EM Spectral Functions from Lattice QCD: Status, Caveats and Perspectives

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Thermal Photons and Dileptons in Heavy-Ion Collisions - TPD2014
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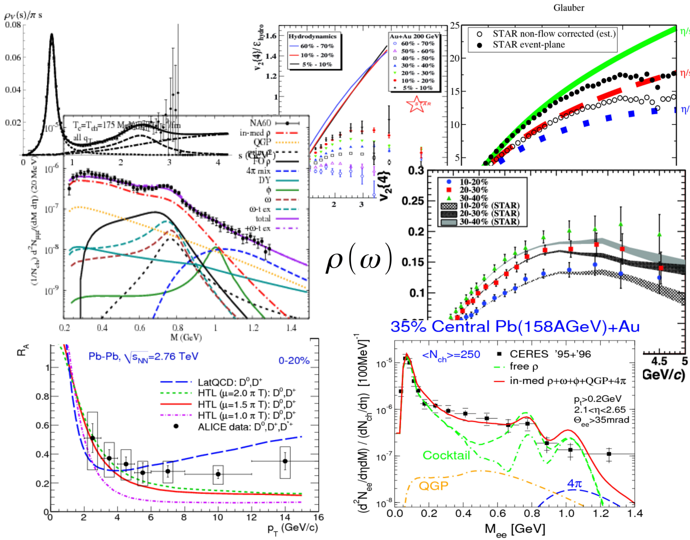
Outline

Introduction



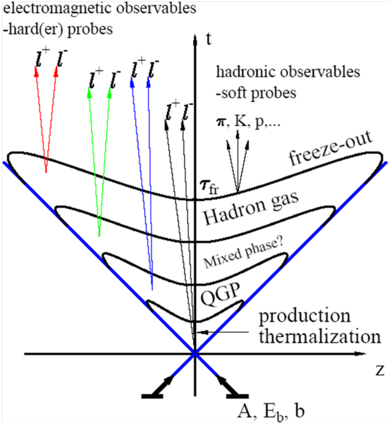
- Many (real-time) observables are linked to spectral functions.

Dileptons and the EM spectral funtion

- Dileptons are produced at every stage of a heavy-ion collision.
- Their production rate $\mathrm{d}N_{l^+l^-}/\mathrm{d}\omega$ is accessible via experiments .
- Their production rate is also available from theory:

$$\frac{\mathrm{d}N_{l^+l^-}}{\mathrm{d}\omega\mathrm{d}^3p} = \dots$$

$$\dots C_{em}\frac{\alpha_{em}^2}{6\pi^3}\frac{\rho(\omega,\vec{p},T)}{(\omega^2-\vec{p}^2)(\mathrm{e}^{\omega/T}-1)}$$



- $\rho(\omega,\vec{p},T)$ is the SPF of the electromagnetic current

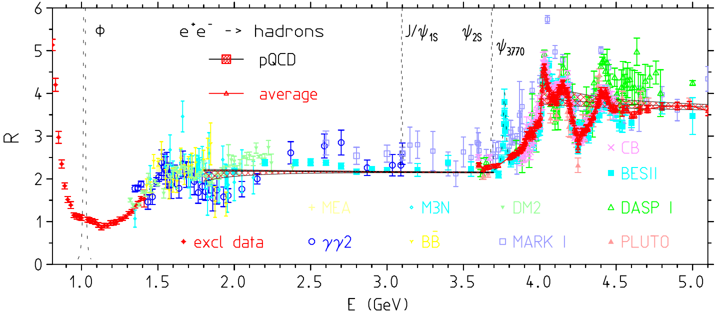
A first look: The vacuum

In vacuum, i.e. in the cold system, the EM SPF can be measured directly in experiment via $R(s) \propto \sigma(e^+e^- \to \mathrm{hadrons})$.

- Optical theorem:

$$\sigma(s) = \frac{R(s)}{12\pi} = \frac{\rho(\omega^2)}{\omega^2}$$

- HICs also include contributions from the hot stage of the event.



[0902.3360]

Cold and hot systems: The lattice perspective

What can we learn about the real-time physics encoded in the spectral functions from the Euclidean formulation of thermal field theory and especially lattice QCD?

How well can we control the reconstruction of the spectral function from non-perturbative lattice input data?

- **Here, I review a number of recent lattice studies using different approaches.**

  [1012.4963], [1109.3941], [1204.4945], [1212-2.4200], [1301-2.7436], [1307.6763], [1310.7466], [1402.6210]

- The meson masses covered will range from bottomonia to light vector mesons, i.e. the ρ.

What can we learn about the real-time physics encoded in the spectral functions from the Euclidean formulation of thermal field theory and especially lattice QCD?

How well can we control the reconstruction of the spectral function from non-perturbative lattice input data?

- **Connected observables in the vacuum...**
  - ...LO hadronic contribution to $(g-2)_\mu$ [1306.2532].
  - ...Time-like pion form factors [1105.1892].
  - not covered here.
- **Connected observables at finite temperature...**
  - ...Quarkonium dissociation [1402.1601], [1204.4945].
  - ...HQ diffusion and electrical conductivity [1311.3759], [1307.6763], [1212-2.4200].
  - ...Dilepton rates [1012.4963].
  - covered here.

Cold and hot systems: The lattice perspective

- The current-current correlator is given by

$$G_{\mu\nu}(\tau, T) = \int d^3x \langle J_\mu(\tau,\vec{x}) J_\nu(0,\vec{0})^\dagger \rangle$$

with the isospin current[1]:

$$J_\mu(\tau,\vec{x}) = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)$$

- For the lattice vector correlator $G_{ii}(\tau, \vec{p}, T = 1/\beta)$ the connection to the spf $\rho(\omega)$ is:

$$G_{ii}(\tau, \vec{p}, T) = \int_0^\infty \frac{d\omega}{2\pi} \frac{\cosh(\omega(\beta/2 - \tau))}{\sinh(\omega\beta/2)} \rho(\omega)$$

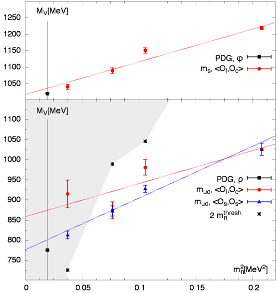
- The $T > 0$ correlator based on the $T = 0$ spf can be computed directly from the $T = 0$ correlator:

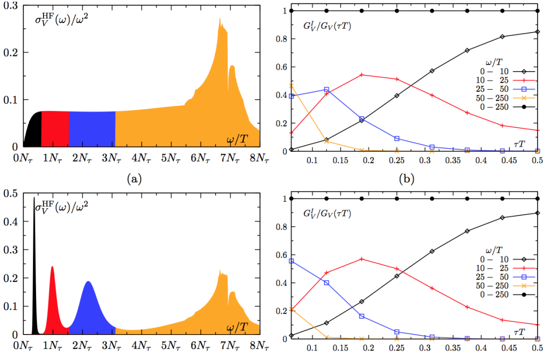
$$G^{rec}(\tau, T; T' = 0) = \sum_{m \in \mathcal{Z}} G(|\tau + m\beta|, T' = 0)$$

[1] Isospin $\approx$ EM: The disconnected part is negligible up to $\sim$ 1.5fm, Lat'14

Noise, lattice effects and the convolution integral

- $T = 0$, the signal is often lost before the ground state dominates $\Rightarrow$ Noise increases $\propto \exp(m_\pi)$ while signal goes $\propto \exp(m_\rho)$.
- $T > 0$, lattice extent is often too short.
- Most lattice setups do not allow for an unstable ρ-meson.





- Correlator is only sensitive to area under SPF.
- Low peak region at midpoint: ~85% for dissociated, ~90% for bound states.

[Wissel'06]

Expectations: Transport phenomena and bound-state dissociation

- A simple $T = 0$ SPF is the minimal hadronic Ansatz (MHA)

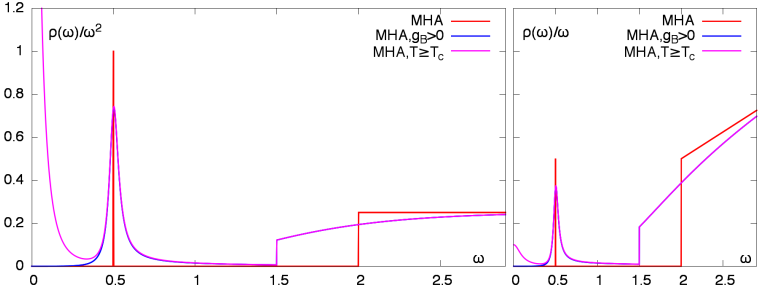
$$\frac{\rho_{MHA}(\omega)}{\omega^2} \sim A\delta(\omega - m) + B\Theta(\omega - s_0)$$

- At finite T: Modification of bound states and continuum threshold

$$\frac{\rho_{MHA}^{g_B>0}(\omega)}{\omega^2} \sim A' \frac{g \tanh(\omega\beta)^3}{(\omega-m)^2+g_B^2} + B'\Theta(\omega - s_0')\tanh(\omega\beta)$$

- Also, emergence of transport peaks around $\omega = 0$

$$\frac{\rho_{MHA}^{T\gtrsim T_c}(\omega)}{\omega^2} \sim A' \frac{g \tanh(\omega\beta)^3}{(\omega-m)^2+g_B^2} + B'\Theta(\omega - s_0')\tanh(\omega\beta) + \frac{C'}{\omega^2}\frac{\tanh(\omega\beta)}{(\omega/g')^2+1}$$



Basics of SPF reconstruction

Basics of SPF reconstruction

The process of reconstruction is an ill-posed problem

- Only a finite number of points is available to approximate a continuous function.
- Recall, in a typical spline interpolation[2]:
  $\Rightarrow$ Approximate F with $I_k = \sum_k c(k)B(k)$ by searching for $[D - I_k] = \min$.
- In the reconstruction only the transformed data $D = \mathcal{L}[F]$ is known:
  $\Rightarrow$ Minimize $[D - I_k] = \min$ with $I_k = \mathcal{L}[\sum_k c(k)B(k)]$ and approximate $F = \mathcal{L}^{-1}[F']$ given the known un-transformed basis functions and weights.
- Two possibilities:
  1. Fix the basis functions and minimize $I_k = \sum_k c(k)\mathcal{L}[B(k)]$.
  2. Define the basis functions by specific Ansätze $A(c(k'))$ that depend on a certain number of parameters, $I_{k'}^{Ansatz} = \mathcal{L}[A(c(k'))]$.

[2] F= True function; D = Data; I = Approximation to F; B(k) = Basis function; c(k) = weights

Basics of SPF reconstruction

- The maximum entropy method (MEM) uses the first epproach.
  - Based on Bayes theorem.
  - The minimization is extended to incorporate a Shannon-Jaynes-Entropy term.
  - The basis functions are fixed to

$$\frac{\rho(\omega)}{\omega} = m(\omega) \exp \sum_k c(k) B(k, \omega)$$

    where the prior information (or default model) $m(\omega)$ is introduced.
- Caveats and problems in the past:
  - Past: Divergent kernel.
  - Past: small lattice sizes, often staggered (further reduction of points).
  - Impact of default model?
  - Accuracy of data enough to converge close to true solution?
  - Interpretation of MEM artefacts in SPF result?

Basics of SPF reconstruction

- Recent analyses started using also the second approach.
  - The minimization is done as highlighted above.
  - The spectral function is determined by a fixed Ansatz depending on a number of parameters $c(k')$

$$\rho(\omega) = \rho_{Ansatz}(\omega, c(k'))$$

- Caveats:
  - Choice of Ansatz?
  - Dependence on Ansatz?
  - Insensitivity of correlator (deviation from δ-functions is very small)?

An incomplete review of recent lattice studies

An incomplete review of recent lattice studies

Lattice calculations have studied the SPF using EFT methods (NRQCD, HQET) and direct computations.

- Access to different phenomena in the SPF's:
  - Bound-states only (color: orange).
  - Transport only (color: brown).
  - Transport and bound-states (color: red).
- In the following recent lattice studies in all three categories are highlighted[3].

[3]Naturally incomplete, my sincere apologies to everybody not mentioned.

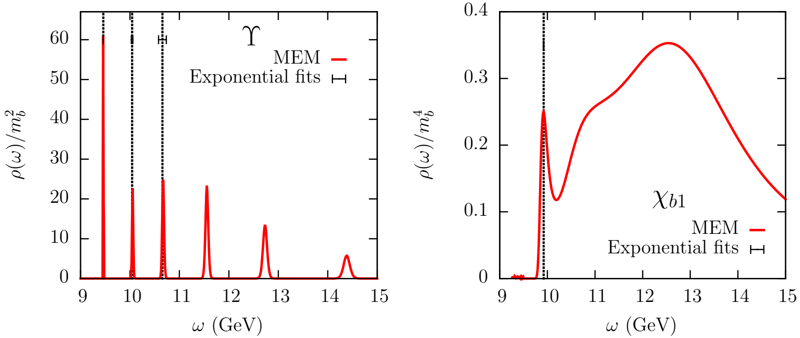
Bottomonia and very heavy systems

Bottomonia from NRQCD [1402.1601]

Bottomonium correlators are calculated using NRQCD on dynamical, anisotropic $N_f = 2 + 1$ Wilson-Clover ensembles.

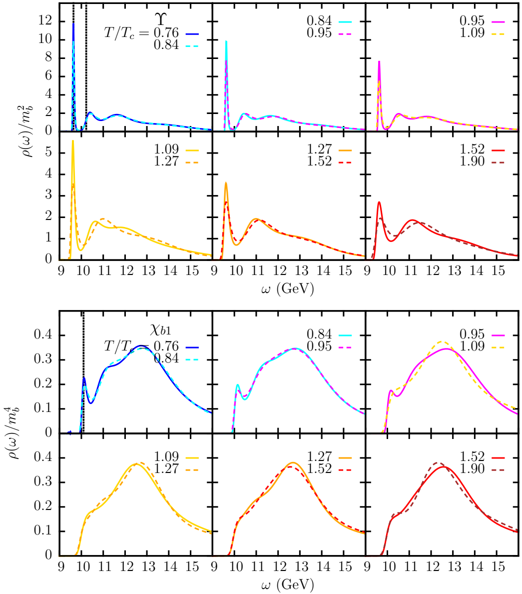
- Not including energies below the bound-state mass excludes transport phenomena.
- Reconstruction method: MEM.
- Pros:
  - Clean probe for bound-state dissociation of very heavy mesons
  - Temperature is varied via fixed-scale approach (N_t is changed in $T = \frac{1}{aN_t}$).
- Cons:
  - Cannot go down in mass to reach e.g. charmonia.
  - Bottomonium masses close to the typical lattice cut-offs, entangled results?

Bottomonia from NRQCD [1402.6210]



- NRQCD has to be tuned: Υ and χ_{b_1} results in the vacuum.
- Reconstruction via MEM.

Bottomonia from NRQCD [1402.6210]

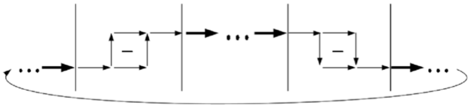


Results at finite T:

- ϒ exhibits ground state peak throughout.
- χ_{b_1} seems to dissociate ⇒ but cut-off close and large.
- Reconstruction via MEM.

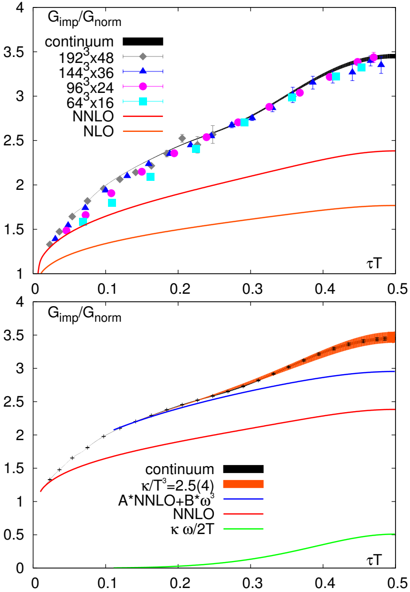
HQ diffusion from EFT [1311.3759], QM'14

The correlator of the electric field is computed on quenched ensembles and extrapolated to the continuum limit.



- The operator is purely gluonic and is defined only in HQET context $\Rightarrow$ No bound-states.
- Reconstruction method: Ansatz.
- Pros:
  - Clean probe for heavy-quark diffusion.
  - Multi-level algorithm can be used to beat exponential noise increase.
- Cons:
  - No clear way to extend to finite quark mass.
  - How to extend to dynamical QCD? How to renormalize different discretizations of the E-field?

HQ diffusion from EFT [1311.3759], QM'14



E-field correlator is extrapolated to the continuum limit.

G_{norm} is the tree-level (lattice spacing) improved, free correlator.

Ansatz for diffusion coefficient κ:

$$\rho(\omega) = \max\left[\mathrm{A}\rho_{\mathrm{NNLO}}(\omega) + \mathrm{B}\mathrm{w}^3, \frac{\omega\kappa}{2\mathrm{T}}\right]$$

- Ansatz based on NNLO and constant contribution for κ.
- Very good description of data.
- $\kappa/T^3 = 2.5(4)$.

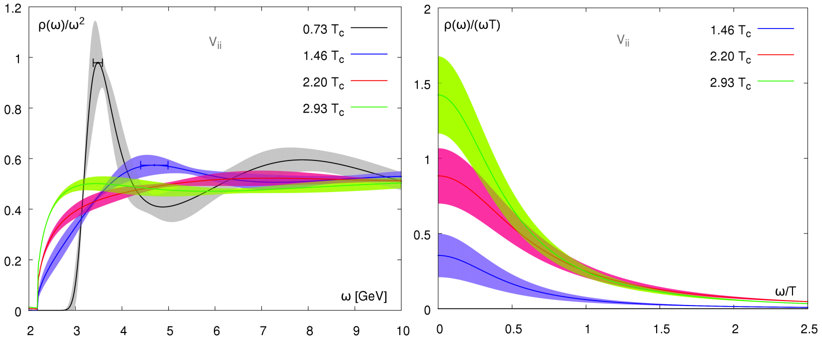
Charmonia

Direct computation of charmonia [1204.4945]

Charmonium correlators are calculated using full QCD on large, quenched ensembles.

- The calculation includes both transport and bound-state phenomena.
- Reconstruction method: MEM.
- Pros:
  - Full non-perturbative calculation.
  - Temperature is varied via fixed-scale approach (N_t is changed in $T = \frac{1}{aN_t}$).
- Cons:
  - Lattice spacing has to be fine enough to resolve charmonia.
  - Small lattice spacing requires large lattice volume $\Rightarrow$ Expensive.
    - Either: Live with quenched.
    - Or: Anisotropic lattices $\Rightarrow$ Entanglement with cut-off?
  - Have to disentangle transport and bound-state effects.

Direct computation of charmonia [1204.4945]



- Careful MEM with Ansatz based cross-checks (not shown here).
- Dissociation of J/Ψ around $1.46\,T_c^{quench}$.
- Emergence of transport peak with $\kappa/T^3 \sim 4-7$.
- Open question: How to reconcile with EFT computation?

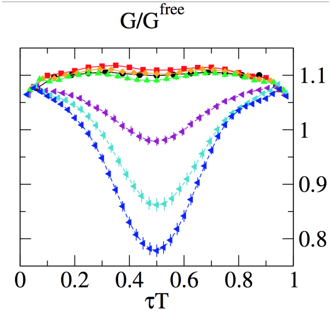
Light mesons

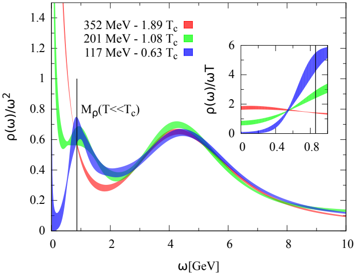
Electrical conductivity accross the phase transition [1307.6763], [1310.7466]

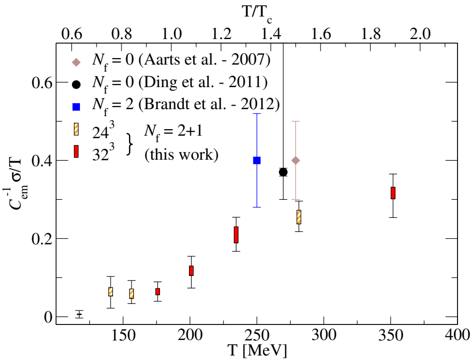
Conserved current correlators are calculated on anisotropic $N_f = 2 + 1$ Wilson-Clover ensembles with $m_\pi \sim 400\text{MeV}$ in $T/T_c \in [0.63 : 1.90]$

- The calculation includes both transport and bound-state phenomena.
- Reconstruction method: MEM.
- Pros:
  - Full non-perturbative calculation.
  - Temperature is varied via fixed-scale approach (N_t is changed in $T = \frac{1}{aN_t}$).
- Cons:
  - Large anisotropy $\xi = 3.5$.
  - Still small time extent $N_t \sim 24$ at $T \sim T_c$.
  - Have to disentangle transport and bound-state effects from small number of points ($N_t/2$).

Electrical conductivity accross the phase transition [1307.6763], [1310.7466]







- High statistics correlator data.
- Analysis can track σ_{el}/T accross the phase transition.
- No detailed info on ρ-dissociation.

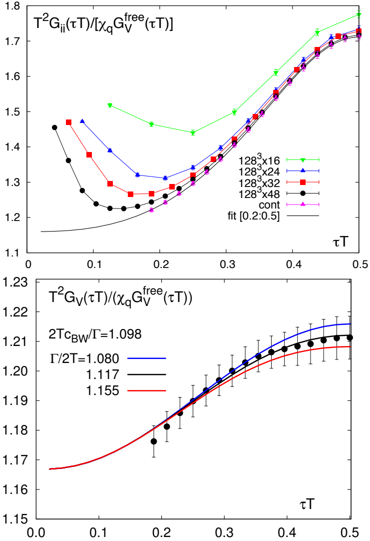
Electrical conductivity in the quenched continuum [1012.4963], [1301-2.7436], Lat'14

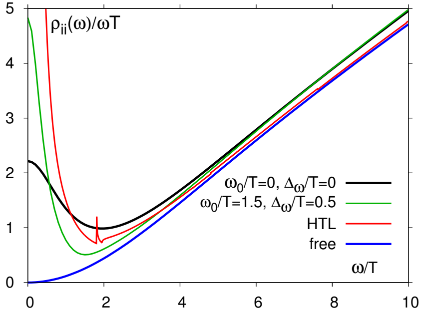
Local current correlators are calculated on quenched ensembles and the continuum limit is taken in $T/T_c \in [1.1 : 1.45]$.

- The calculation includes both transport and bound-state phenomena.
- Reconstruction method: Ansatz.
- Pros:
  - Full non-perturbative calculation.
  - Continuum limit is taken.
  - Very large lattices $N_t \in [48 : 64]$ and $N_s/N_t \in [2.5 : 3]$.
- Cons:
  - Quenched.
  - Only temperatures above T_c available and feasible.

Electrical conductivity in the quenched continuum [1012.4963], [1301-2.7436], Lat'14

First study to use the Ansatz at $T/T_c \sim 1.45$ in the continuum [1012.4963].





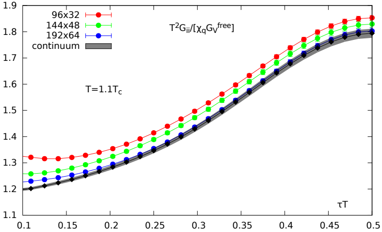
Ansatz for SPF:

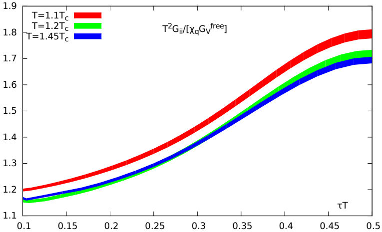
$$\rho(\omega) = \rho_{peak}(\omega) + k\rho_{free}(\omega).$$

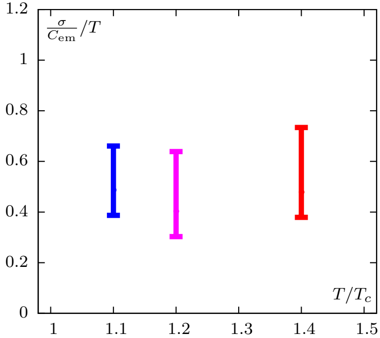
- Reliable calculation of σ_{el}/T.

Electrical conductivity in the quenched continuum [1012.4963], [1301-2.7436], Lat'14

Extended to larger lattices with $T \sim 1.1T_c$ and $T \sim 1.2T_c$ [1301-2.7436], Lat'14







Ansatz for SPF:

$$\rho(\omega) = \rho_{peak}(\omega) + k\rho_{free}(\omega).$$

- Reliable calculation of σ_{el}/T.
- No clear signature of ρ-meson.

EM current and sum rules [1212-2.4200]

Local current correlators are calculated on $N_f = 2$ Wilson-Clover ensembles with $m_\pi = 270$MeV and $T \sim 1.2T_c$.

- The calculation includes both transport and bound-state phenomena.
- A sum rule and $T = 0$ lattice data used to constrain the SPF.
- Reconstruction method: Ansatz.
- Pros:
  - Thermal modification directly in terms of deviation from vacuum(!)
  - Very large lattices $N_t = 16$ and $N_s/N_t = 4$ (for dynamical).
  - Low m_π, large $m_\pi L \sim 4$.
- Cons:
  - Only one lattice spacing.
  - So far, only one temperature ($T > T_c$) analyzed.

EM current and sum rules [1212-2.4200]

Local current correlators are calculated on $N_f = 2$ Wilson-Clover ensembles with $m_\pi = 270$MeV and $T \sim 1.2T_c$.

- The subtracted vector spf obeys the sum rule:

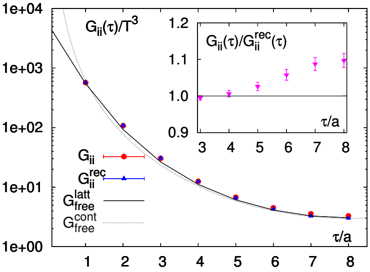
$$0 \equiv \int_{-\infty}^{\infty} \frac{d\omega}{\omega}(\rho_{ii}(\omega, T) - \rho_{ii}(\omega, 0)) = \int_{-\infty}^{\infty} \frac{d\omega}{\omega} \Delta\rho(\omega, T)$$

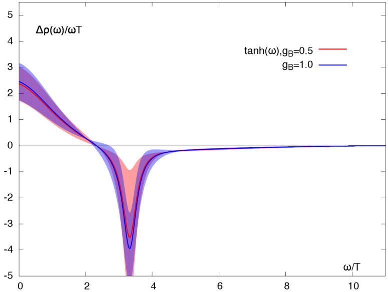
- The spf, $\rho(\omega, T = 0)$, in the reconstructed correlator constrains the shape of the finite temperature spf, $\rho(\omega, T > 0)$.
- Use this sum rule as additional constraint and fit to:

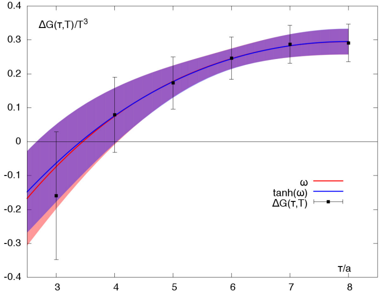
$$\Delta\rho(\omega, T) = \rho_{transport}(\omega, T) - \rho_{particle}(\omega, T) + \Delta\rho_{free}(\omega, T; T = 0)$$

- At this point: All thermal modification of the ρ-meson is forced into $\rho_{particle}(\omega, T) \Rightarrow$ Will be extended soon.

EM current and sum rules [1212-2.4200]







- Use the reconstructed correlator to form difference $\Delta G(\tau, T)$.
- In the fit: Width and mass of $\rho_{particle}(\omega, T)$ is fixed from $T = 0$ data.
- Reliable determination of σ_{el}/T.
- Significant spectral weight in the ρ-region, model-dependent!

Summary

Summary

Status

- A number of lattice groups are attacking the problem of spf reconstruction (at finite temperature).
- Electrical conductivities show good agreement over a number of lattice setups and reconstruction methods.
- HQ diffusion needs to be further explored, but new studies are underway.
- First results on bottomonium dissociation from lattice (NR)QCD.

Caveats

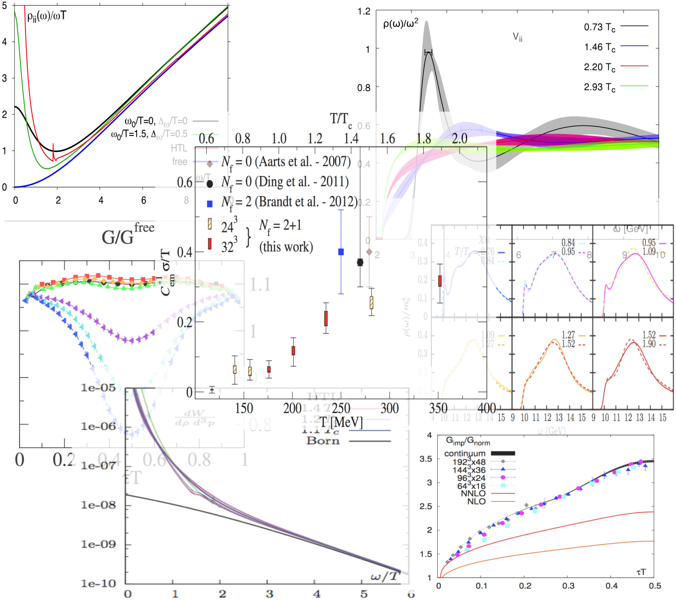
- SPF reconstruction is the central difficulty.
- Need high accuracy data and a large lattice sizes.
- Large scale (, expensive) lattice calculations required.

Summary

Perspectives

- New calculations on larger lattices are under way:
  - anisotropic $N_f = 2 + 1$, charmonia, shown at Lat'14.
  - quenched, isotropic bottomonium using full QCD, shown at Lat'14.
  - $N_f = 2$, $N_t = 24, 20, 12$ shown at QM'14.
- New reconstruction methods are being developed, shown at QM'14 and Lat'14.
- Improved codes to boost accuracy at reasonable computational cost.
- Theoretical understanding of finite momentum SPF is extending, shown at MITP 07 2014 $\Rightarrow$ Extend also lattice calculations.
- Also: Updates in vacuum vector spectroscopy from lattice QCD.

Lots to do!



ρii(ω)/ωT
ω0/T=0, Δω/T=0
ω0/T=1.5, Δω/T=0.5
HTL
free
ρ(ω)/ω²
Vii
0.73 Tc
1.46 Tc
2.20 Tc
2.93 Tc
T/Tc
Nf = 0 (Aarts et al. - 2007)
Nf = 0 (Ding et al. - 2011)
Nf = 2 (Brandt et al. - 2012)
24³
32³
Nf = 2+1
(this work)
C_em^-1 σ/T
T [MeV]
G/G^free
Born
ω/T
Gimp/Gnorm
continuum
192³x48
144³x36
96³x24
64³x16
NNLO
NLO
τT