Lattice QCD calculation of vector spectral functions and EM emission rates

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based on work with

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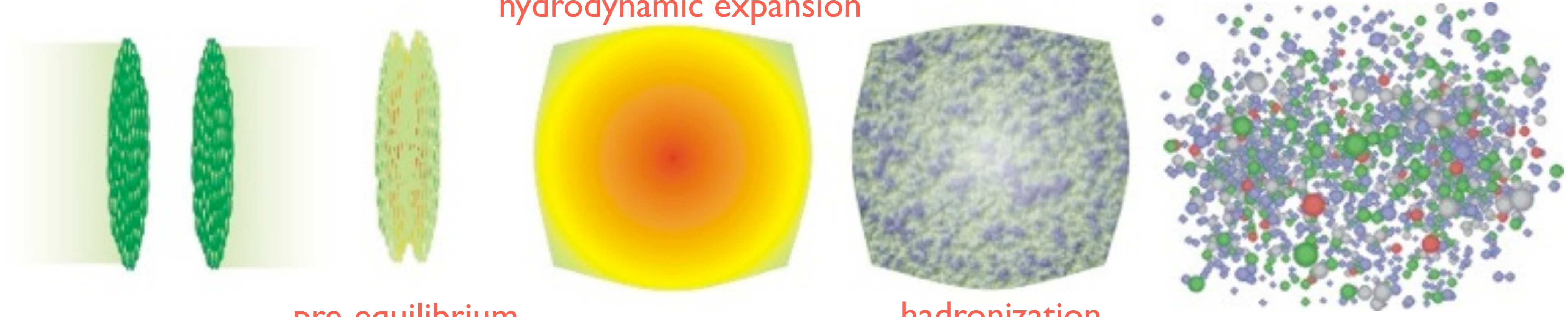
J.Phys.G38:124178,2011, arXiv:1109.4054

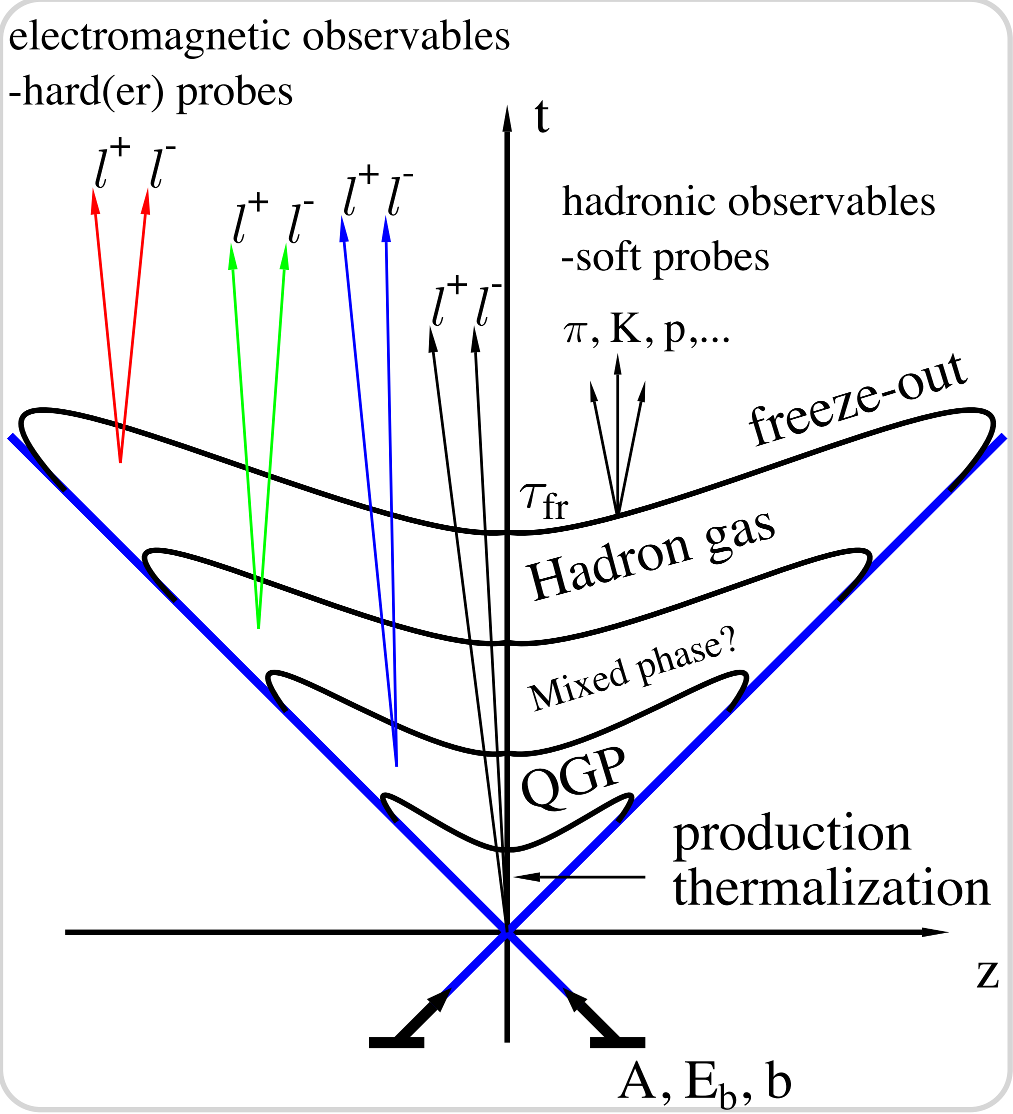
Outline

- Introduction & Motivation
  - thermal dilepton & photon emission rates, electrical conductivity
  - Euclidean correlation and spectral functions
- Vector correlation functions on the lattice
  - finite volume & cut-off effects
  - continuum extrapolation
- Thermal dilepton emission rate and electric conductivity
  - continuum extrapolated results at $T \simeq 1.45 T_c$
  - preliminary results at $T \simeq 1.1 T_c$
- Thermal photon emission rate
  - continuum extrapolated vector correlators at finite p at $T \simeq 1.1 T_c$
- Conclusions & Outlook

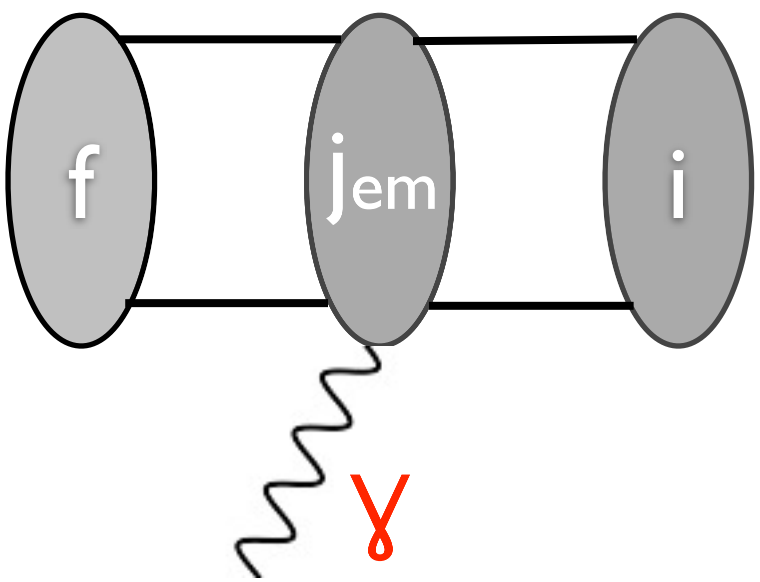
Electromagnetic observables in HIC



- dileptons/photons: penetrating probes
- produced in the all stages of HIC
- models needed to describe the evolution of the system
- understanding of contributions from all sources



photon/dilepton production rates



Transition amplitude between |i> and |f> with a on shell photon

$$S_{fi}^{(\lambda)} = -ie \int \mathrm{d}^4x \, \exp(ipx) \, \epsilon_\mu^{(\lambda)}(p) \, \langle f | j_{em}^\mu(x) | i \rangle$$

Electromagnetic current operator

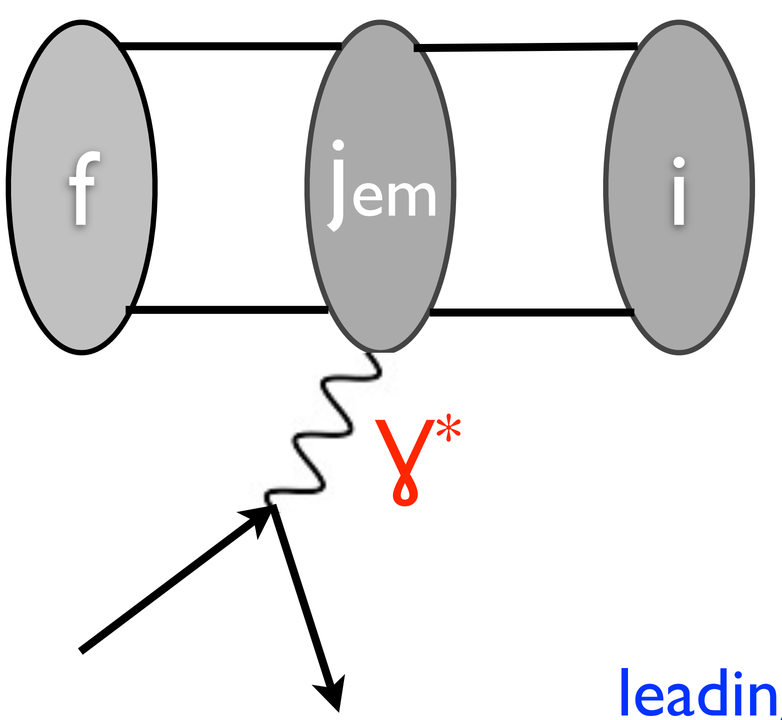
$$j_{em}^\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \cdots$$

Number of photons emitted per unit proper volume

$$R_\gamma = \frac{\mathrm{d}^4 N_\gamma}{\mathrm{d}^4 x} = \frac{1}{\int \mathrm{d}^4 x} \int \frac{\mathrm{d}^3 p}{2\omega(2\pi)^3} \frac{1}{Z} \sum_{f,i,\lambda} e^{-(E_i - \mu N_i)/T} \left| S_{fi}^{(\lambda)} \right|^2$$

Photon emission rate

$$\omega \frac{\mathrm{d}R_\gamma}{\mathrm{d}^3 p} = \sum_f Q_f^2 \frac{\alpha_{em}}{4\pi^2} \frac{\rho_T(\omega = |\vec{p}|, \vec{p})}{\exp(\omega/T) - 1}$$



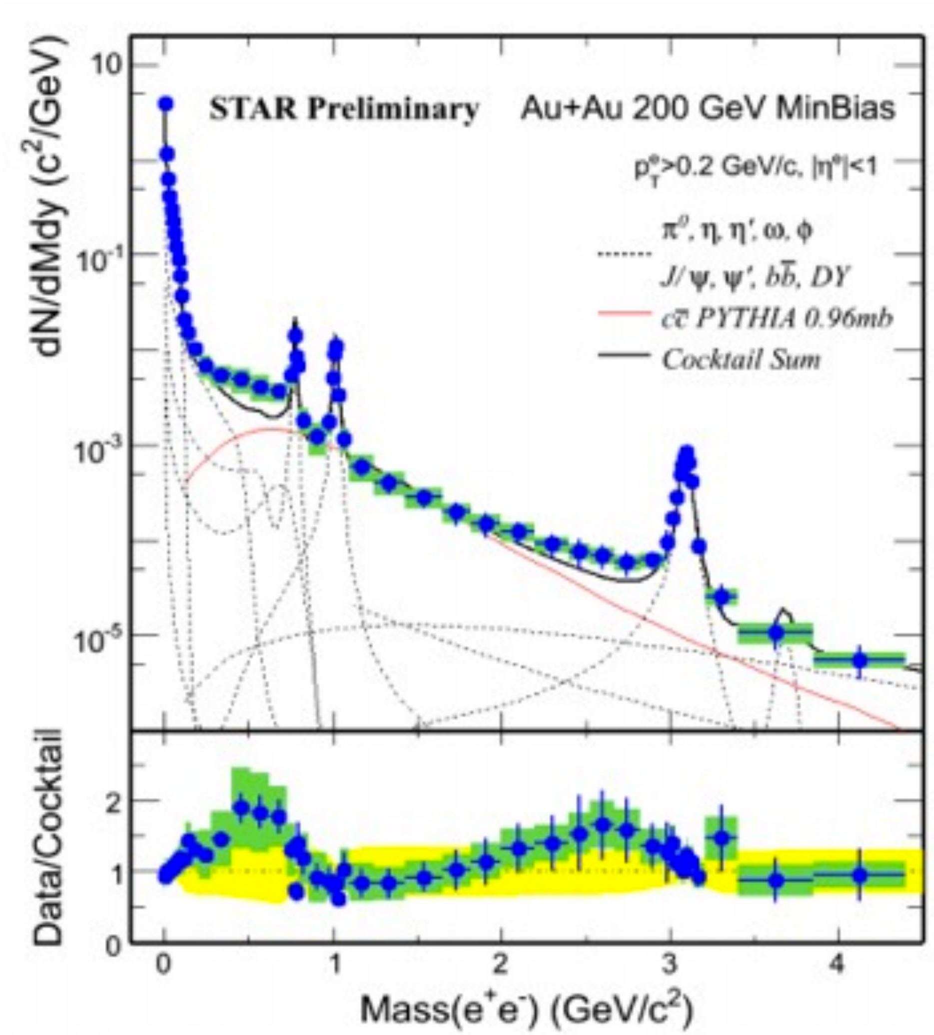
Emission rate of dilepton from virtual photons

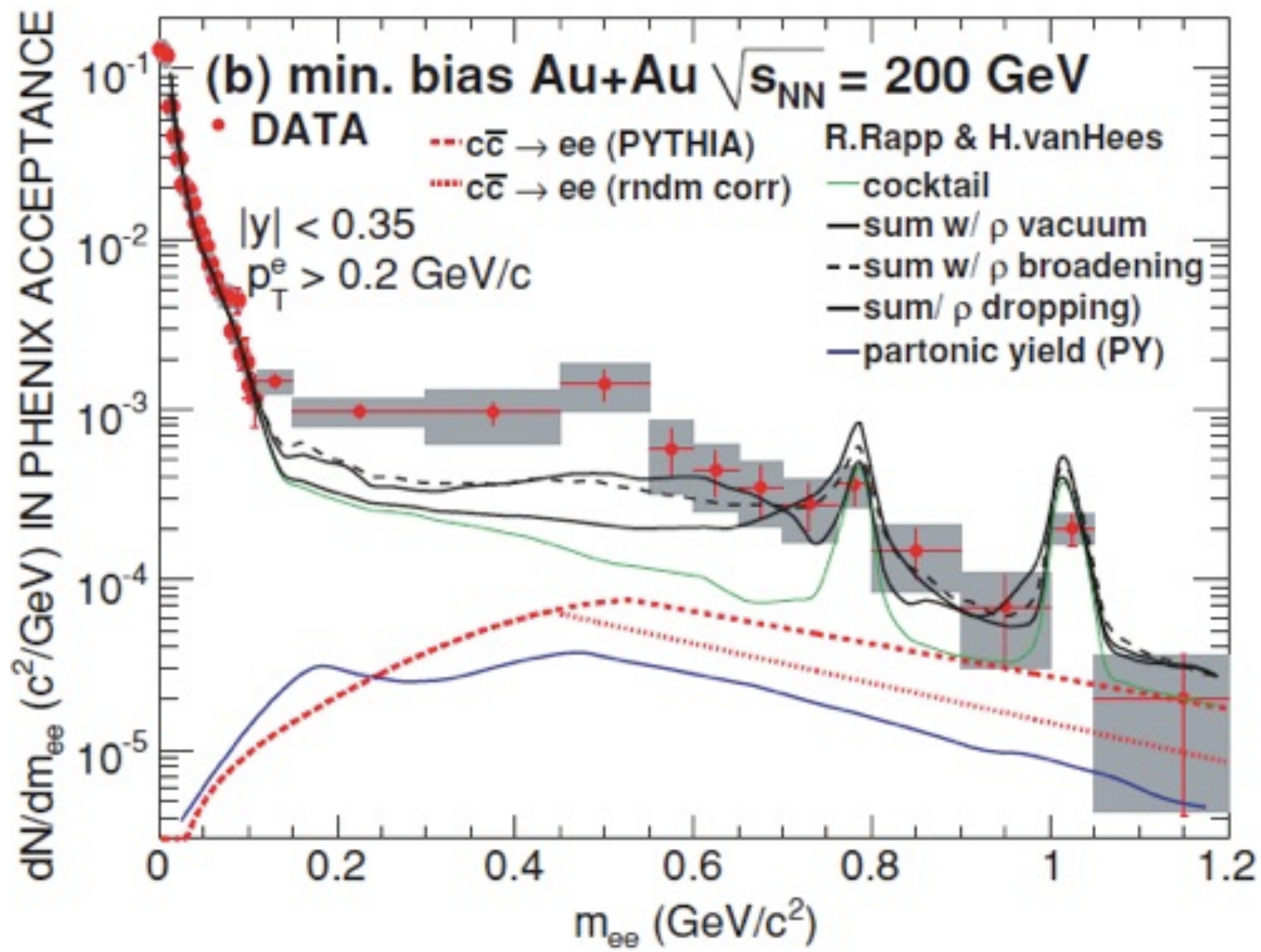
$$\frac{\mathrm{d}R_{l^+l^-}}{\mathrm{d}\omega \mathrm{d}^3 p} = \sum_f Q_f^2 \frac{\alpha_{em}^2}{6\pi^3} \frac{2\rho_T(\omega, \vec{p}) + \rho_L(\omega, \vec{p})}{(\omega^2 - \vec{p}^2)(\exp(\omega/T) - 1)}$$

Above EM emission rate formulae are valid in
leading order of α_{em} and exact to all orders in strong coupling

Experimental results on dilepton rates

- dilepton rates: $$\frac{dN_{l^+l^-}}{d\omega d^3p} = C_{em}\frac{\alpha_{em}^2}{6\pi^3}\frac{\rho_V(\omega,\vec{p},T)}{(\omega^2-\vec{p}^2)(e^{\omega/T}-1)} \qquad C_{em}=e^2\sum_{f=1}^{n_f}Q_f^2$$

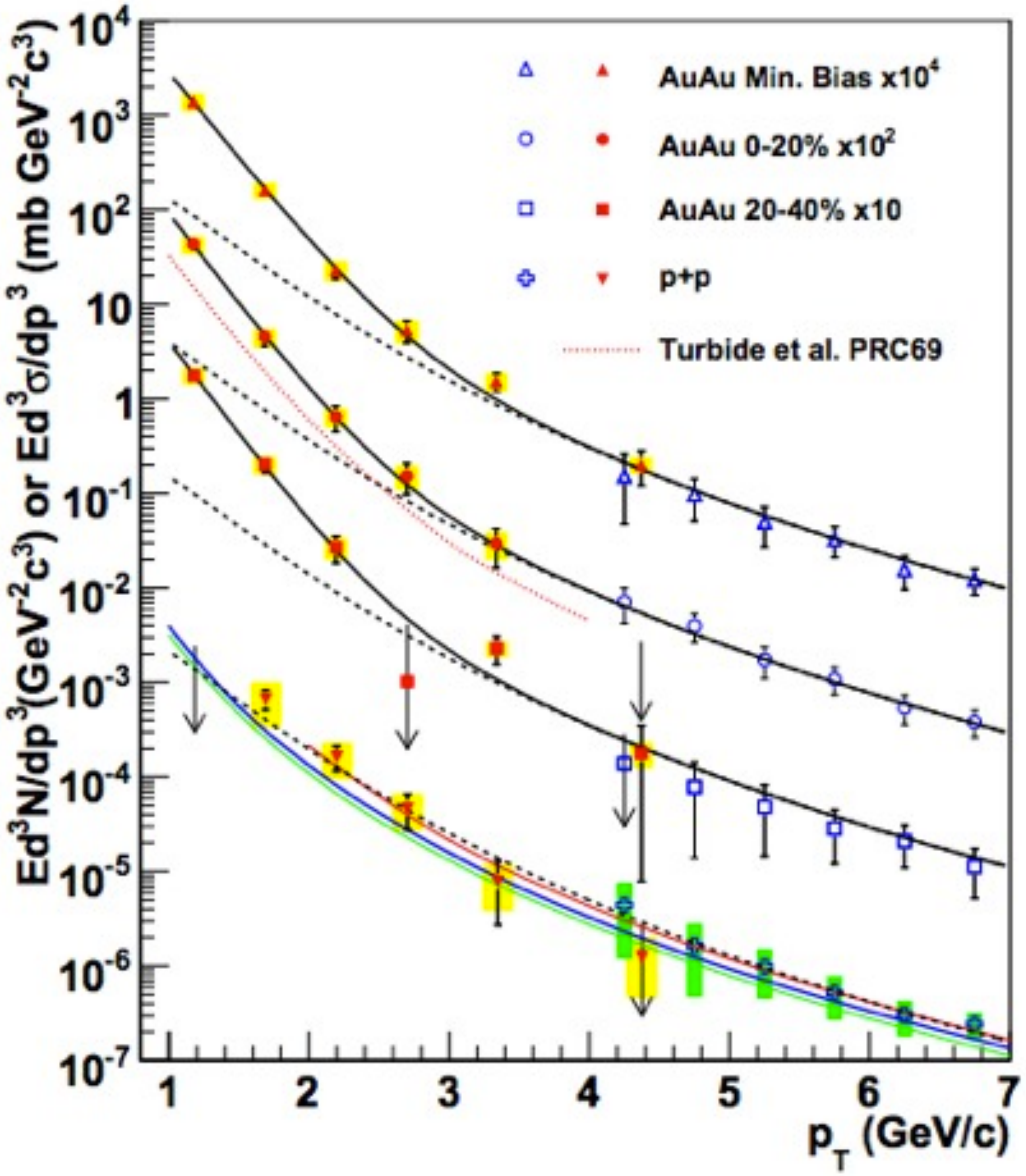




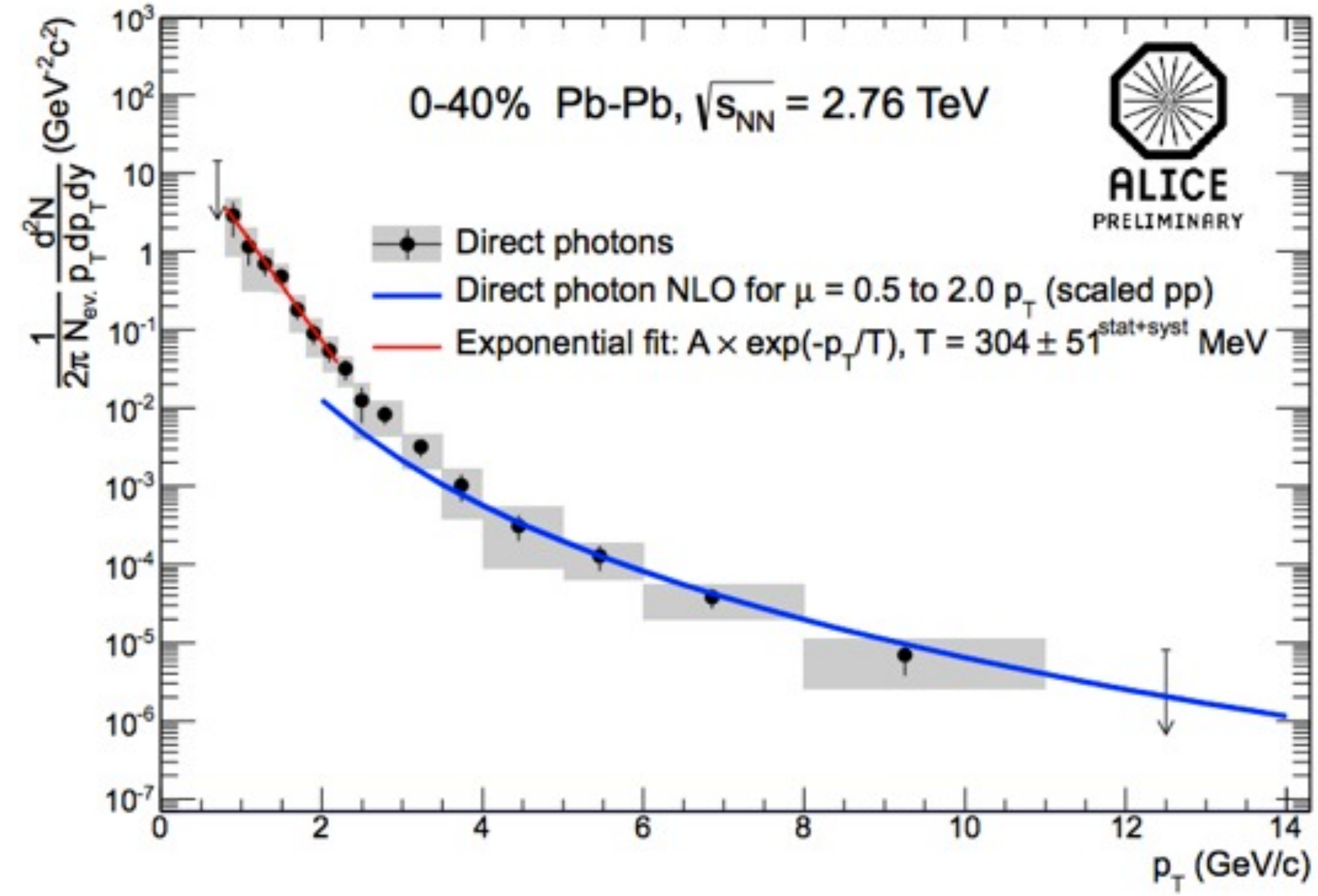
PHENIX: Phys. Rev. C 81 (2010) 034911

- Controversy between STAR and PHENIX in the low mass low pt region
- If PHENIX is right it may imply new mechanism for dilepton production

Direct photon spectra



PHENIX: Phys.Rev.Lett.104:132301,2010



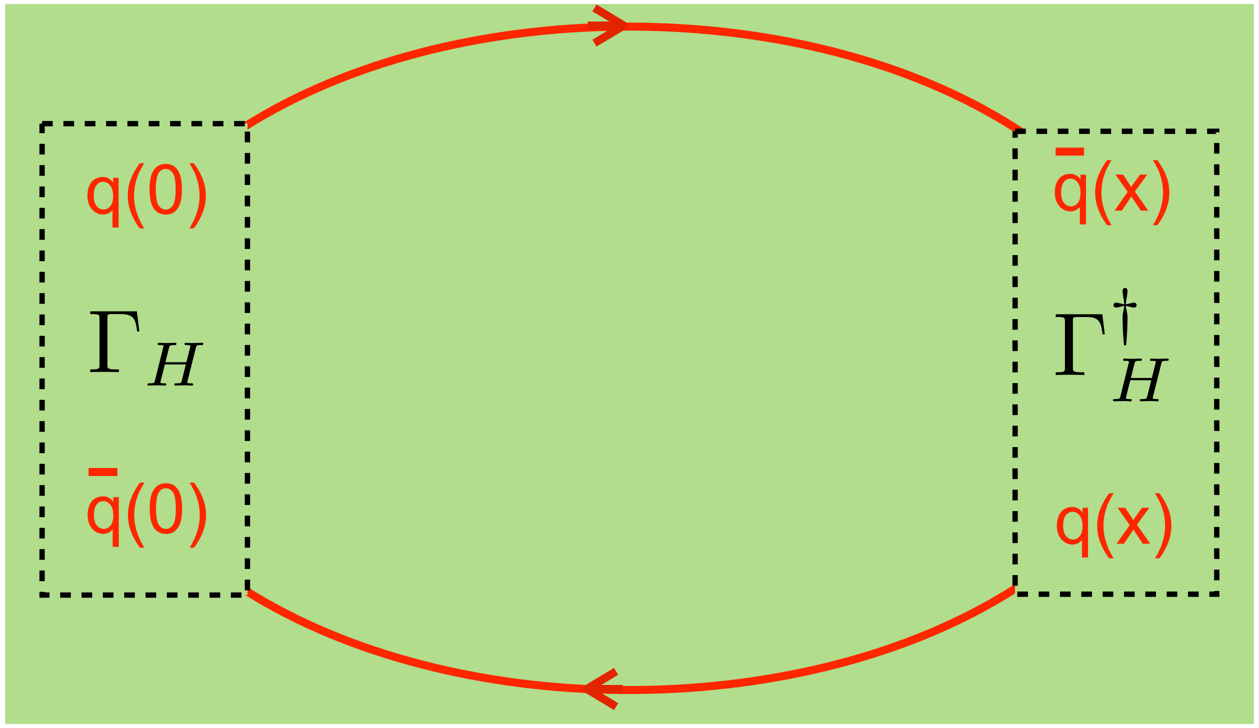
ALICE, M. Wilde, arXiv:1210.5958

- Enhanced direct photon production in AuAu/PbPb collisions at pt<2.5 GeV/c

- photon emission rate:

$$\omega \frac{\mathrm{d}R_\gamma}{\mathrm{d}^3 p} = C_{em} \frac{\alpha_{em}}{4\pi^2} \frac{\rho_T(\omega = |\vec{p}|, T)}{\exp(\omega/T) - 1}$$

Vector correlation & spectral functions



Euclidean correlation function

$$G_{\mu\nu}(\tau,\vec{p}) = \int \mathrm{d}^3x \langle J_\mu(\tau,\vec{x}) J_\nu^\dagger(0,\vec{0})\rangle\, e^{i\vec{p}\cdot\vec{x}}$$

$$J_\mu(\tau,\vec{x}) \equiv \bar{q}(\tau,\vec{x})\gamma_\mu q(\tau,\vec{x})$$

Spectral function

$$\rho(\omega,\vec{p}) = D^+(\omega,\vec{p}) - D^-(\omega,\vec{p}) = 2\,\mathrm{Im} D_R(\omega,\vec{p})$$

Relation between spectral function and correlator

$$G(\tau,\vec{p}) = \int \mathrm{d}^3x\, e^{-i\vec{p}\cdot\vec{x}}\, D^+(-i\tau,\vec{x})\,, \qquad D^+(t,\vec{x}) = D^-(t+i\beta,\vec{x})$$

$$G_H(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\, \rho_H(\omega,\vec{p},T)\, \frac{\cosh(\omega(\tau-1/2T))}{\sinh(\omega/2T)} \quad , \qquad H = 00,\ ii,\ V\ .$$

Vector correlation function

$$G_H(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\, \rho_H(\omega,\vec{p},T)\, \frac{\cosh(\omega(\tau-1/2T))}{\sinh(\omega/2T)} \quad , \qquad H = 00,\ ii,\ V\ .$$

time like correlator G_{00} and space like correlator G_{ii}

$$G_V(\tau,\vec{p},T) = G_{ii}(\tau,\vec{p},T) + G_{00}(\tau,\vec{p},T)$$

conserved current, J_0, gives τ-independent correlator G_{00}

$$G_{00}(T) \equiv -\chi_q T + \mathcal{O}(a^2)$$

the local, non-conserved current needs to be renormalized

$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V \bar{\psi}(\tau,\vec{x})\gamma_\mu\psi(\tau,\vec{x})$$

avoid ambiguities of renormalization

$$R(\tau) \equiv \frac{G_V(\tau)}{G_{00}(\tau)} \quad ; \quad R(\tau) \equiv \frac{G_V(\tau)}{G_{00}(\tau) G_V^{free}(\tau T)}$$

Prior information on spectral functions

- free vector spectral function (in the infinite temperature limit)

$$\rho_{00}^{free}(\omega) = -2\pi T^2 \omega \delta(\omega)$$

$$\rho_{ii}^{free}(\omega) = 2\pi T^2 \omega \delta(\omega) + \frac{3}{2\pi}\omega^2 \tanh(\frac{\omega}{4T})$$

  - ✦ δ-functions cancel in $\rho_V(\omega) \equiv \rho_{00}(\omega) + \rho_{ii}(\omega)$

- vector spectral function at T<∞

  - ✦ δ-function in ρ_{00} is protected

$$\rho_{00}(\omega, T) = -2\pi \chi_q \omega \delta(\omega)$$

  - ✦ δ-function in ρ_{ii} is smeared out

possible form: Breit-Wigner (BW) form + modified continuum

$$\rho_{ii}(\omega, T) = \chi_q c_{BW} \frac{\omega\Gamma}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi}\left(1 + \frac{\alpha_s}{\pi}\right)\omega^2 \tanh(\frac{\omega}{4T})$$

3-4 parameters: $(\chi_q), c_{BW}, \Gamma, \alpha_s$

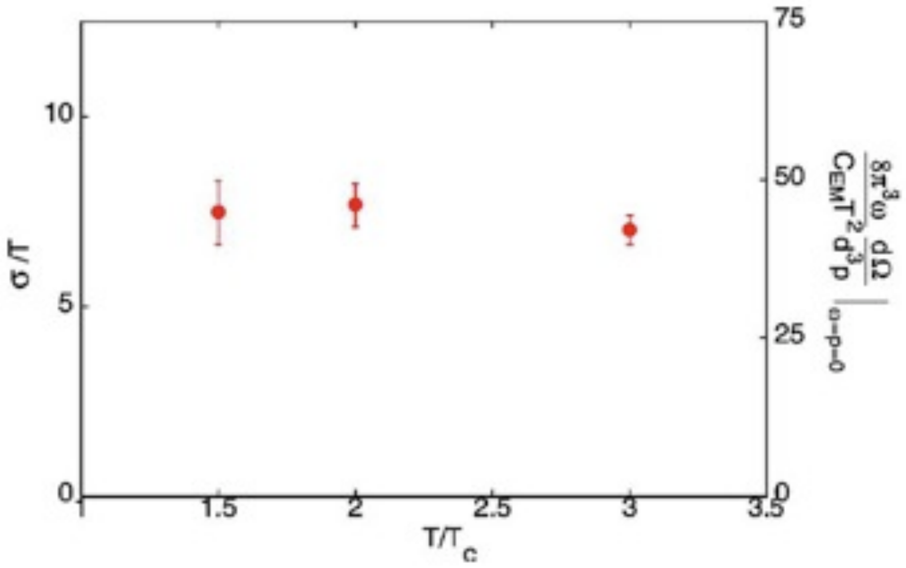
previous lattice results on electrical conductivity

Electrical conductivity: $$\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega)}{\omega T}$$

The emission rate of soft photons: $$\lim_{\omega \to 0} \omega \frac{\mathrm{d}R_\gamma}{\mathrm{d}^3 p} = \lim_{\omega \to 0} C_{em} \frac{\alpha_{em}}{4\pi^2} \frac{\rho_T(\omega = |\vec{p}|, T)}{e^{\omega/T} - 1} = \frac{3}{2\pi^2} \sigma(T) T \alpha_{em}$$

Lattice calculations with unrenormalized currents produced unrenormalized electrical conductivity:

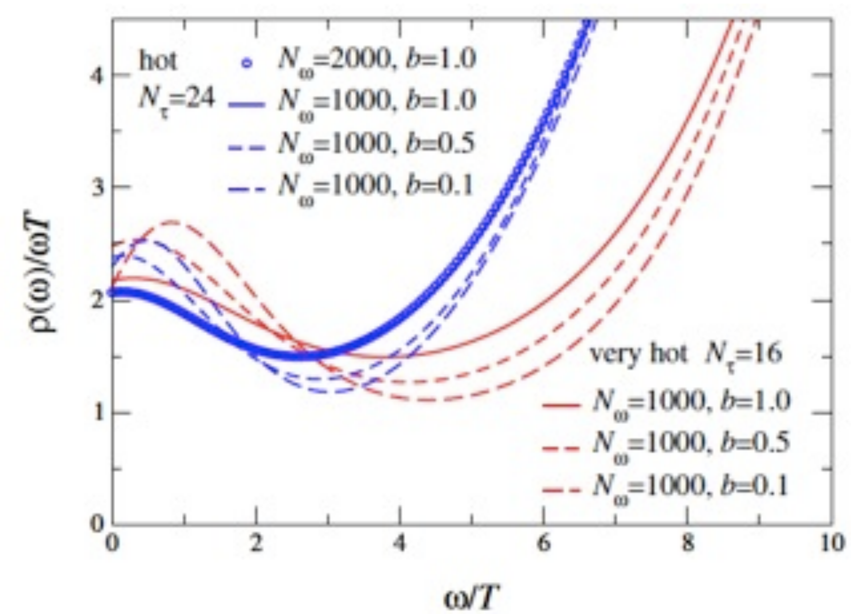
$$\sigma/T \simeq 7 C_{em}$$



S. Gupta PLB 597(2004)

$N_\tau = 8 - 14,\ N_\sigma \leq 44$

$$\sigma/T \simeq (0.4 \pm 0.1) C_{em}$$



G.Aarts et al., PRL 99(2007)

$N_\tau = 16, 24,\ N_\sigma = 64$

staggered fermions used

ρ_{even} and ρ_{odd} need to be distinguished

Vector correlation functions on large & fine lattices at 1.45Tc

- SU(3) gauge configurations at $T/T_c \simeq 1.45$
- lattice size $N_\sigma^3 \times N_\tau$ with N_σ = 32-128 & N_τ=16,24,32,48
- Non-perturbatively clover O(a) improved Wilson fermions
- Quark masses close to chiral limit $\kappa \simeq \kappa_c$

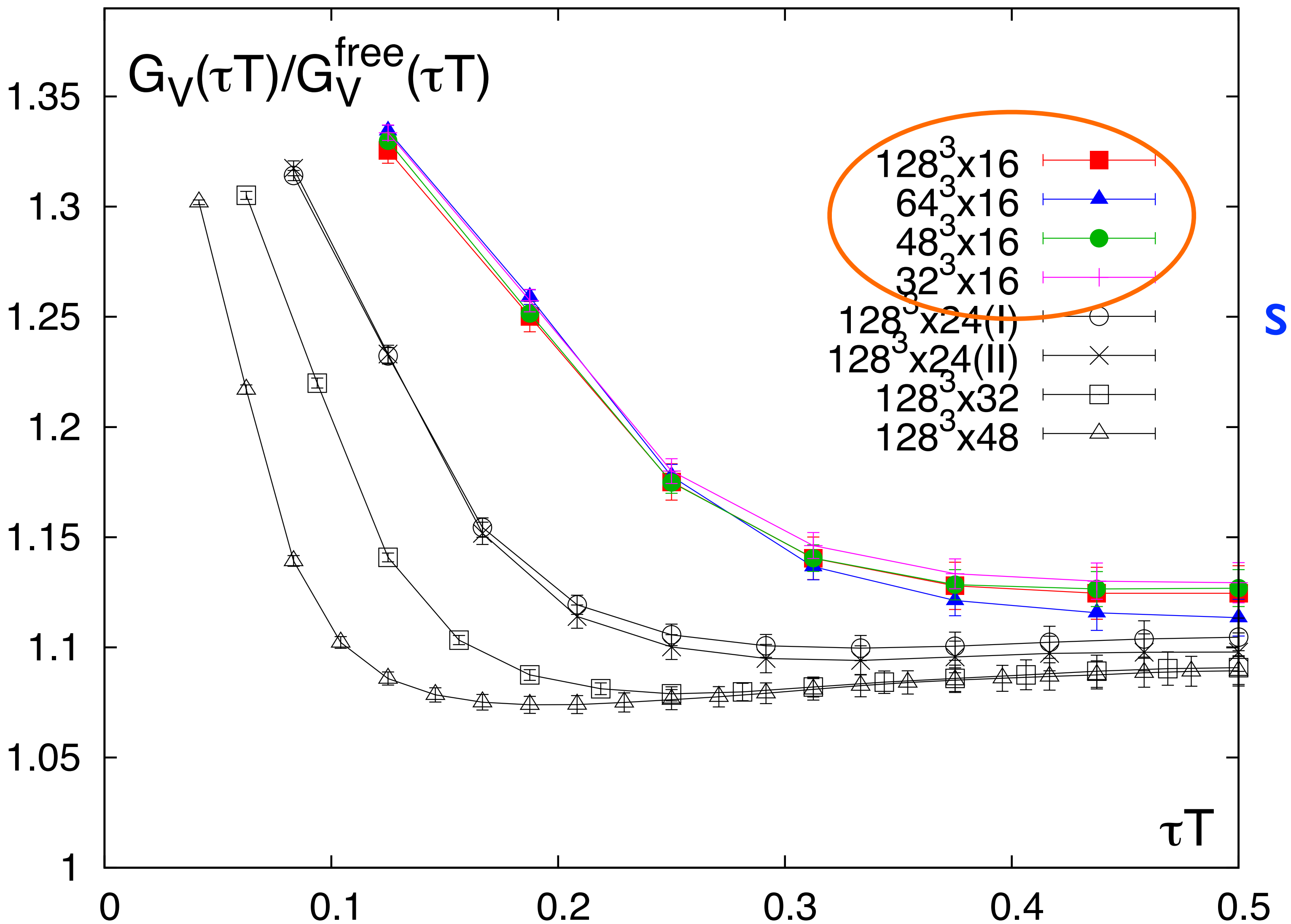
volume dependence

N_τ	N_σ	β	c_{SW}	κ	Z_V	a^{-1}[GeV]	a[fm]	#conf
16	32	6.872	1.4125	0.13495	0.829	6.43	0.031	251
16	48	6.872	1.4125	0.13495	0.829	6.43	0.031	229
16	64	6.872	1.4125	0.13495	0.829	6.43	0.031	191
16	128	6.872	1.4125	0.13495	0.829	6.43	0.031	191
24	128	7.192	1.3673	0.13431	0.842	9.65	0.020	340
	128	7.192	1.3673	0.13440	0.842	9.65	0.020	156
32	128	7.457	1.3389	0.13390	0.851	12.86	0.015	255
48	128	7.793	1.3104	0.13340	0.861	18.97	0.010	451

cut-off dep. & continuum extrapolation

close to continuum

Volume & cut-off dep. of vector corr. function



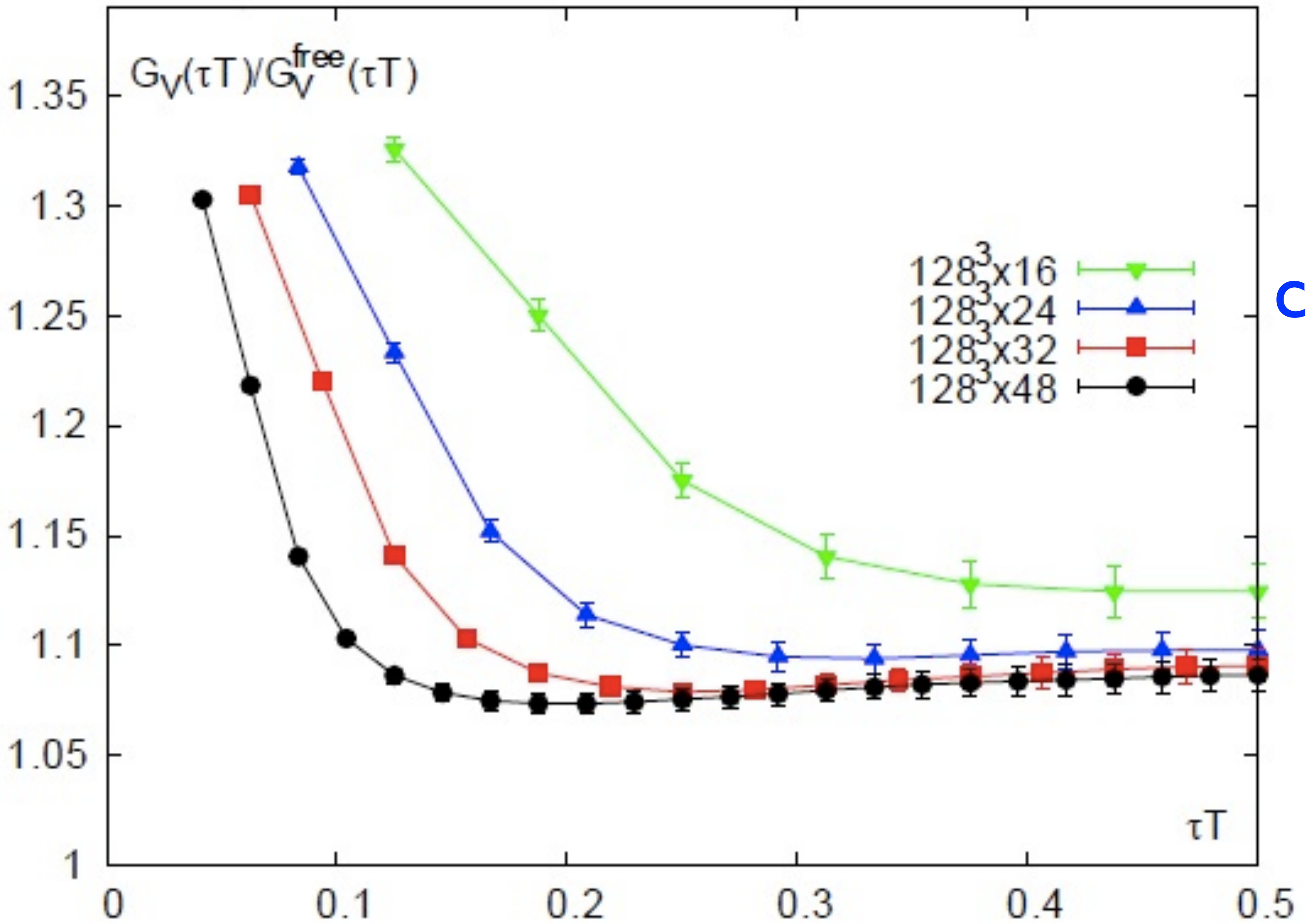
small volume dependence

weak τT dep. near 1/2

Normalized by free correlators in the continuum $G_V^{free}(\tau T)$

$$G_V^{free}(\tau T) = 6T^3 \left(\pi\ (1 - 2\tau T) \frac{1 + \cos^2(2\pi\tau T)}{\sin^3(2\pi\tau T)} + 2\ \frac{\cos(2\pi\tau T)}{\sin^2(2\pi\tau T)} \right)$$

Volume & cut-off dep. of vector corr. function



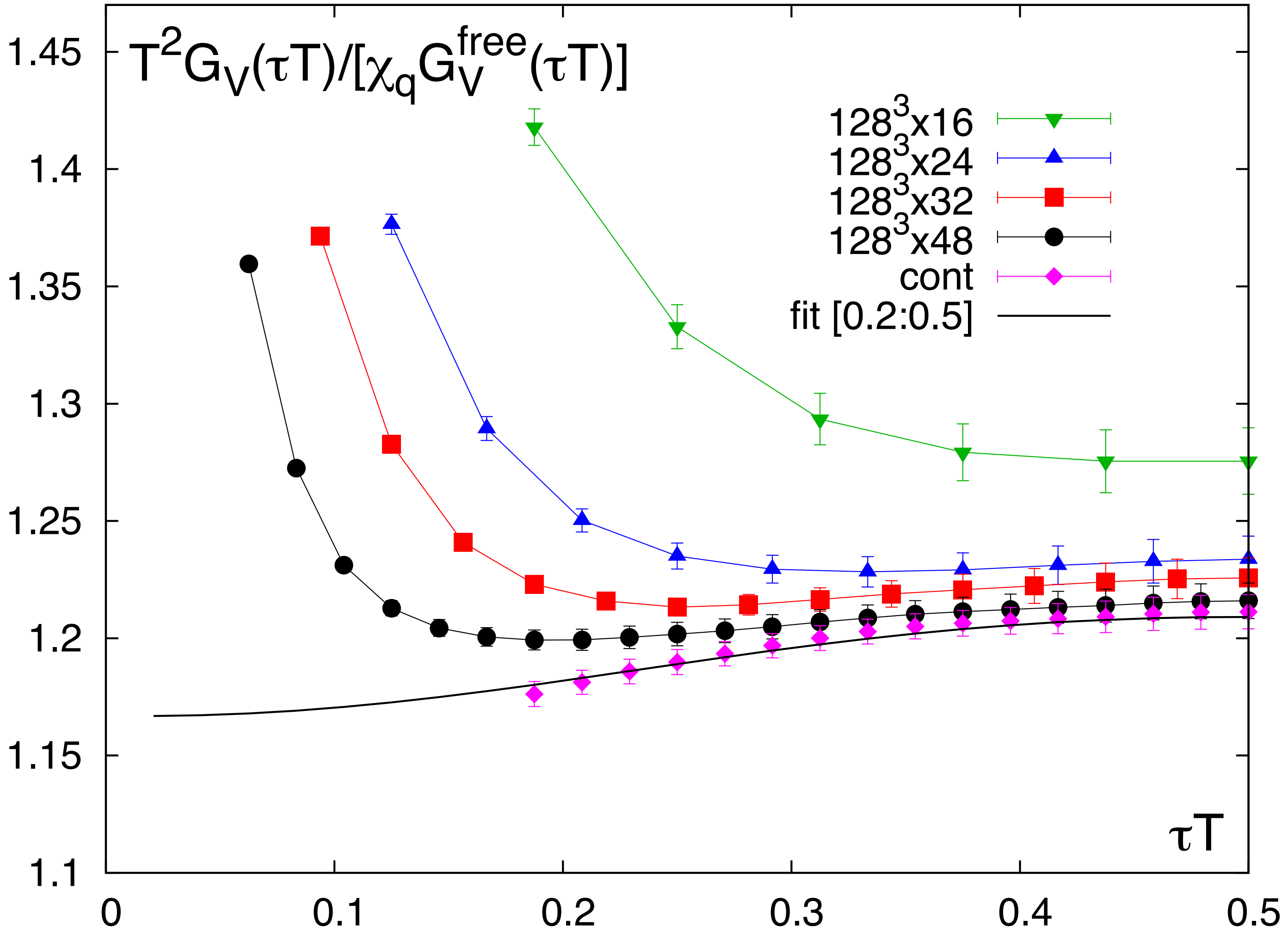
cut-off effects are more server than finite volume effects

large N_τ needed to perform continuum extrapolation

$G_V(\tau T)$ is close to the free case at large τT

incomplete cancelation between $G_{00}(\tau T)$ and BW-contribution to $G_{ii}(\tau T)$?

Continuum extrapolation



$$\frac{G_V(1/2)}{G_V^{free}(1/2)} = 1.086 \pm 0.008\,,$$

$$\frac{G_V(1/4)}{G_V^{free}(1/4)} = (0.982 \pm 0.005)\frac{G_V(1/2)}{G_V^{free}(1/2)}$$

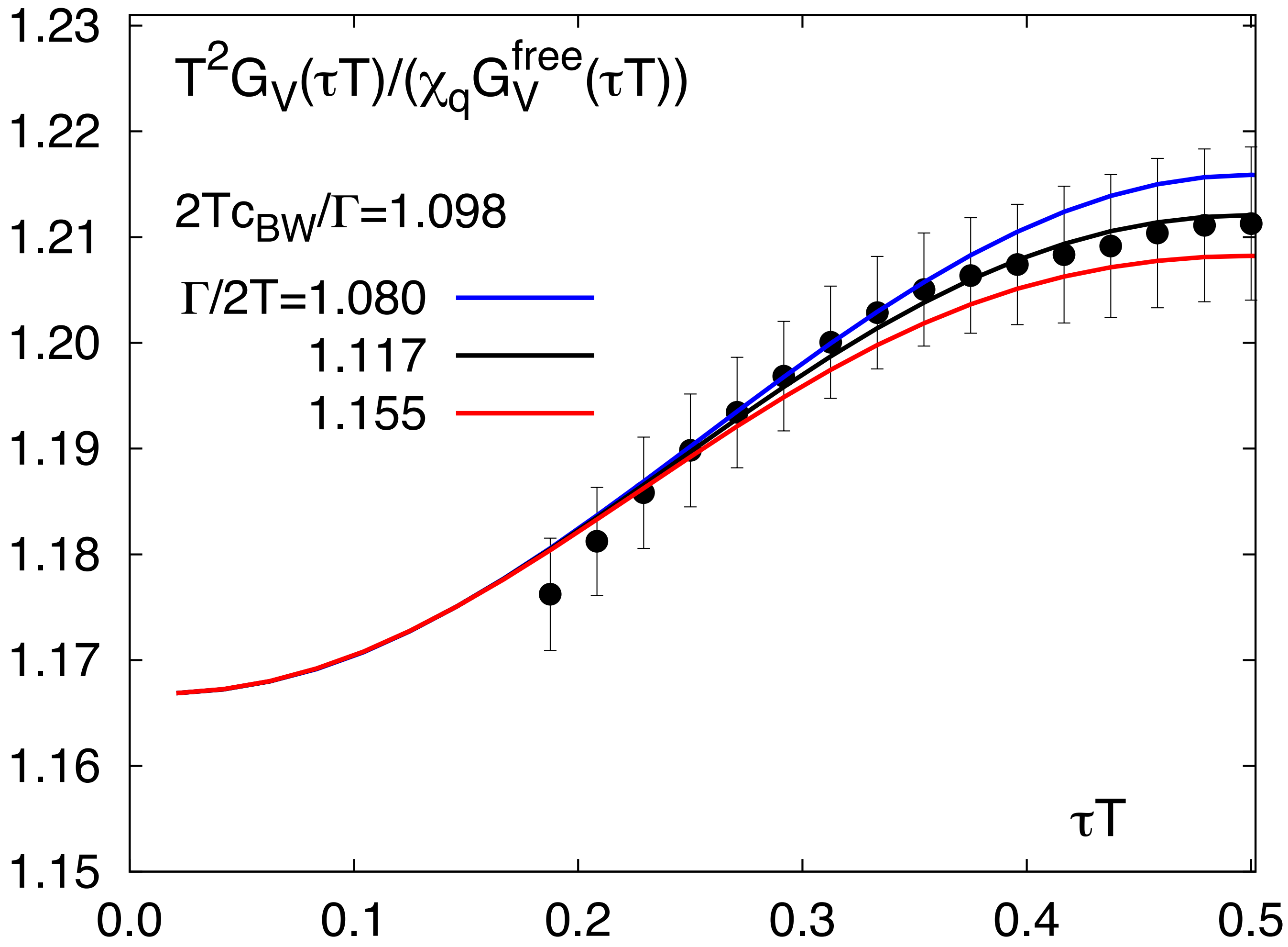
- Increase of $G_V(\tau T)/G_V^{free}(\tau T)$ with τT is obvious
- The rise with τT indicates that vector spectral function in the low frequency region is different from the free case
- Motivation for the Breit-Wigner type ansatz fitting

Breit-Wigner + continuum Ansatz

$$\widetilde{\rho}_{ii}(\widetilde{\omega}) = \frac{2c_{BW}\widetilde{\chi}_q}{\widetilde{\Gamma}} \frac{2\widetilde{\omega}(\widetilde{\Gamma}/2)^2}{\widetilde{\omega}^2 + (\widetilde{\Gamma}/2)^2} + \frac{3}{2\pi}(1+k)\ \widetilde{\omega}^2\ \tanh(\frac{\widetilde{\omega}}{4})$$

$$k = 0.0465(30)\ ,\ \widetilde{\Gamma} = 2.235(75)\ ,\ 2c_{BW}\widetilde{\chi}_q/\widetilde{\Gamma} = 1.098(27)$$

→ vary width Γ with the other two parameters fixed

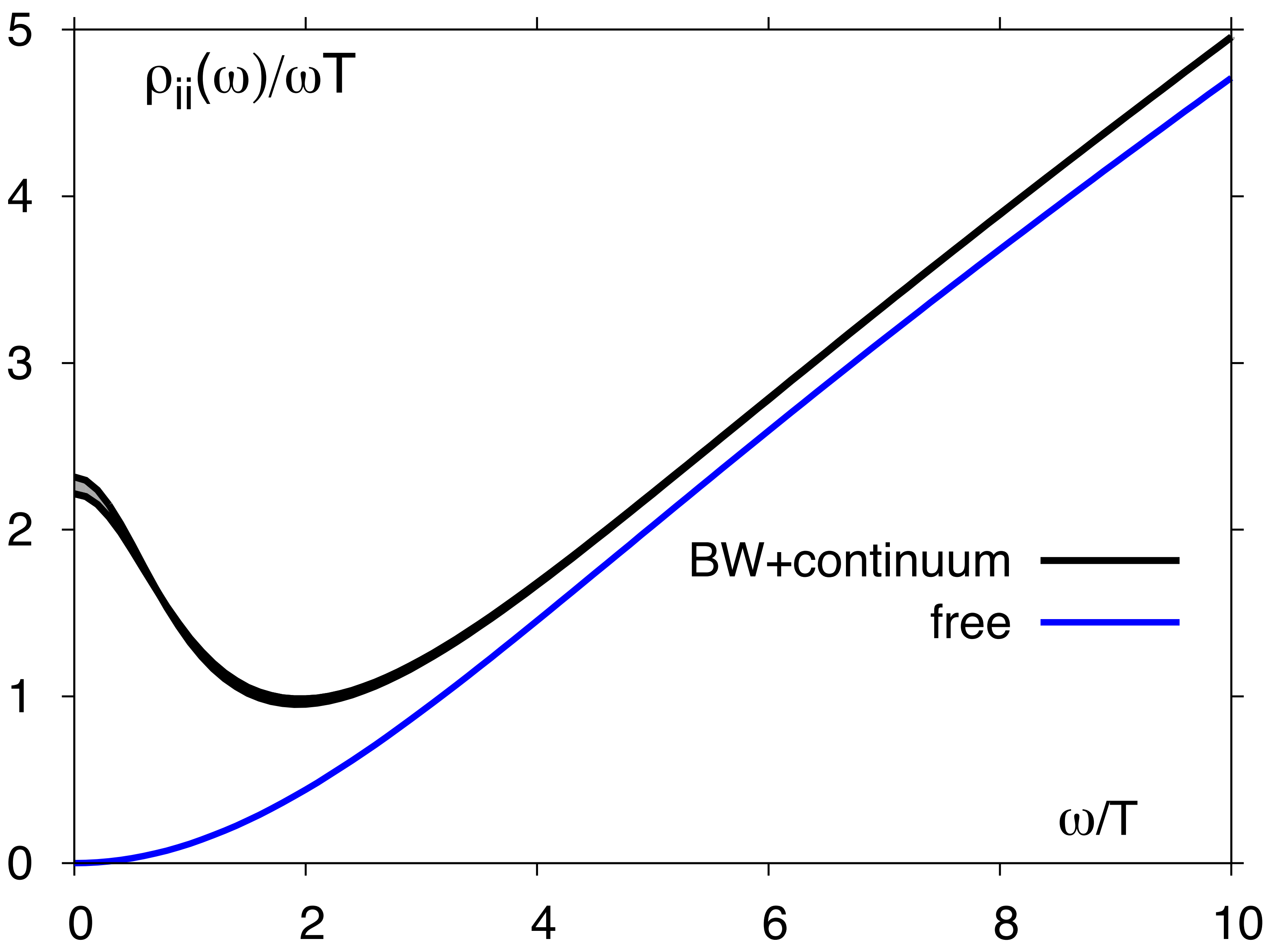


- vector correlation function is sensitive to the low energy, Breit-Wigner contribution only for distance τT ≳ 0.25

Estimate of electrical conductivity

$$\widetilde{\rho}_{ii}(\widetilde{\omega}) = \frac{2c_{BW}\widetilde{\chi}_q}{\widetilde{\Gamma}} \frac{2\widetilde{\omega}(\widetilde{\Gamma}/2)^2}{\widetilde{\omega}^2 + (\widetilde{\Gamma}/2)^2} + \frac{3}{2\pi}(1+k)\ \widetilde{\omega}^2\ \tanh(\frac{\widetilde{\omega}}{4})$$

$$k = 0.0465(30)\ ,\ \widetilde{\Gamma} = 2.235(75)\ ,\ 2c_{BW}\widetilde{\chi}_q/\widetilde{\Gamma} = 1.098(27)$$



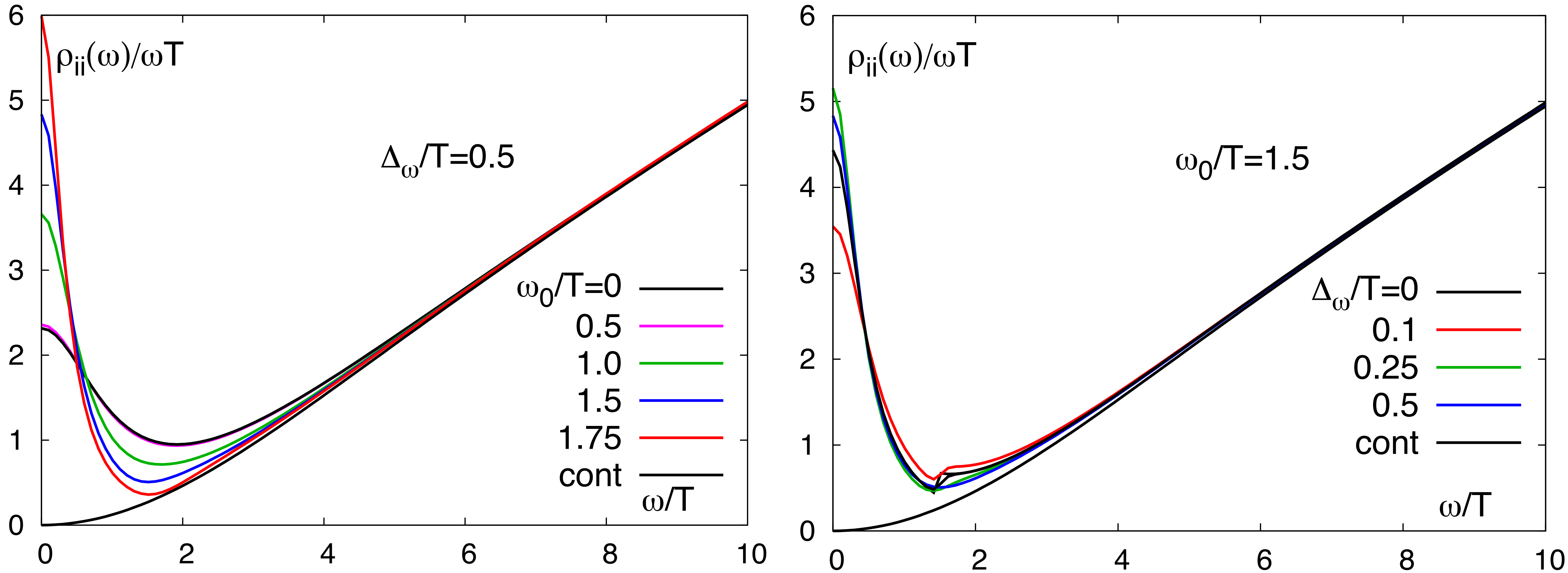
$$\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega\to 0} \frac{\rho_{ii}(\omega)}{\omega T}$$
$$= \frac{C_{em}}{3} \frac{2c_{BW}\widetilde{\chi}_q}{\widetilde{\Gamma}}$$
$$= (0.37 \pm 0.01)C_{em}$$

Breit-Wigner + truncated continuum Ansatz

$$\rho_{ii}(\omega) = 2\chi_q c_{BW} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi}(1+k)\ \omega^2\ \tanh(\frac{\omega}{4T})\,\Theta(\omega_0, \Delta_\omega)$$

$$\Theta(\omega_0, \Delta_\omega) = \left(1 + \mathrm{e}^{(\omega_0^2 - \omega^2)/\omega\Delta_\omega}\right)^{-1}$$

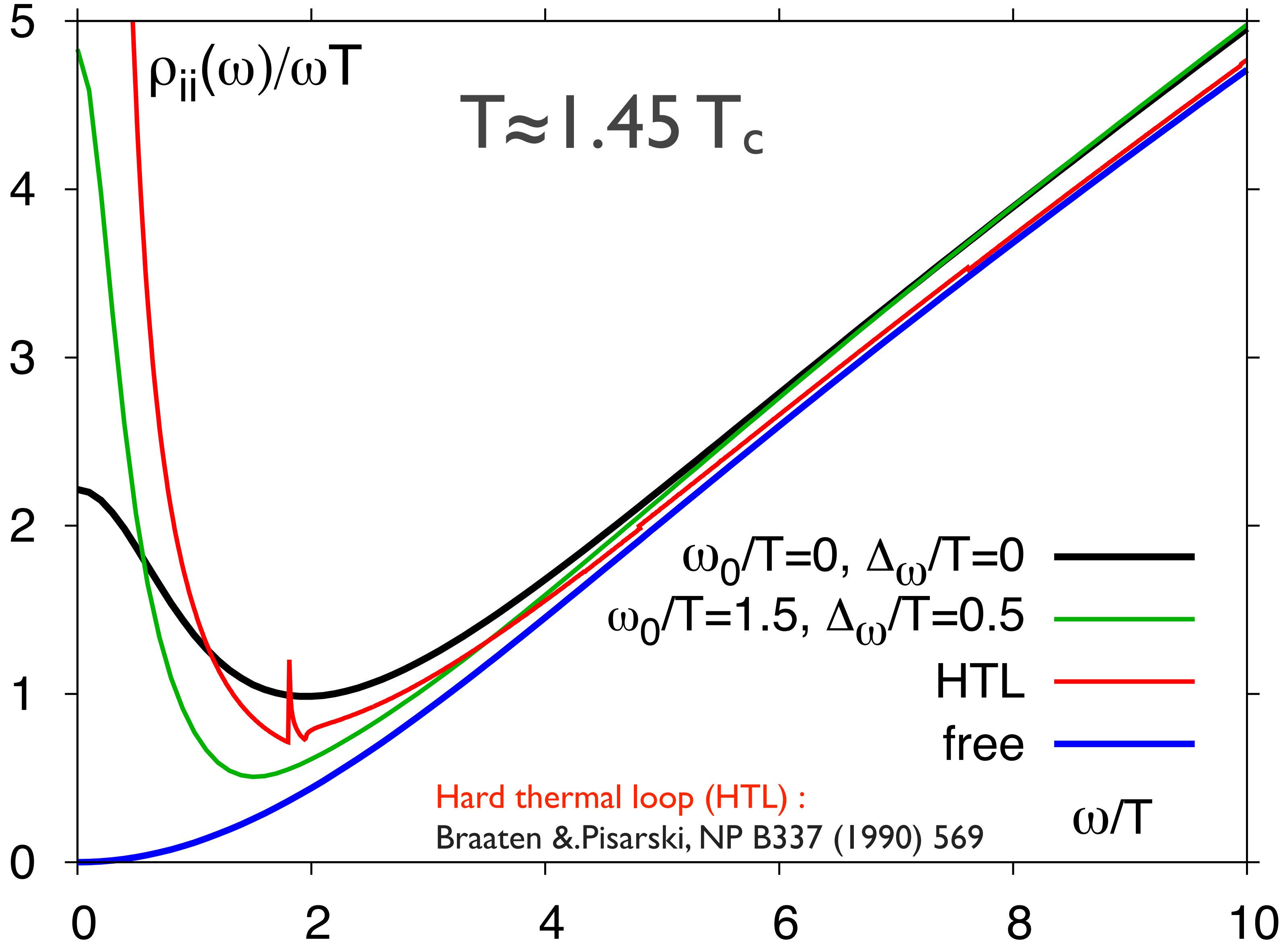
delay the onset (ω_0) of the continuum part



- Rise of BW peaks compensate for the cut from continuum parts
- Fits become worse with increasing ω_0 and/or increasing Δ_ω

Electrical conductivity at 1.45 Tc

$$\rho_{ii}(\omega) = 2\chi_q c_{BW} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi}(1+k)\ \omega^2\ \tanh(\frac{\omega}{4T})\,\Theta(\omega_0, \Delta_\omega)$$



HTL conductivity is divergent at w~0

$$\Theta(\omega_0, \Delta_\omega) = \left(1 + e^{(\omega_0^2 - \omega^2)/\omega\Delta_\omega}\right)^{-1}$$

electrical conductivity

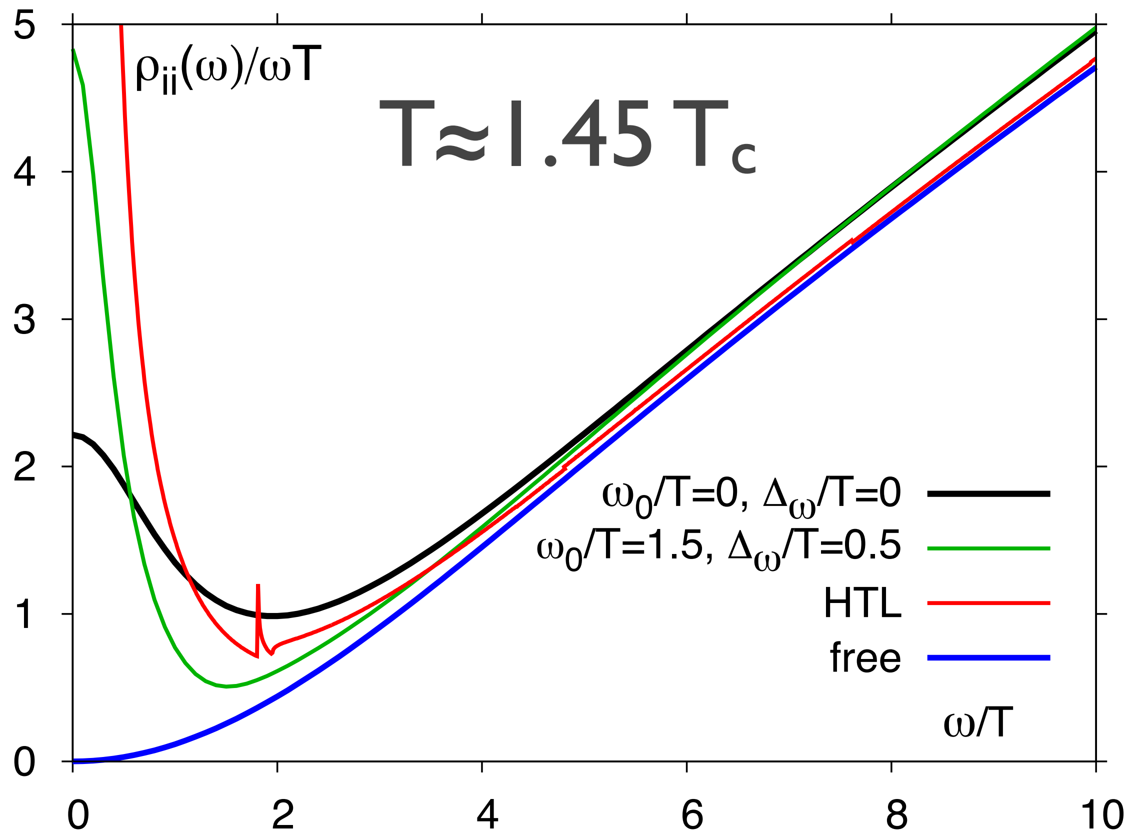
$$1/3 \lesssim \frac{1}{C_{em}}\frac{\sigma}{T} \lesssim 1$$

Soft photon emission rate

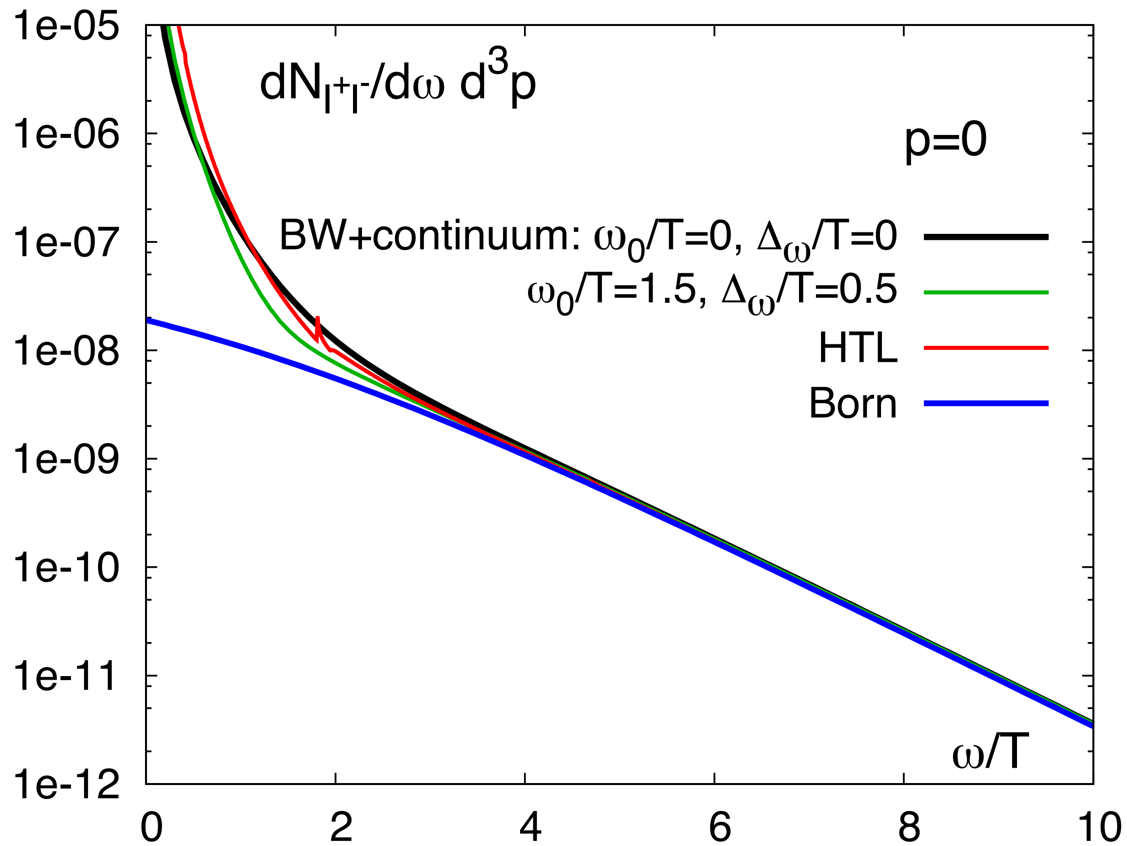
$$\lim_{\omega\to 0} \omega \frac{dR_\gamma}{d^3p} = (0.0004 \ - \ 0.0013)\, T_c^2$$

Thermal dilepton rates at 1.45 T_c

$$\frac{dN_{l^+l^-}}{d\omega d^3p} = C_{em}\frac{\alpha_{em}^2}{6\pi^3}\frac{\rho_V(\omega,\vec{p},T)}{(\omega^2-\vec{p}^2)(e^{\omega/T}-1)}$$



Hard thermal loop (HTL): Braaten &.Pisarski, NP B337 (1990) 569



HTD, Francis, Kaczmarek,Karsch, Laermann, Soeldner, Phys.Rev. D83 (2011) 034504

- thermal dilepton rate approaches leading order Born rate at w/T ≳ 4
- enhancement at small w/T

Vector correlation function at $T=1.1T_c$

- SU(3) gauge configurations at $T/T_c \simeq 1.1$

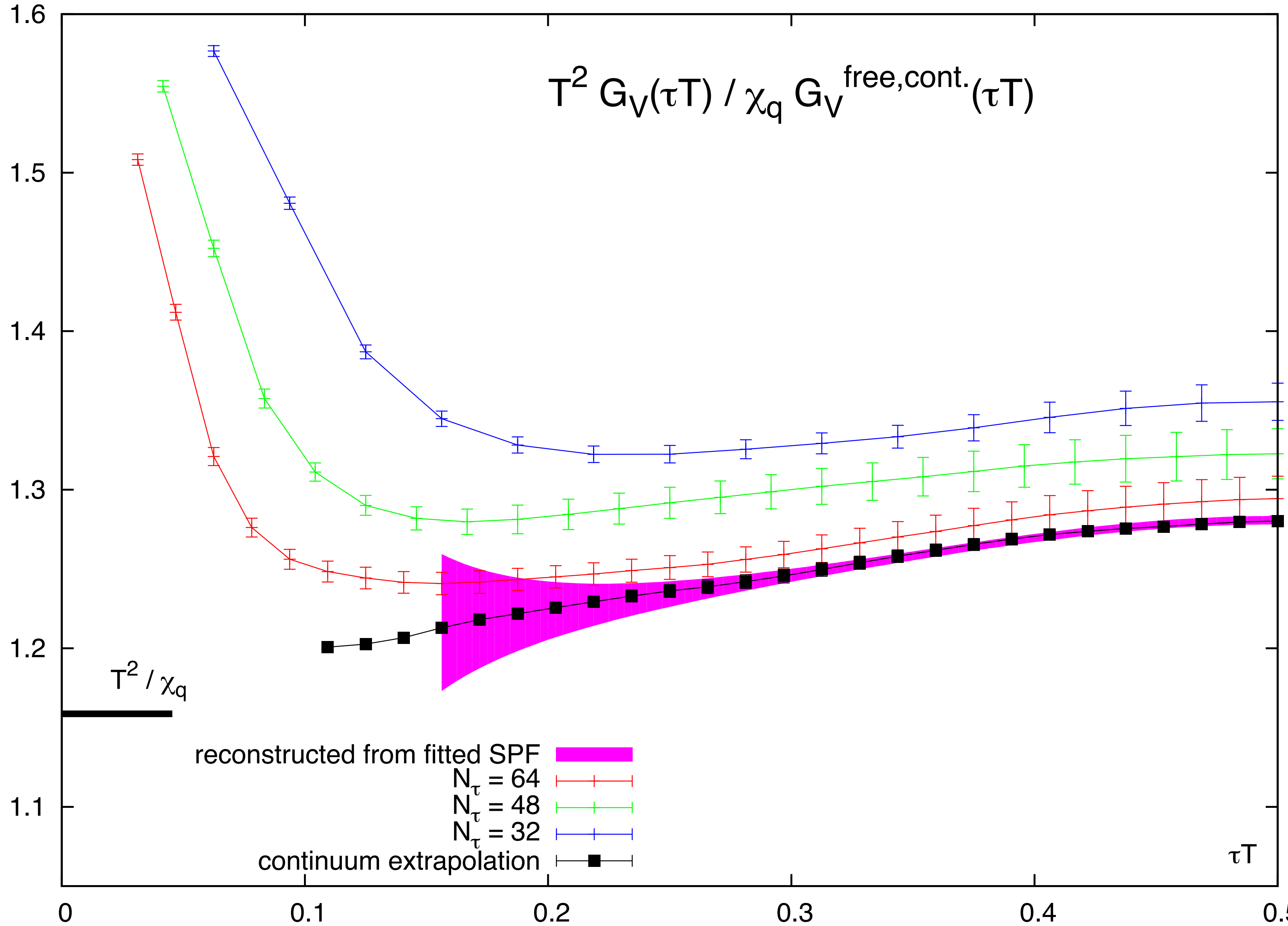
N_τ	N_σ	β	κ	$1/a$[GeV]	a[fm]	#conf
32	96	7.192	0.13440	9.65	0.020	223
48	144	7.544	0.13383	14.21	0.15	226
64	192	7.793	0.13345	19.30	0.010	165

- Allows to study the T-dependence of thermal dilepton and electrical conductivity
- Fixed aspect ratio $N_\sigma/N_\tau=3$ allows the continuum extrapolation of the correlation function at finite momentum

$$\frac{\vec{p}}{T} = 2\pi \frac{N_\tau}{N_\sigma} \vec{n}$$

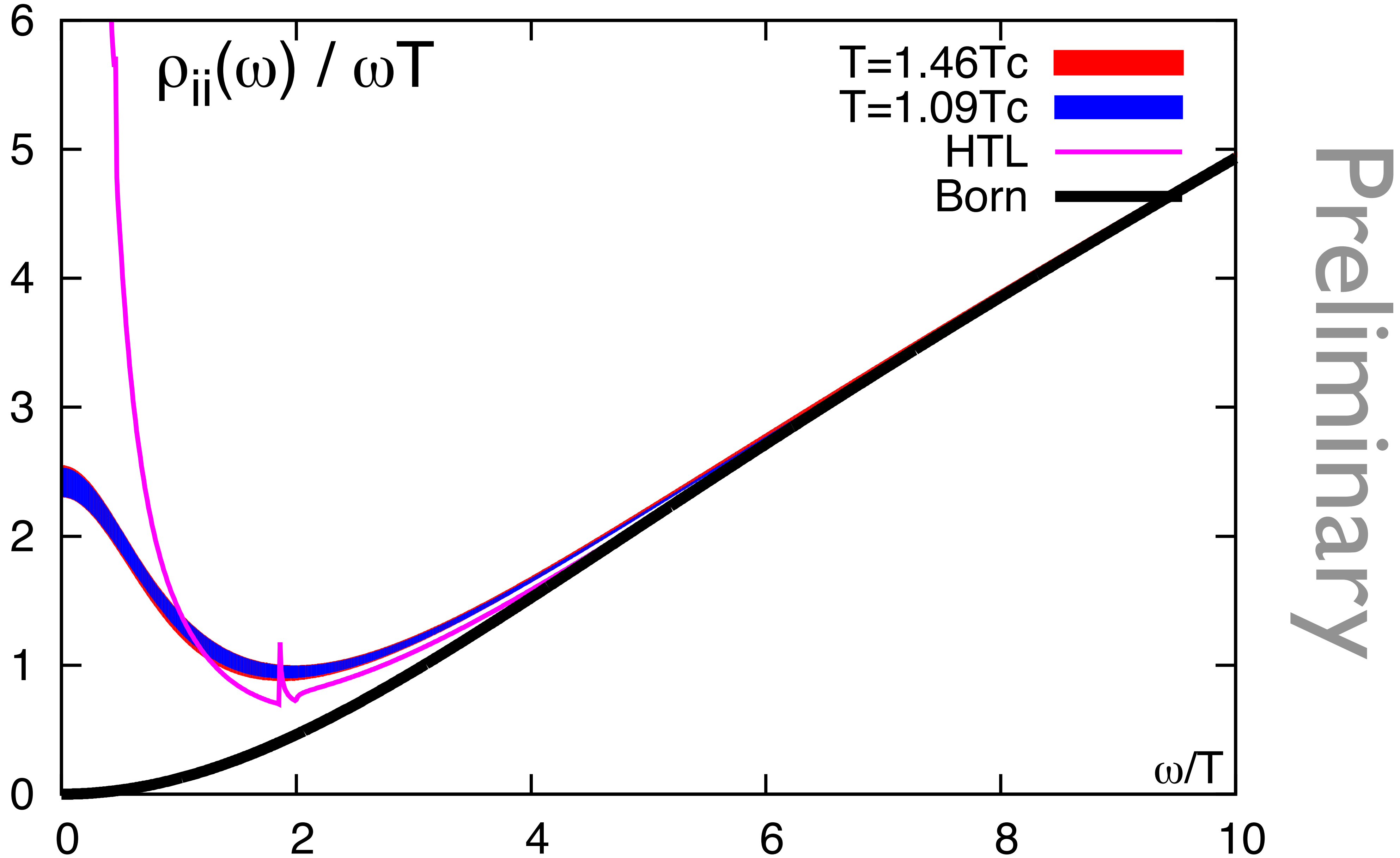
- 15 values of p from 0 up to 3.5 GeV allow the extraction of the thermal photon rate

continuum extrapolation of vector corr. at 1.1 T_c



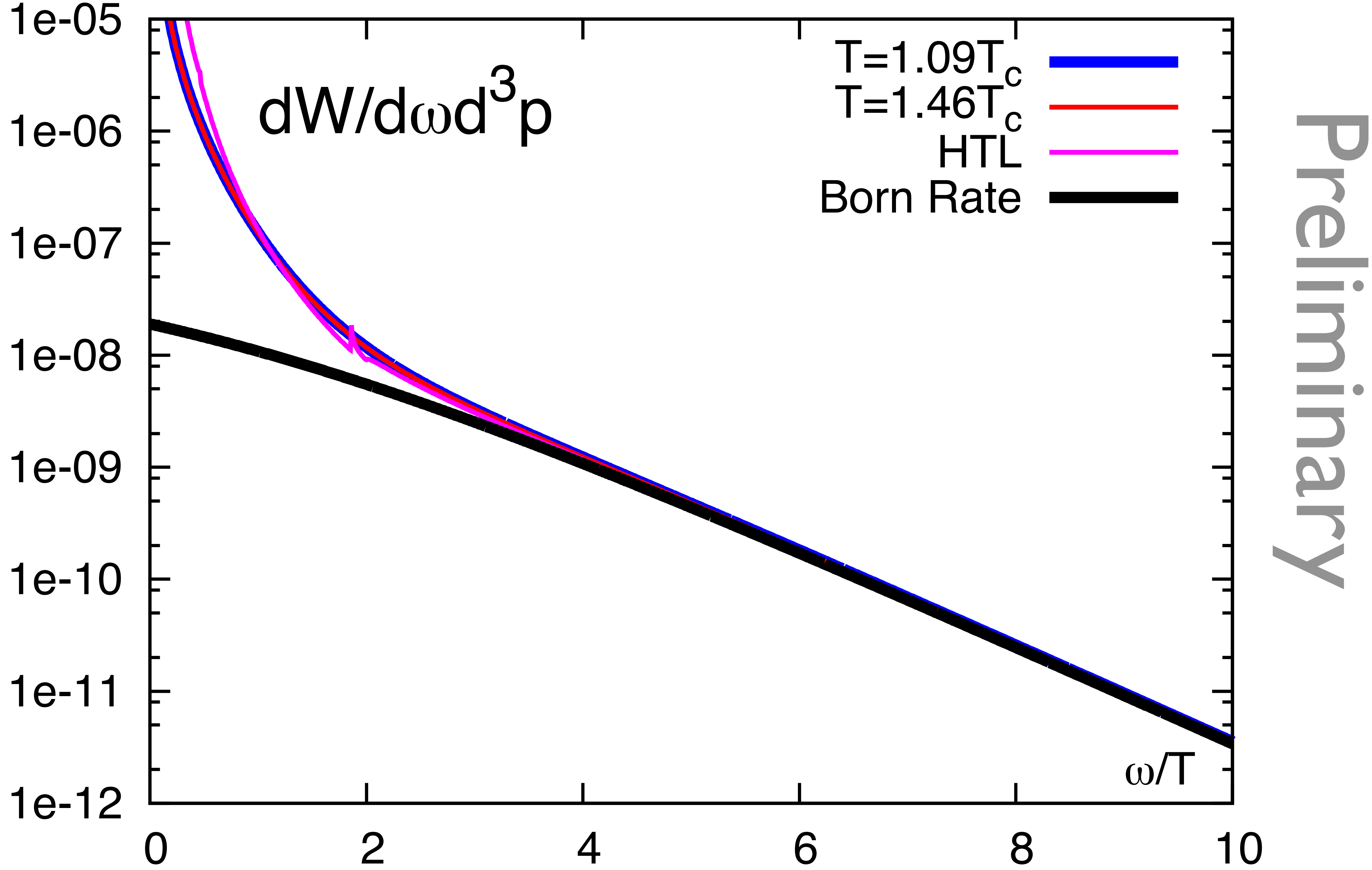
- reliable continuum extrapolation from τT larger than 0.2
- used to extract vector spectral function at 1.1 T_c

vector spectral functions



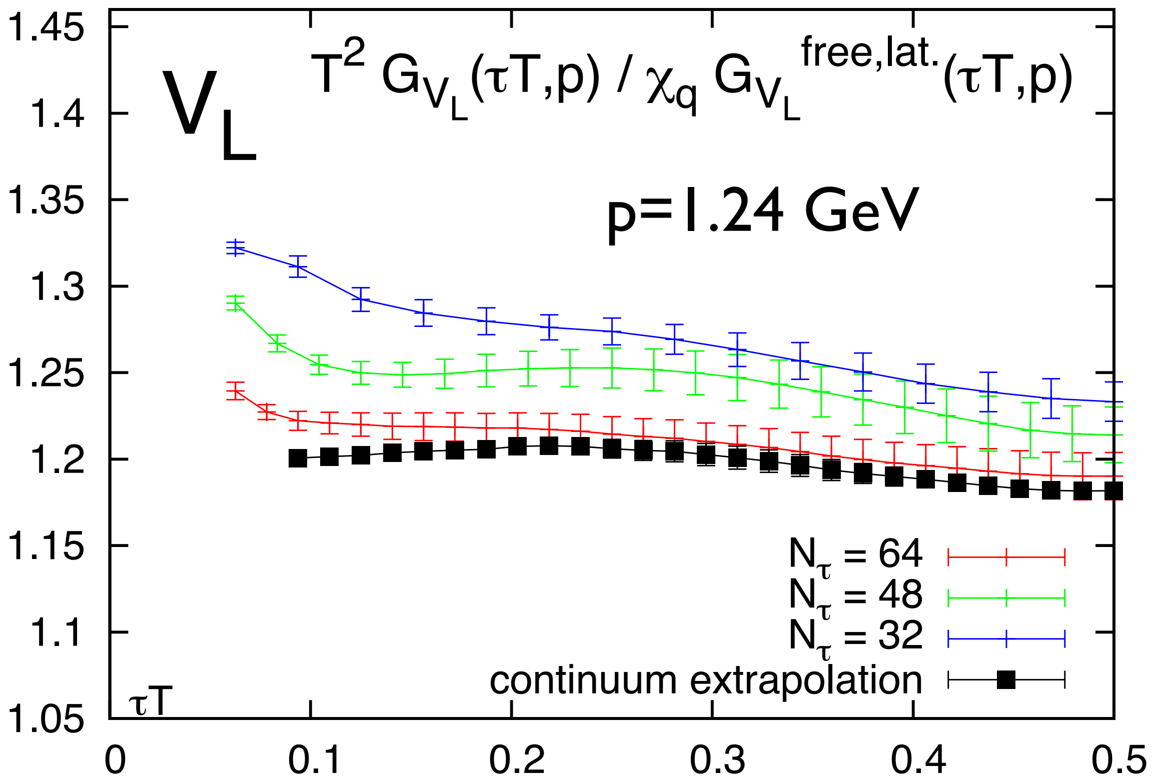
- spf at 1.1 T_c: use the same ansatz as at $T=1.45T_c$
- almost no change from $1.45T_c$ to $1.1T_c$, e.g. similar σ/T
- uncertainties of spf at 1.1 T_c need to be checked

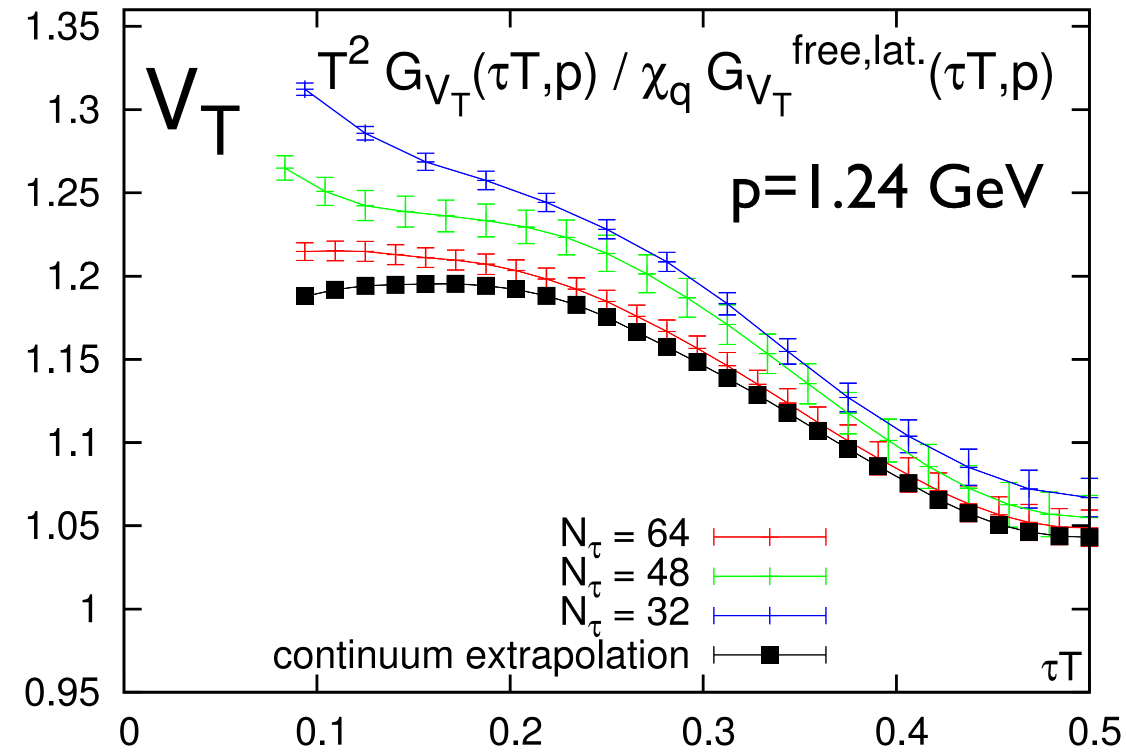
thermal dilepton emission rate



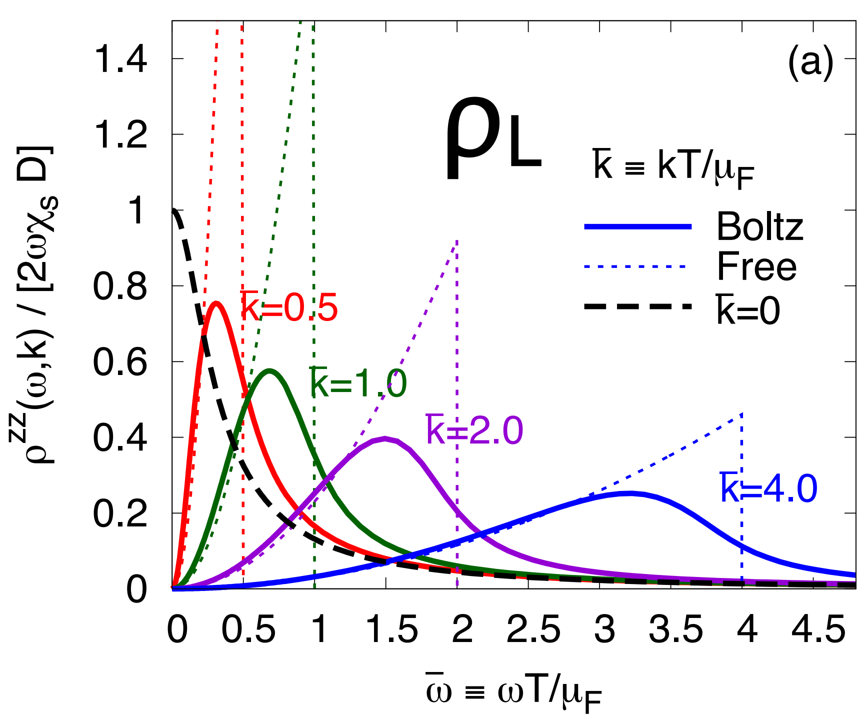
- uncertainties of spf at 1.1 T_c need to be checked

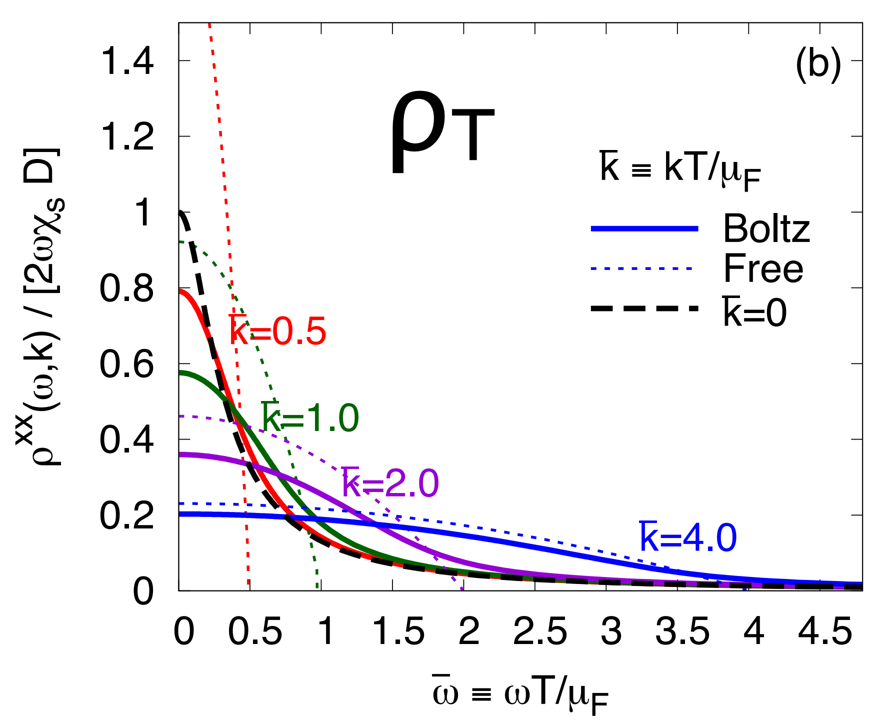
Polarized vector correlation function at finite p





Nontrivial structure seen in the low energy region of finite p spectral function

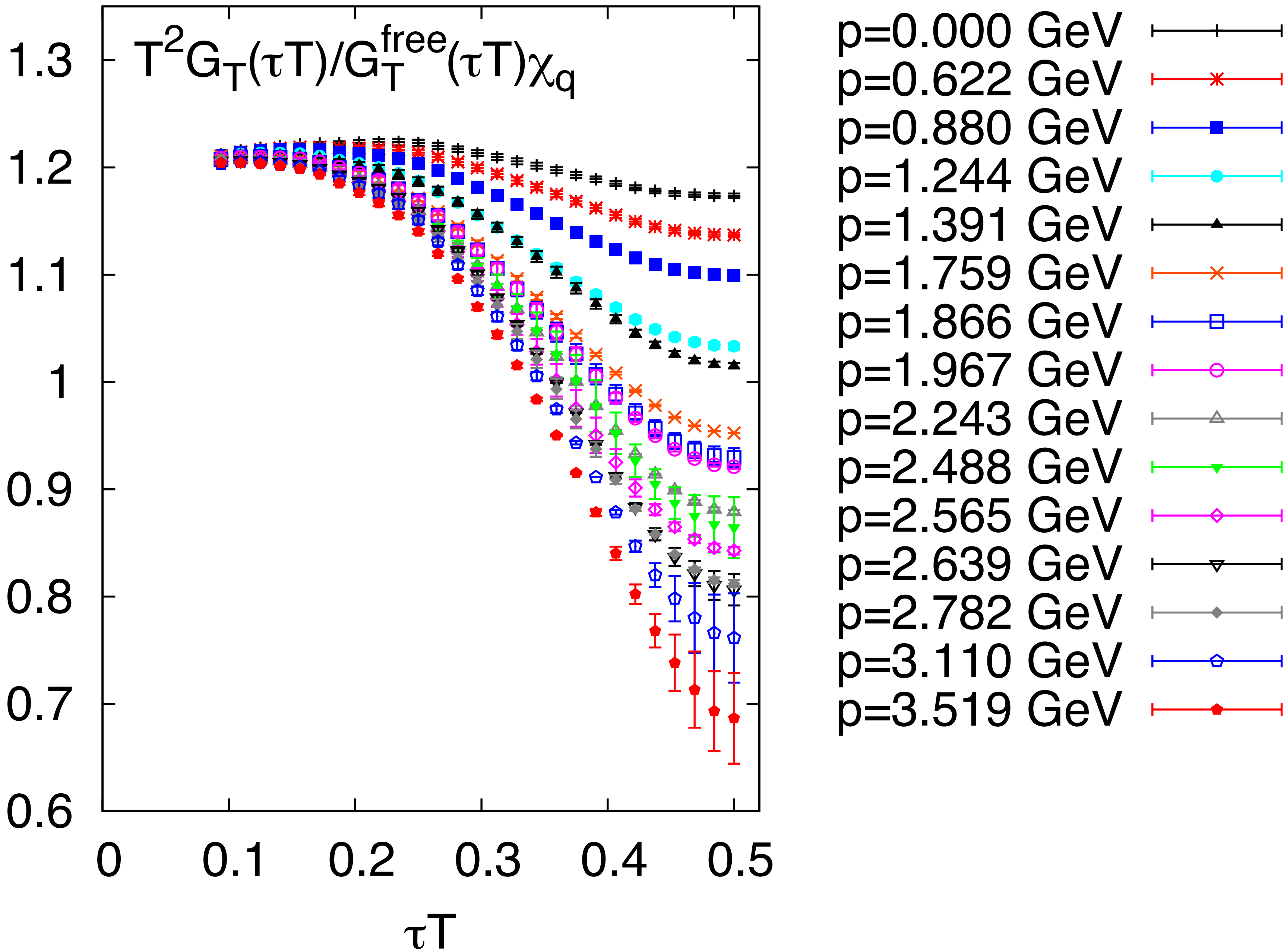




J. Hong and D. Teaney, PRC 82(2010)044908

Thermal photon rate

continuum extrapolated transverse vector correlation functions at 1.1T_c



$$\omega \frac{\mathrm{d}R_\gamma}{\mathrm{d}^3p} = C_{em} \frac{\alpha_{em}}{4\pi^2} \frac{\rho_T(\omega = |\vec{p}|, T)}{\exp(\omega/T) - 1}$$

Conclusions & Outlook

- We calculated the vector correlation function at $T \simeq 1.45T_c$ and 1.1 T_c in quenched lattice QCD and performed a continuum extrapolation

$T \simeq 1.45T_c$

- Electrical conductivity $1/3 \lesssim \frac{1}{C_{em}}\frac{\sigma}{T} \lesssim 1$
- Dilepton rate approaches leading order Born rate at $\omega/T \gtrsim 4$

$T \simeq 1.1T_c$

- Similar thermal dilepton rate and electrical cond. as that at 1.45 T_c
- Uncertainties need to be checked
- Continuum extrapolated results for vector correlator at finite p
- Extraction of thermal photon rate is on the way...

comparison of corr. at 1.1 and 1.5 T_c