1 - \frac{1}{R_\gamma}) \cdot \gamma_{\text{inc}}$$

- Inclusive photons: measure all photons that are produced
- Decay photons: calculated from measured particle spectra with photon decay branches (π^0, η, ...)

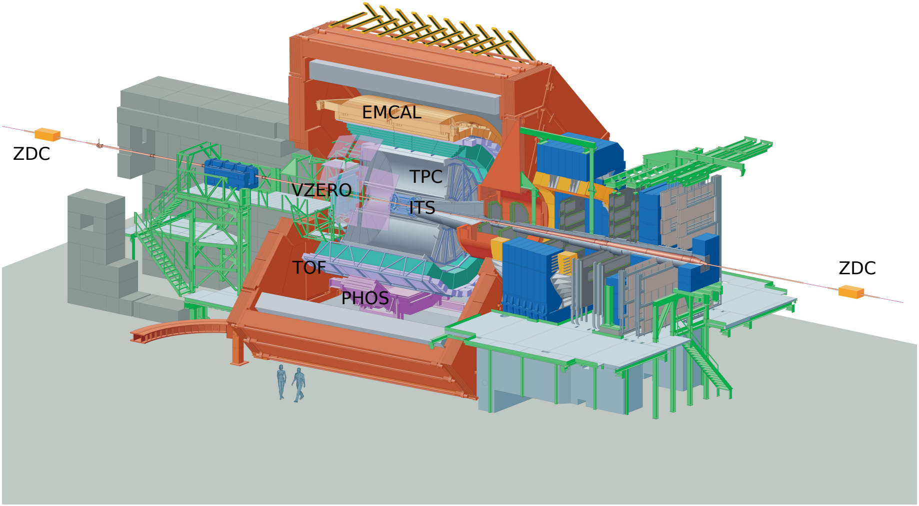
Double Ratio:

$$R_\gamma = \frac{\gamma_{\text{inc}}}{\pi^0} \Big/ \frac{\gamma_{\text{decay}}}{\pi^0_{\text{param}}} \approx \frac{\gamma_{\text{inc}}}{\gamma_{\text{decay}}} \qquad \text{if} > 1 \text{ direct photon signal}$$

→ advantage of ratio method: cancellation of uncertainties

- Numerator: Inclusive γ spectrum per π^0
- Denominator: Sum of all decay photons per π^0
  Decay photons are obtained by a cocktail calculation

Measuring photons, π^0 and η Mesons in ALICE



Measuring Photons, π^0 and η Mesons with PCM

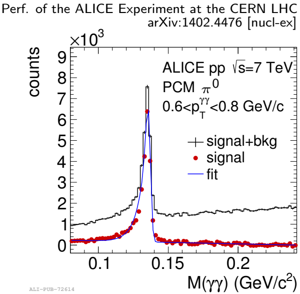
Photon Conversion Method (PCM)

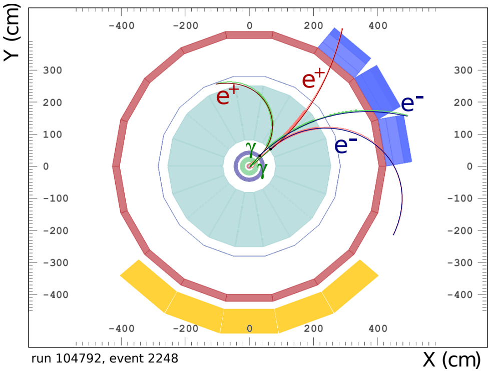
$pp \to \pi^0 + X_n$, $\pi^0 \to \gamma\gamma$, $\gamma\gamma \to e^+e^- e^+e^-$

$(m_{\pi^0} = 0.135 \text{ GeV}/c^2, BR_{\gamma\gamma} = 0.988)$

$pp \to \eta + X_n$, $\eta \to \gamma\gamma$, $\gamma\gamma \to e^+e^- e^+e^-$

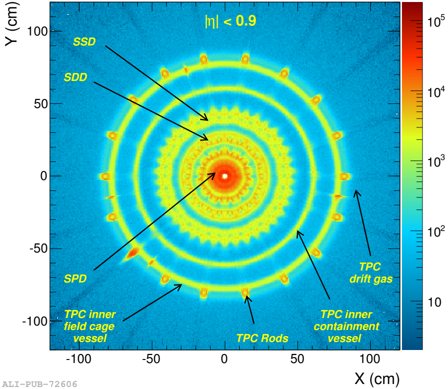
$(m_{\eta} = 0.548 \text{ GeV}/c^2, BR_{\gamma\gamma} = 0.393)$

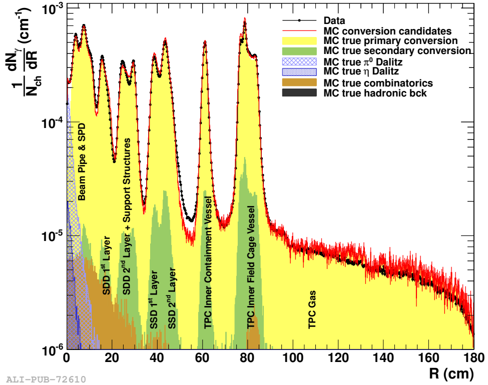




- High resolution ($\sigma_{\pi^0} < 2$ MeV$/c^2$) at very low p_T ($0.3 < p_T < 2$ GeV$/c$)
- High momentum reach limited only by statistics
- Conversion probability ($\sim 8.5\%$), acceptance: $|\eta| < 0.9$, $0 < \varphi < 2\pi$

γ - Ray Tomography of ALICE

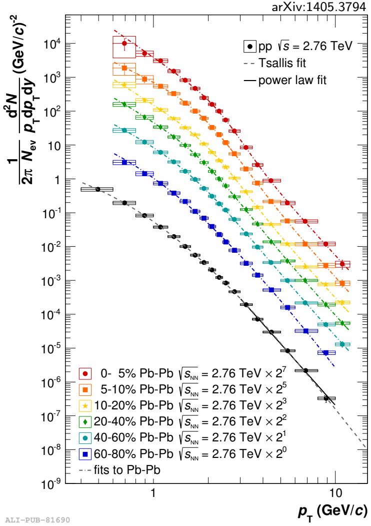


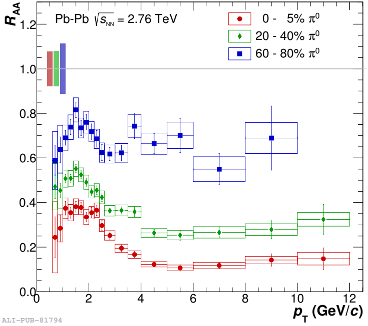


Performance of the ALICE Experiment at the CERN LHC
arXiv:1402.4476 [nucl-ex]

- Very useful tool to check the material budget:
  - Effective radiation length: $X/X_0 = 0.114 \pm 0.005$ $(|\eta| < 0.9, R < 180\,\mathrm{cm})$
  - Final systematic error is $\sim 4.5\%$
- Cuts on the decay topology of photons and electron track properties
  $\rightarrow$ Purity at 90% at $2\,\mathrm{GeV}/c$ for 0-40% Pb–Pb events
- Background is mainly combinatorial - Strange particle contribution negligible

π^0 Transverse Momentum Spectra & R_{AA}





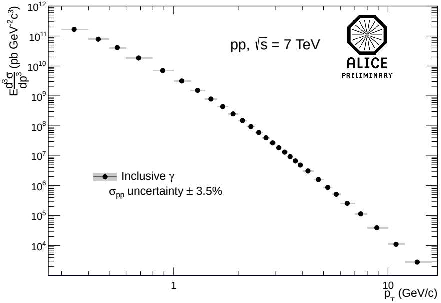
- π^0 measurement needed as input for R_γ,
- Statistical & systematic uncertainties of π^0 measurement dominate uncertainties on the R_γ
- Size of excess in R_γ depends on R_{AA} of π^0 $\rightarrow$ suppression of main source of decay γ
- Extraction of direct photons easier in more central events

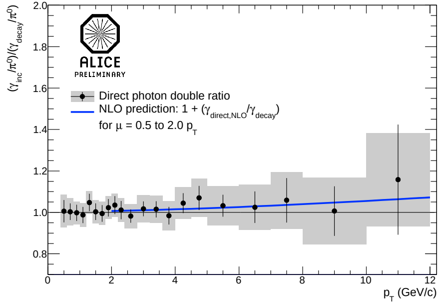
Direct photons in pp collisions at $\sqrt{s} = 7$ TeV

- Inclusive γ spectrum corrected for:
  - purity ($\mathcal{P}$), efficiency ($\mathcal{E}$), conversion probability ($\mathcal{C}$), secondary photon candidates
- In the ratio uncertainties related to:
  - normalization, π^0 measurement, rec. efficiency

  partially or exactly canceled
- The NLO double ratio prediction is plotted as $\mathcal{R}_{NLO} = 1 + \frac{\gamma_{direct,NLO}}{\gamma^{decay}_{cocktail}}$
- Measurement is consistent with the expected direct photon signal
- Integrated luminosity for measurement ~ 5 nb^{-1}

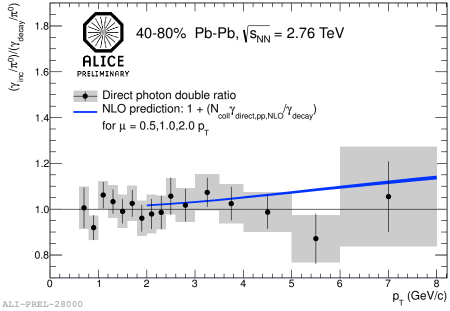
Direct photon signal in pp at 7 TeV is consistent with zero





Nucl.Phys. A904-905 (2013) 573c-576c

40-80%

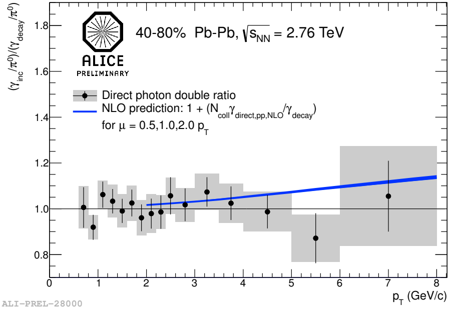


Double ratio for peripheral events shows no excess at any value of p_T

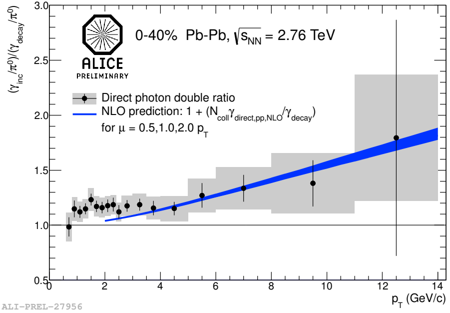
- Measurement is consistent with the expected direct photon signal
- pp NLO predictions scaled with N_{coll}

Double Ratio - Pb–Pb 2.76 TeV

40-80%



0-40%



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- Measurement is consistent with the expected direct photon signal
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Excess of $20\% \pm 5\%^{stat} \pm 10\%^{syst}$ for $p_T < 4\,\mathrm{GeV/c}$

- N_{coll} scaled pp NLO in agreement with high p_T direct photons

Definition of Excess in Central Pb–Pb collisions

Experimental definition of Direct Photons:

- Every photon which is not directly produced by: π^0, η, ω, η', ϕ, ρ^0 and Σ^0
- Decay photons simulated via a cocktail calculation based on measured yield of π^0 (Pb–Pb, pp) and η (pp), remaining spectra are obtained from m_T scaling of measured π^0

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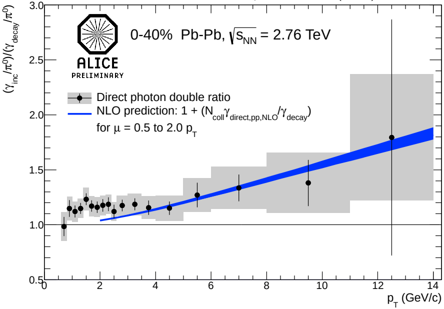

Experimental measurement of π^0:

- Published π^0 measurements contain feed-down from higher mass particles going to π^0, except π^0 from K^0_s
- Measured spectra are taken as input for cocktail calculation

Cocktail Generation

Decay photon spectra are obtained via calculation

- Based on a fit to measured π^0 (Pb–Pb, pp) and η (pp)
- Other particle spectra obtained via m_T-scaling of measured π^0
- Incorporated mesons: π^0, η, η', ω, ϕ, ρ_0 and the Σ^0 baryon

m_T-Scaling:
Same shape of cross sections, $f(m_\mathrm{T})$, of various mesons

$$E\frac{d^3\sigma_m}{dp^3} = C_m \cdot f(m_\mathrm{T})$$

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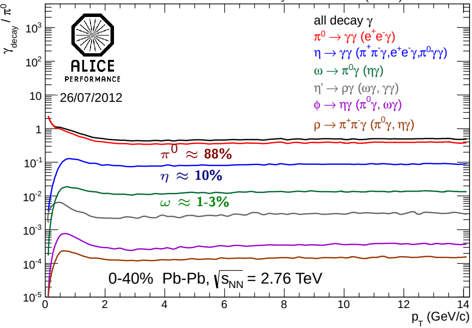
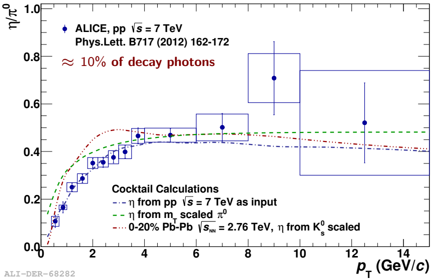


Meson (C_m)	meas.	Mass	Decay Branch	B. Ratio
π^0	pp, Pb–Pb	134.98	$\gamma\gamma$ $e^+e^-\gamma$	98.789% 1.198%
η (0.48)	pp	547.3	$\gamma\gamma$ $\pi^+\pi^-\gamma$ $e^+e^-\gamma$	39.21% 4.77% $4.9 \cdot 10^{-3}$
ρ^0 (1.0)		770.0	$\pi^+\pi^-\gamma$ $\pi^0\gamma$	$9.9 \cdot 10^{-3}$ $7.9 \cdot 10^{-4}$
ω (0.9)	pp	781.9	$\pi^0\gamma$ $\eta\gamma$	8.5% $6.5 \cdot 10^{-4}$
η' (0.25)		957.8	$\rho^0\gamma$ $\omega\gamma$ $\gamma\gamma$	30.2% 3.01% 2.11%
ϕ (0.35)	pp, Pb–Pb	1019.5	$\eta\gamma$ $\pi^0\gamma$ $\omega\gamma$	1.3% $1.25 \cdot 10{-}3$ $<$ 5%
Σ^0 (1.0)		1192.6	$\Lambda\gamma$	100%

Phys. Rev. C (arXiv:1110.3929)

Test of Assumptions for Cocktail



- η & ω meson only measured in pp, φ meson measured in pp & 0-10% Pb–Pb collisions
- m_T scaling overestimates yield at low p_T consistently for all 3 mesons
- Collective flow in Pb–Pb collisions modifies shape of spectra, thus m_T scaling might not be a valid approximation especially at low p_T
- Systematic uncertainties on cocktail 5-10%
- Aim to measure η & ω meson at low p_T in Pb–Pb collisions

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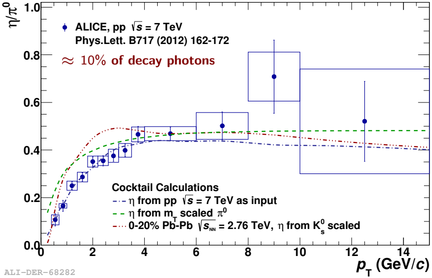

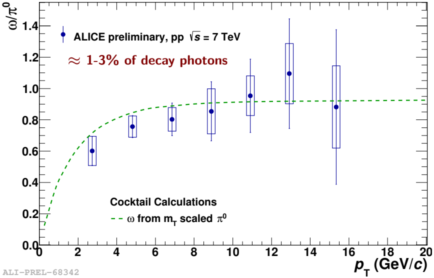

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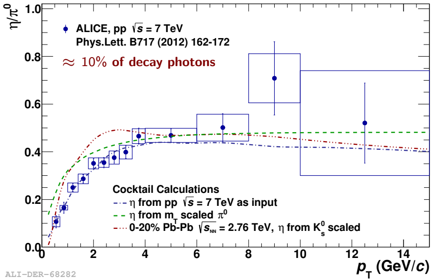

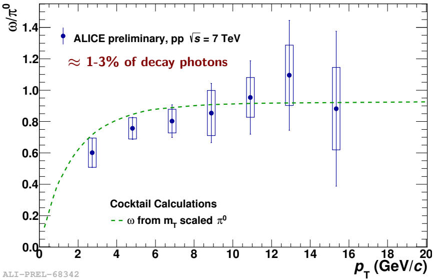

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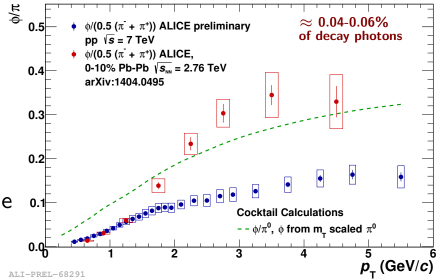

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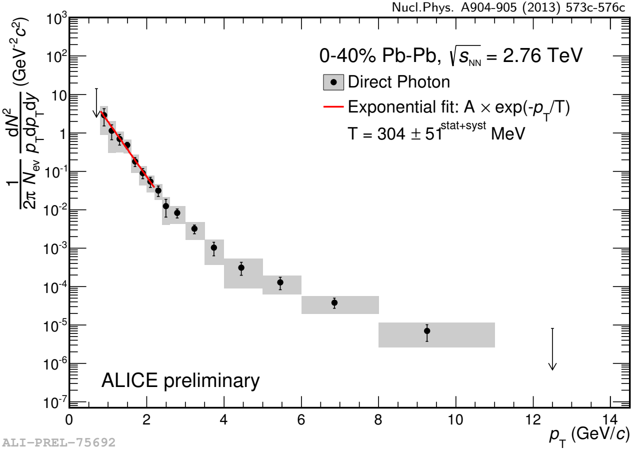
Results of Pb–Pb Direct Photons at 2.76 TeV



Direct Photon Spectrum for central Pb–Pb events

Spectrum derived from double ratio by:

$$\gamma_{direct} = \left(1 - \frac{1}{R_\gamma}\right) \cdot \gamma_{inc}$$

- Systematic uncertainties on the double ratio are partially correlated in p_T, Significance of direct photon signal depends on degree of correlation
- Easiest example for fully correlated uncertainties: Material bugdet uncertainty (absolute 4.5% of double ratio)

Direct Photon Flow

$$v_2^{\text{direct }\gamma} = \frac{R_\gamma \cdot v_2^{\text{inc }\gamma} - v_2^{\text{decay }\gamma}}{R_\gamma - 1}$$

- $R_\gamma \cdot v_2^{\text{inc }\gamma}$: weighted inclusive photon v_2 due to extra photons compared to background
- $v_2^{\text{decay }\gamma}$: calculated decay photon v_2 from cocktail calculation

Inclusive Photon v_2 Analysis Method

Initial azimuthal asymmetry in coordinate space in non-central A+A
$\Rightarrow$ asymmetry in momentum space

$$\frac{\mathrm{d}N}{\mathrm{d}\phi} = \frac{1}{2\pi}\left(1 + 2\sum_{n\geq 1} v_n \cos(n(\phi - \Psi_n^{RP}))\right)$$

v_2 given by photon production with respect to event plane

$$v_2 = \langle\cos(2(\phi - \Psi_2^{RP}))\rangle$$

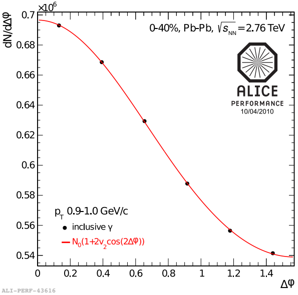
Event Plane angle determined by using the VZERO detector

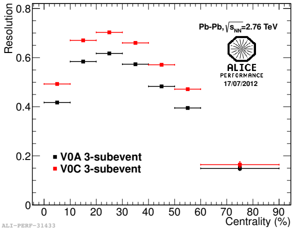
- VZEROA: $2.8 < \eta < 5.1$
- VZEROC: $-3.7 < \eta < -1.7$

Reaction plane resolution obtained by the three sub-event method

Resolution correction for EP:

$$v_2 = \frac{v_2^{EP}}{\langle\cos(2\Psi_2^{EP} - \Psi_2^{RP}))\rangle} = \frac{v_2^{\mathrm{raw}}}{\text{resolution}}$$





J.Phys.Conf.Ser. 446 (2013) 012028

Cocktail Simulation of Decay Photon v_2

Decay photon v_2:

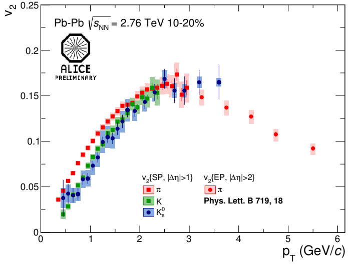
- KE_T scaling: v_2 of mesons scales with KE_T
  $KE_T = m_T - m = \sqrt{p_T^2 + m^2} - m$

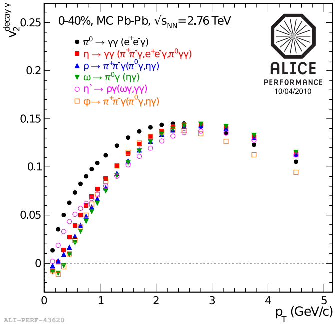
$\Rightarrow$ $v_2^{\pi^0} \approx v_2^{\pi^\pm}$ $(m^{\pi^0} \approx m^{\pi^\pm})$

$\rightarrow$ v_2 of various mesons (X) calculated via KE_T (quark number) scaling from $v_2^{\pi^\pm}$

$$v_2^X(p_T^X) = v_2^{\pi^\pm}\left(\sqrt{(KE_T^X + m^{\pi^\pm})^2 - (m^{\pi^\pm})^2}\right)$$

- Decay photon v_2 from different mesons obtained from cocktail calculation





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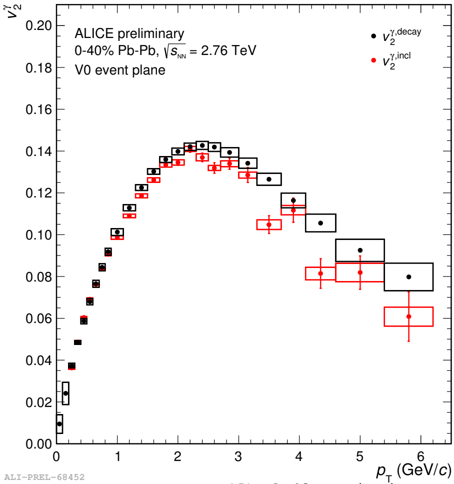
Comparison of Inclusive and Decay v_2

- Above 3 GeV/c inclusive photons significantly smaller than decay photons

→ Direct photon v_2 contribution with $v_2^{\mathrm{direct}} < v_2^{\mathrm{decay}}$

- Below 3 GeV/c consistent within uncertainties

→ Either contribution of direct photons with similar v_2 or no direct photons



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Direct Photon v_2 0-40%

Direct photon v_2:

$$v_2^{\text{direct }\gamma} = \frac{R_\gamma \cdot v_2^{\text{inc }\gamma} - v_2^{\text{decay }\gamma}}{R_\gamma - 1}$$

- $R_\gamma \cdot v_2^{\text{inc }\gamma}$: weighted inclusive photon v_2 due to extra photons compared to background
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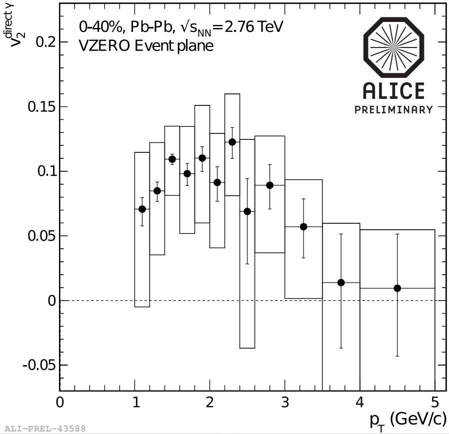

- Large direct photon v_2 for $p_T < 3\,\mathrm{GeV}/c$ measured
- Magnitude of v_2 comparable to hadrons
- Result points to late production times of direct photons after flow is established

Direct Photon Yield and Flow - Puzzle ?

- Central points for direct photon yield and v_2 underestimated by most theoretical calculations by factors of 2-10
- No significant deviation beyond 2σ

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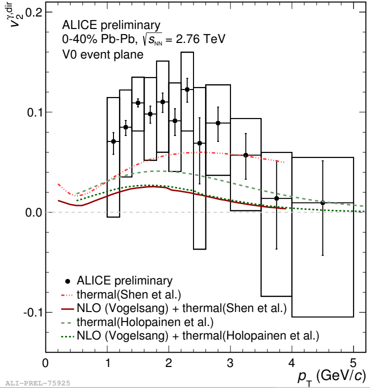

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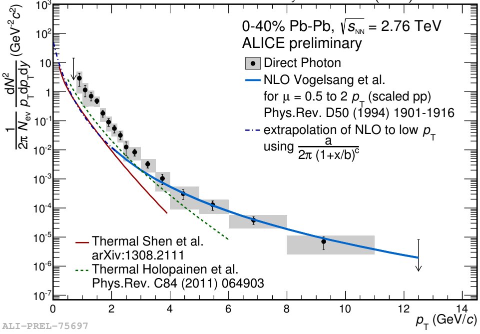


- Both measurements are coupled via R_γ, critical assessment of uncertainties and their correlations needed
- Theory curves composed out of different sources, experimentally not possible to distinguish those

Propagation and Correlation of Errors on the R_γ

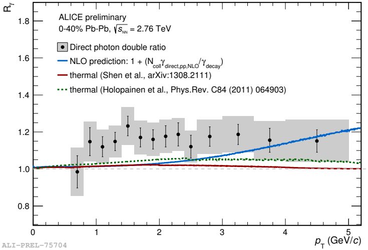
- Measured R_γ less than $2\sigma_{\mathrm{sys}}$ deviation from 1
- Gaussian error propagation only applicable if:
  a) Relation between observable and input observables is linear or
  b) Uncertainties sufficiently small

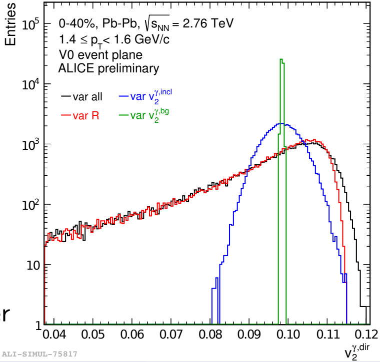
  both conditions not fulfilled

$$\frac{\partial v_n^{\gamma,\,\mathrm{dir}}}{\partial R_\gamma} = \frac{v_n^{\gamma,\,\mathrm{decay}} - v_n^{\gamma,\,\mathrm{inc}}}{(R_\gamma - 1)^2}$$

- Errors for $v_n^{\gamma,\,\mathrm{dir}}(p_\mathrm{T})$ calculated using MC simulation with probability distributions according to $R_\gamma(p_\mathrm{T})$, $v_n^{\gamma,\,\mathrm{decay}}(p_\mathrm{T})$, $v_n^{\gamma,\,\mathrm{inc}}(p_\mathrm{T})$ within 4 $\sigma(p_\mathrm{T})$ of respective uncertainties
- p_T correlated uncertainty, like material budget (4.5%), complicates error propagation

→ Evaluation of significance of R_γ and $v_n^{\gamma,\,\mathrm{dir}}$ under investigation





Alternative Representation of Direct Photon Flow

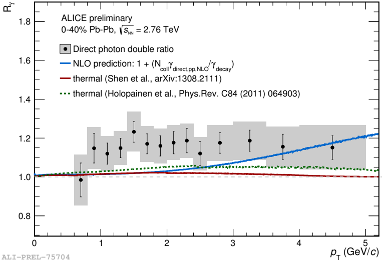
Comparison of

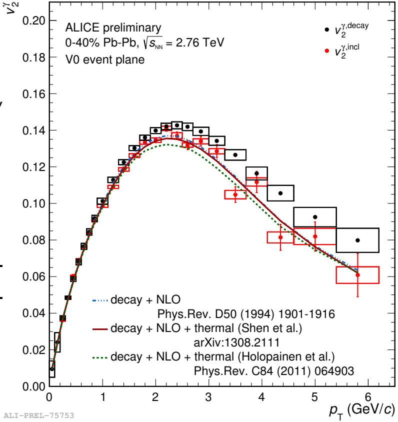
$$\left(v_{n,\ \text{measured}}^{\text{incl}\ \gamma} - v_{n}^{\text{model}\ \gamma}\right)/\sigma^{\text{tot.}}$$

for various models, where model could be:

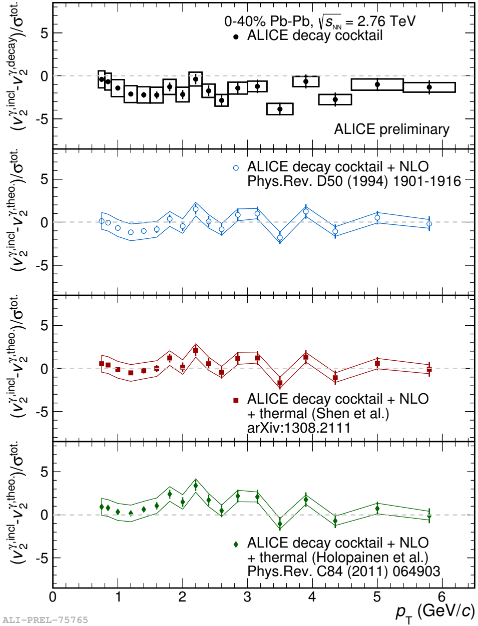
- $v_{n,\ \text{decay}}$ based on measured π data
- $v_{n,\ \text{decay}}$ based on measured π data $\cdot\, w_{\gamma,\ \text{decay}}$
  $+\ v_{n,\ \text{NLO}} \cdot w_{\gamma,NLO}$
  $+\ v_{n,\ \text{thermal}} \cdot w_{\gamma,\ \text{thermal}}$
- $v_{n,\ \text{incl}}$ from full theory calculation

Allows decoupling of measured R_{γ} from comparison, large discrepancy of central points in R_{γ} between theory and data taken out



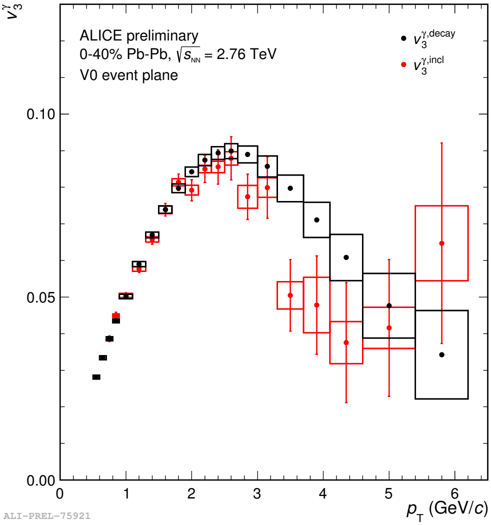


Comparison of Inclusive Photon v_2



- Deviations from 0 for data, mainly explained by contribution from prompt photons
- Region of interest for thermal sources: 1-3 GeV/c
  Large systematic uncertainties
- No statement on the existence of direct photon puzzle can be made by ALICE at this stage

New Measurement: Inclusive Photon v_3



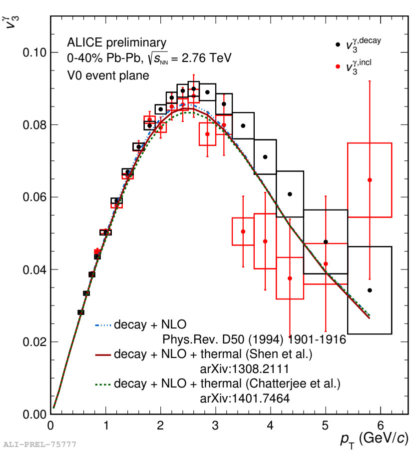
- First measurement of inclusive photon v_3 at LHC
- Above 3 GeV/c inclusive photons consistently smaller than decay photons, with large statistical uncertainties

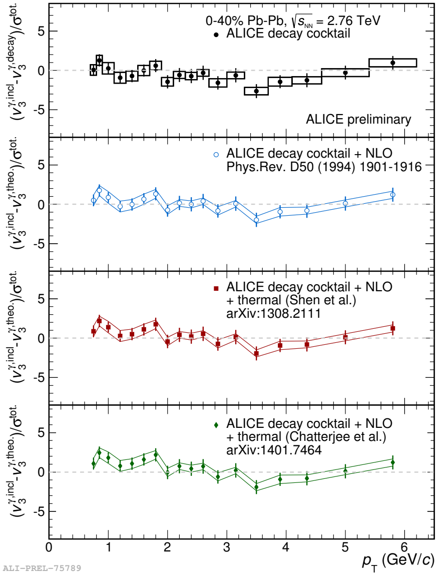
→ Direct photon v_3 contribution with $v_3^{\mathrm{direct}} < v_3^{\mathrm{decay}}$ as expected for prompt photons

- Below 3 GeV/c mostly consistent within uncertainties

→ Either contribution of direct photons with similar v_3 or no direct photons

Comparison of Inclusive Photon v_3





- Very small contribution from thermal v_3
- No significant deviation from 0 in region of interest between 1-3 GeV/c

Summary

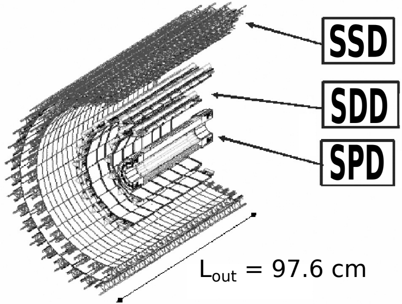
- $R_\gamma \approx 1.2 \pm 0.05^{\text{stat}} \pm 0.1^{\text{syst}}$ has been measured by ALICE in 0-40% Pb–Pb collisions
- Direct photon yield extracted with an exponential slope of $T = 304 \pm 51^{\mathrm{stat+syst}}\,\mathrm{MeV}$
- Direct photon v_2 which is of similar size as the charged hadron flow has been measured in 0-40% Pb–Pb collisions
- First measurement of inclusive photon v_3 at the LHC in 0-40% Pb–Pb collisions
- Current uncertainties on R_γ, $v_n^{\gamma\text{incl}}$ & $v_n^{\gamma\text{decay}}$ do not allow statement on the existence of a direct photon puzzle at LHC energies

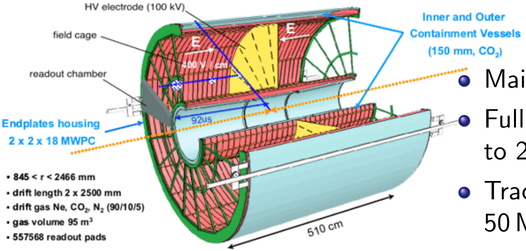
Backup Slides

Closer Look at the Central Barrel - ITS and TPC

Inner Tracking System - ITS

- Full azimuth coverage, six cylindrical layers
- Three different detector types: silicon pixel / drift / stripes
- Designed for primary / secondary vertex finding (inner radius $R_{BP} = 2.94\,\mathrm{cm}$)
- Tracks charged particles down to $p_{\mathrm{T}} = 100\,\mathrm{MeV/c}$





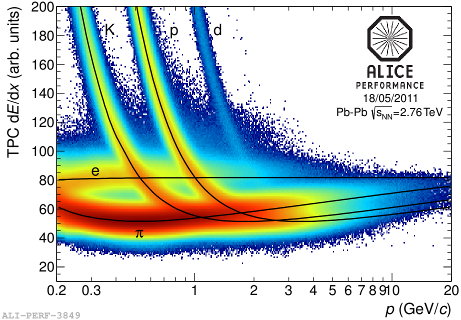
Time Projection Chamber - TPC

- Main tracking and PID detector
- Full azimuth coverage, $R = 84.8\,\mathrm{cm}$ up to $246.6\,\mathrm{cm}$
- Tracking: 100 MeV/c (primary) or 50 MeV/c (secondary) up to 100 GeV/c

Electron Selection Criteria

Global Electron Selection Criteria

- Both tracks originate from the same V0 candidate
- No kinks
- Opposite charge
- Small *R* cut ($R < 5\,\mathrm{cm}$)
- TPC refit condition
- Minimum momentum of 50 MeV/c
- Minimum fraction of the TPC clusters with respect to findable clusters due to conversion radius



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PID Based Selection Criteria

- nσ around electron energy loss hypothesis in the TPC $\mathrm{d}E/\mathrm{d}x$
- TOF electron nσ selection

  (if information available)
- After PID $\sim$ 80% pure photon sample

Photon χ^2/ndf:

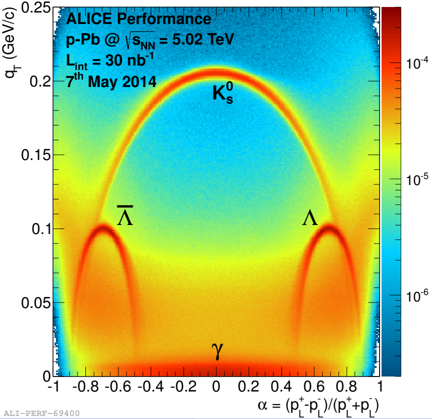
- Based on a Kalman-Filter (AliKFParticle package)
- Measure for conversion likelihood: includes: zero V0 mass, pointing to primary vertex, correct electron mass, mutual secondary vertex

Further Photon Selection Criteria:

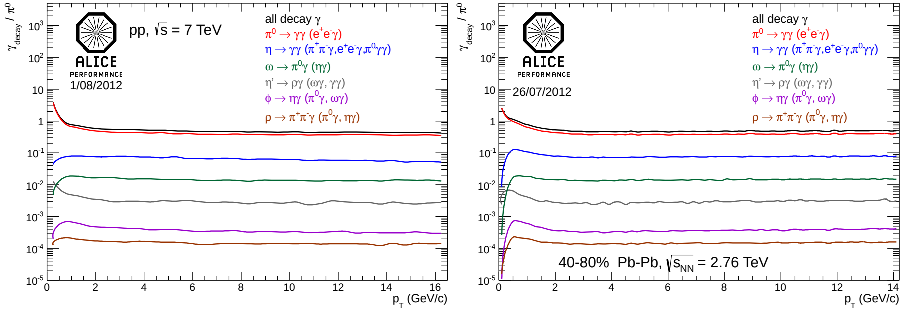
- Crosschecks for std. photon criteria
- Psi-Pair angle
  opening angle perpendicular to B field
- Cosine of pointing angle
  pointing to the primary vertex

Photon q_T:

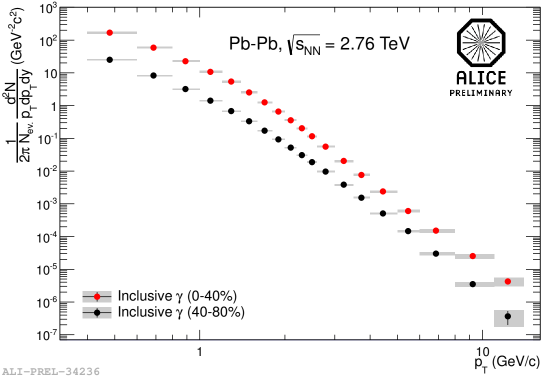
- Transv. mom. component of daughter relative to the V0
  $q_T = p \times \sin(\Theta_{\mathrm{mother-daughter}})$
- Clear separation of γ, Λ and K^0_s



Cocktail for peripheral Pb–Pb and pp collisions



Two centrality selections: 0-40% and 40-80%(central and peripheral)



Inclusive γ spectrum corrected for:

- purity ($\mathcal{P}$),
- efficiency ($\mathcal{E}$),
- conversion probability ($\mathcal{C}$),
- secondary photon candidates

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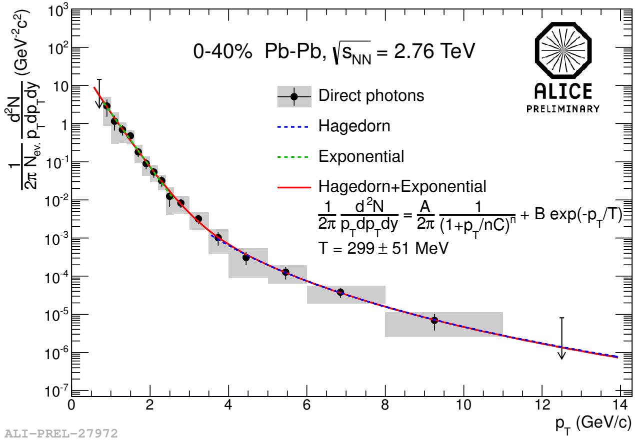

Combined fit (Hagedorn + Exponential) gives similar result for the inverse slope parameter T as for the exponential only fit

Systematic Error Sources R_γ pp

- Cut Variations for γ and π^0:

Cut Name	Std. value	Variation 1	Variation 2	Variation 3
Electron dEdx	-4,5σ	-4,4σ	-3,4σ	-
Pion dEdx	1,-10σ	2,1σ	2,0.5σ	2,0.5σ
Min. p e^+/e^-	0.4 GeV/c	0.4 GeV/c	0.4 GeV/c	0.3 GeV/c
Find. Cls. TPC	0.35	0.6	-	-
Photon χ^2	20	30	10	-
q_t	0.05	0.07	0.03	-
min. p_t e^+/e^-	50 MeV/c	75 MeV/c	100 MeV/c	-
photon η, π^0 y	0.9, 0.8	0.8, 0.7	1.2, 0.9	-
min. R	5 cm - 180 cm	2.8 cm - 180 cm	10 cm - 180 cm	-

  - V0s with shared electrons rejected
  - Purity for different centralities used
  - TOF and α cut not used for pp
  - R cut already considered for material budget

- π^0 yield extraction:
  - Three different integration windows
  - Different Numbers of mixed events for bg, different mixed event bins (n V0s, n tracks)

- Cocktail simulation:
  - Two different fits
  - Variation of the m_t scaling factors (η measured)

Systematic Error Sources R_γ Pb–Pb

- Cut Variations for γ and π^0:

Cut Name	Std. value	Variation 1	Variation 2	Variation 3
Electron dEdx	-3,5σ	-4,5σ	-2.5,4σ	-
Pion dEdx	3,-10σ	2.5,-10σ	3.5,-10σ	3,-10σ
Min. p e^+/e^-	0.4 GeV/c	0.4 GeV/c	0.4 GeV/c	0.3 GeV/c
Find. Cls. TPC	0.6	0.7	0.35	-
Photon χ^2	10	5	20	-
q_t	0.05	0.03	0.07	-
min. p_t e^+/e^-	50 MeV/c	75 MeV/c	100 MeV/c	-
photon η, π^0 y	0.75, 0.7	0.9, 0.8	0.8, 0.7	-
min. R	5 cm - 180 cm	2.8 cm - 180 cm	10 cm - 180 cm	-
α meson central	0.65	1.00	-	-
α meson peripheral	0.8	1.00	-	-
TOF	-5,-5σ	-3,-5σ	-2,-5σ	-

  - V0s with shared electrons rejected
  - Purity for different centralities used

- π^0 yield extraction:
  - Three different integration windows
  - Different Numbers of mixed events for bg, different mixed event bins (n V0s, n tracks)

- Cocktail simulation:
  - Two different fits, with and without blast wave
  - Variation of the m_t scaling factors