Connections between dilepton data and chiral symmetry restoration

Paul Hohler
Texas A&M University

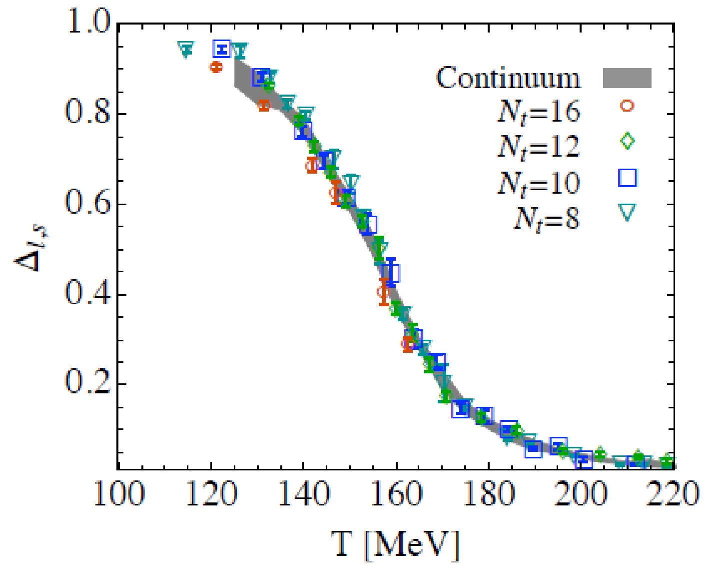
Thermal Radiation Workshop
Brookhaven National Lab
December 6, 2012

Chiral symmetry restoration

- Observing chiral symmetry restoration experimentally may be the most important outstanding problem in heavy-ion physics.

- Ideally, one would measure a chiral order parameter, such as the quark condensate.

On the lattice:



Wuppental-Budapest Collaboration

However, the chiral condensate is not directly measureable.

Need a probe: chiral partners

- Chiral partners
  - Hadronic states which transform into one another through chiral transformations (s wave pion).
  - Iso-vector vector and axial-vector states (ρ and a_1) $a_1 \leftrightarrow \rho + \pi$
  - The relative differences between chiral partners are sensitive to chiral order parameters.
- Determine the in-medium properties of ρ and a_1 mesons.
  - Vector: Thermal dileptons in heavy ion collisions $\rho \rightarrow \gamma \rightarrow e^+e^-$
  - Axial-vector: Background too large $a_1 \rightarrow \gamma\pi$

Couple the a_1 spectral function to the rho spectral function and quark condensate within one framework.

Then measure the rho spectral function and infer the a_1 spectral function and probe chiral symmetry restoration.

Techniques to connect vector and axial-vector channels

- Sum Rules
  - Relate spectral functions to operator product expansion (OPE)
- Hadronic effective field theories
  - ρ, a1, and π are dynamical degrees of freedom.
  - Couple in-medium resonances

Outline for systematic study

1. Vacuum
2. Rigorous low temperature predictions
3. Extend to higher temperatures
4. Effective field theory

Sum rules

- Weinberg type sum rules:
  - Moments of the difference between vector and axial-vector SFs
  - Directly related to chiral symmetry breaking.

$$\int ds(\rho_V - \rho_A)s^n = f_n$$

$$f_{-2} = \frac{1}{3} f_\pi^2 \langle r_\pi^2 \rangle - F_A, f_{-1} = f_\pi^2, f_0 = -m_q \langle \bar{q}q \rangle, f_1 = -2\pi\alpha_s \langle \mathcal{O}_4^{\chi SB} \rangle$$

Weinberg, 1967; Das, Mathur, and Okubo, 1967; Kapusta and Shuryak 1994

- QCD sum rules (with Borel transform):
  - Constrains vector or axial-vector SFs individually.

Shiffman, Vainshtein, Zakharov, 1979

$$\frac{1}{M^2}\int ds \frac{\rho_V(s)}{s} e^{-s/M^2} = \frac{1}{8\pi^2}\left(1+\frac{\alpha_s}{\pi}\right) + \frac{m_q\langle \bar{q}q\rangle}{M^4} + \frac{1}{24M^4}\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \rangle - \frac{56\pi\alpha_s}{81M^6}\langle \mathcal{O}_4^V \rangle$$

$$\frac{1}{M^2}\int ds \frac{\bar{\rho}_A(s)}{s} e^{-s/M^2} = \frac{1}{8\pi^2}\left(1+\frac{\alpha_s}{\pi}\right) + \frac{m_q\langle \bar{q}q\rangle}{M^4} + \frac{1}{24M^4}\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \rangle + \frac{88\pi\alpha_s}{81M^6}\langle \mathcal{O}_4^A \rangle$$

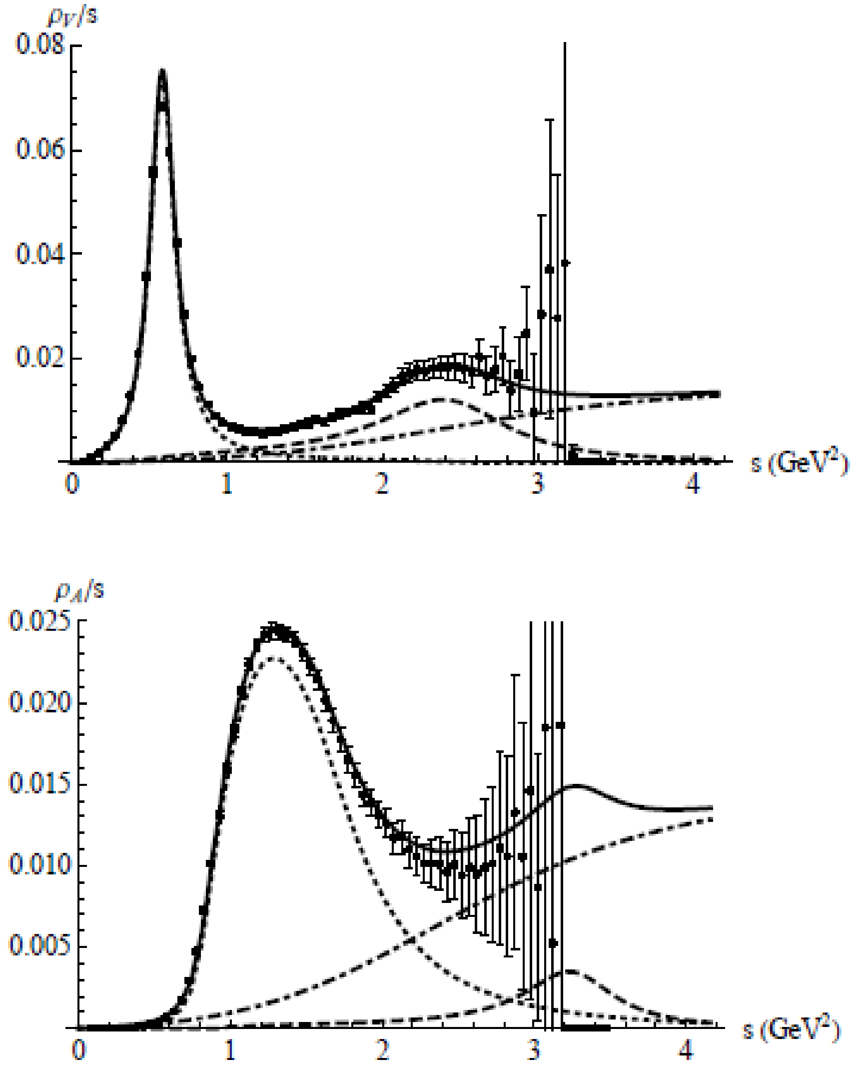
Step 1: Vacuum

- Consider a phenomenological model of the spectral functions for both the vector and axial-vector mesons.
  - Constrain the parameters by the ALEPH data (τ decay) and the Weinberg sum rules (0-2).

- Key and novel features:
  - Rho: microscopic calculation   Rapp and Wambach (1999)
  - Identical continuum in both channels
    - Smooth continuum pushes "threshold" to energies higher than previous considered
  - Include the ρ' resonance.

  - Agreement with Weinberg sum rules <u>requires</u> an excited axial vector resonance state.

Spectral functions in vacuum



PMH and Rapp, 2012

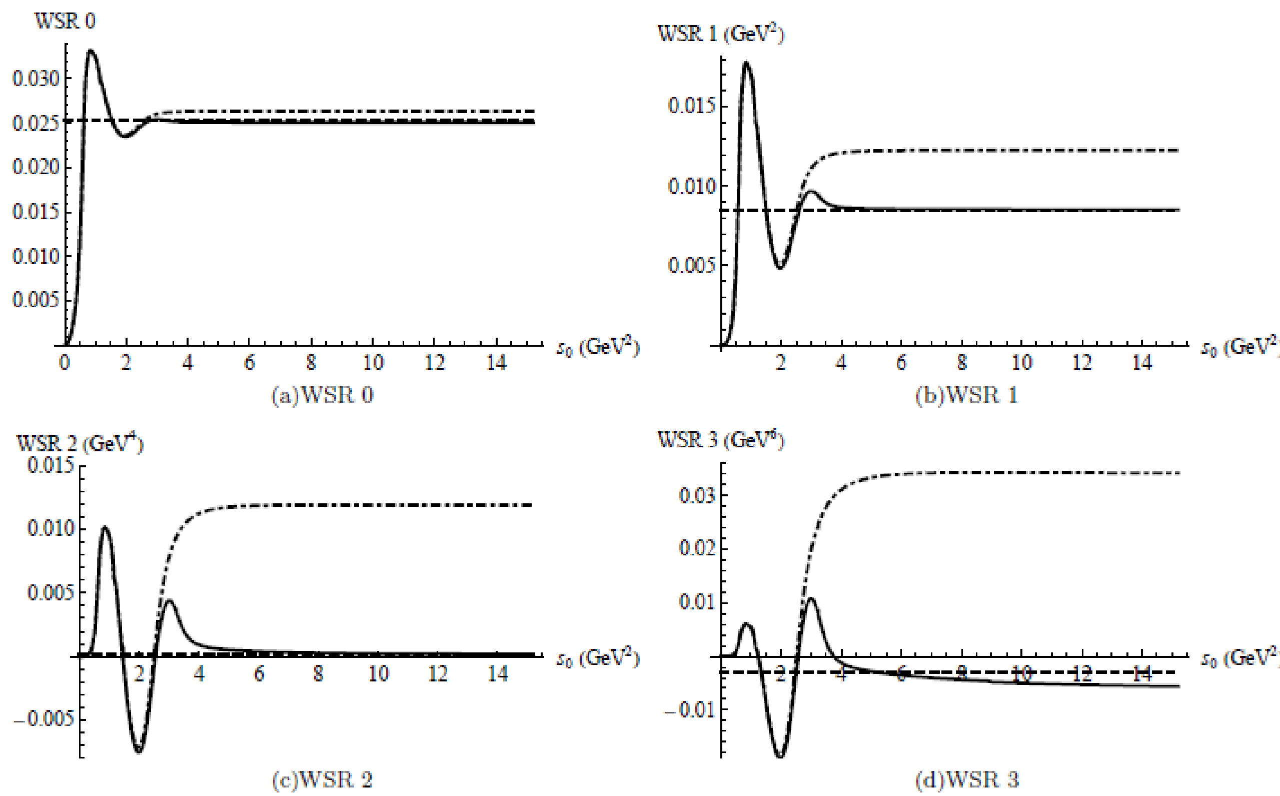
Data from ALEPH (Barate et al. 1998)

Some parameters of interest

	Mass (GeV)	Width (GeV)
ρ’	1.56	0.32
a1	1.24	0.61
a1’	1.80	0.2

How well are the sum rules satisfied?

Weinberg-type sum rules



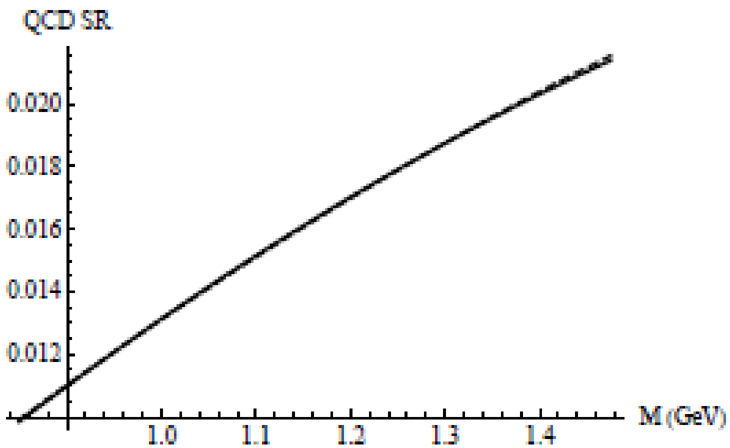
(a)WSR 0 (b)WSR 1 (c)WSR 2 (d)WSR 3

WSR 0	WSR 1	WSR 2	WSR 3
1.28%	~0%	~0%	-96%

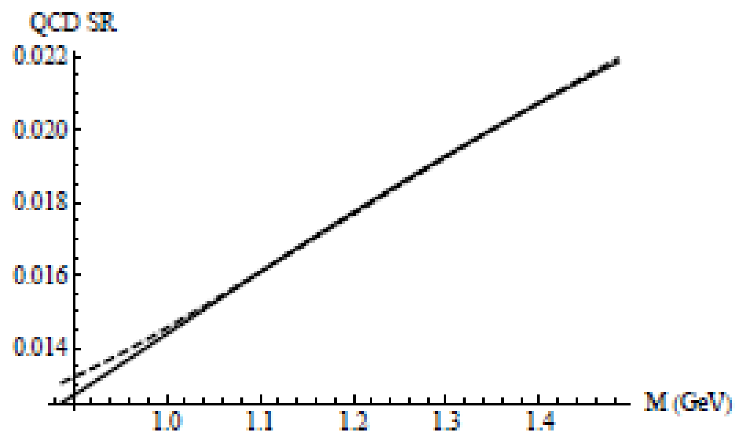
QCD sum rules

Chose the values for κ and the gluon condensate so that sum rules are satisfied.

$$\langle \mathcal{O}_4^V \rangle_0 = \langle \mathcal{O}_4^A \rangle_0 = \kappa \langle \bar{q} q \rangle_0^2 \qquad \kappa = 2.1^{+.3}_{-.2} \qquad \langle \frac{\alpha_s}{\pi} G^2 \rangle = .022 \pm .002 \mathrm{GeV}^4$$



(a)Vector Channel



(b)Axial-Vector Channel

$$d_V = 0.24\% \qquad d_A = 0.56\%$$

In-medium

- Condensates develop a temperature dependence and new non-scalar operators become available for the OPE.
  - Input needed for analysis
- The sum rules then translate these changes of the condensates into modifications of the spectral function.

$$\frac{1}{M^2}\int ds \frac{\rho_V(s)}{s} e^{-s/M^2} = \frac{1}{8\pi^2}\left(1+\frac{\alpha_s}{\pi}\right) + \frac{m_q\langle\bar{q}q\rangle}{M^4} + \frac{1}{24M^4}\langle\frac{\alpha_s}{\pi}G^2_{\mu\nu}\rangle - \frac{56\pi\alpha_s}{81M^6}\langle\mathcal{O}_4^V\rangle$$

$$\frac{1}{M^2}\int ds \frac{\bar{\rho}_A(s)}{s} e^{-s/M^2} = \frac{1}{8\pi^2}\left(1+\frac{\alpha_s}{\pi}\right) + \frac{m_q\langle\bar{q}q\rangle}{M^4} + \frac{1}{24M^4}\langle\frac{\alpha_s}{\pi}G^2_{\mu\nu}\rangle + \frac{88\pi\alpha_s}{81M^6}\langle\mathcal{O}_4^A\rangle$$

Reduction of condensates produces a need for more lower energy spectral strength.

Step 2: Rigorous low temperature prediction

- At low temperatures, in-medium effects are dominated by interaction with thermal pions.

Dey, Eletsky, Ioffe, 1990

$$\rho_V(T) = \rho_V(T=0)(1-\epsilon) + \rho_A(T=0)\epsilon$$

$$\rho_A(T) = \rho_A(T=0)(1-\epsilon) + \rho_V(T=0)\epsilon$$

$$\epsilon = \frac{2}{f_\pi^2}\int \frac{d^3p}{(2\pi)^3 E_p} n_B(E_p)$$

- Temperature dependence of condensates also govern by pions.

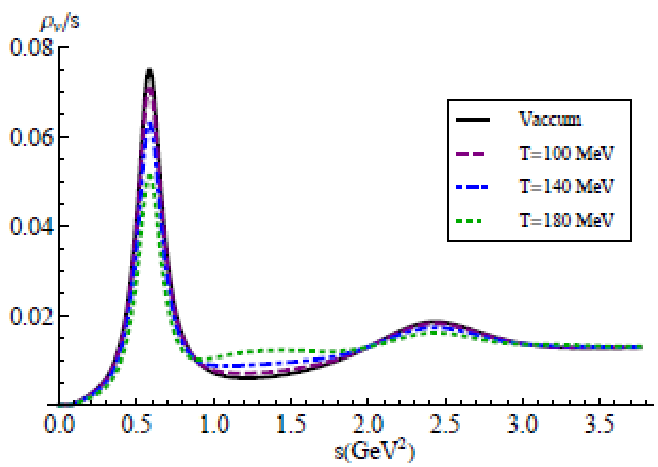
$$\langle \bar{q}q\rangle(T) = \langle \bar{q}q\rangle(0)\left(1 - \frac{3}{4}\epsilon\right)$$

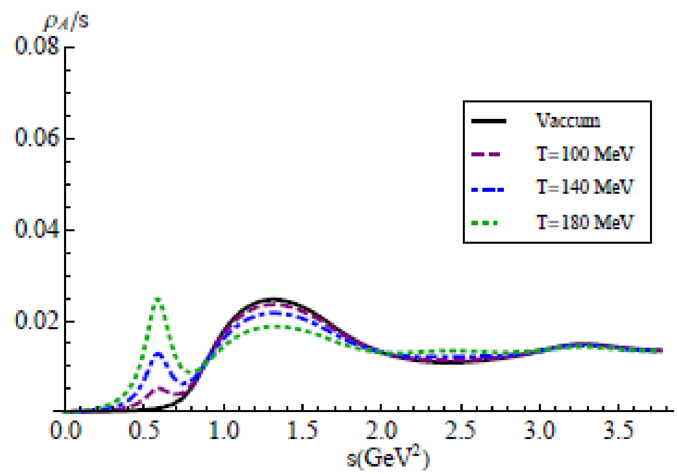
Hatsuda, Koike, Lee, 1993; Steele, Yamagishi, Zahed, 1996; Chanfrey, Delorme, Erison, 1998; Krippa 1998; Marco, Hoffman, Weise, 2002; Kwon, Sasaki, Weise, 2010;Etc.

How does one implement chiral mixing with different continuum thresholds?

What effect does a finite pion mass have on analysis?

With smooth continuum, no ambiguity in mixing.





Holt, PMH, Rapp, 2012

- Flattening
- Trend to one-another

How high in T can analysis be taken?

- $m_\pi = 0$

Marco, Hoffman, Weise, 2002

  - Both WSR and QCDSR are exactly satisfied to order ε
  - Low temperature prescription persists to high (all) temperatures.

- $m_\pi \neq 0$
  - WSR is still satisfied
  - Numerical evaluation is need for QCDSR

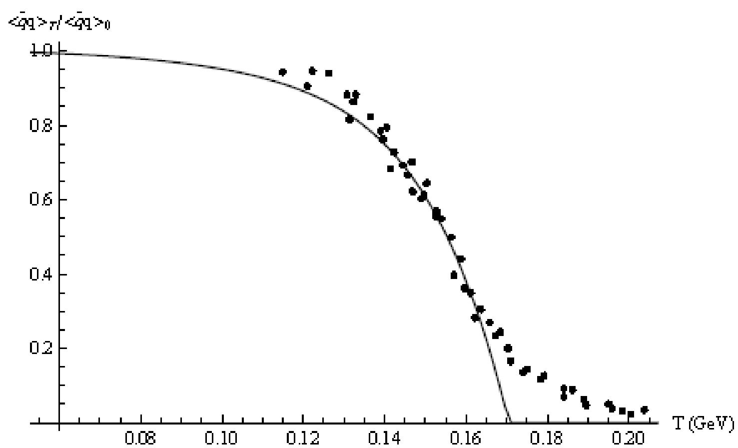
T (MeV)	0	100	120	140	160
ε	0	.06	.1	.16	.23
dV (%)	.24	.32	.48	.85	1.43
dA (%)	.56	.65	.78	1.05	1.6

QCDSRs self-limiting

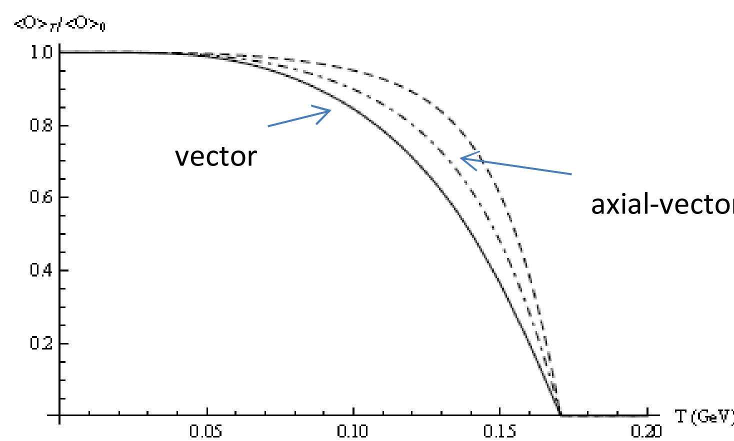
Need additional physics beyond $T{\sim}m_\pi$

Step 3: Beyond low temperatures

- Low temperature study revealed a need for more resonances
  - Model the temperature dependence of condensates on a modified Hadron Resonance Gas
    - All established resonances with mass less than 2 GeV included.



Quark condensate

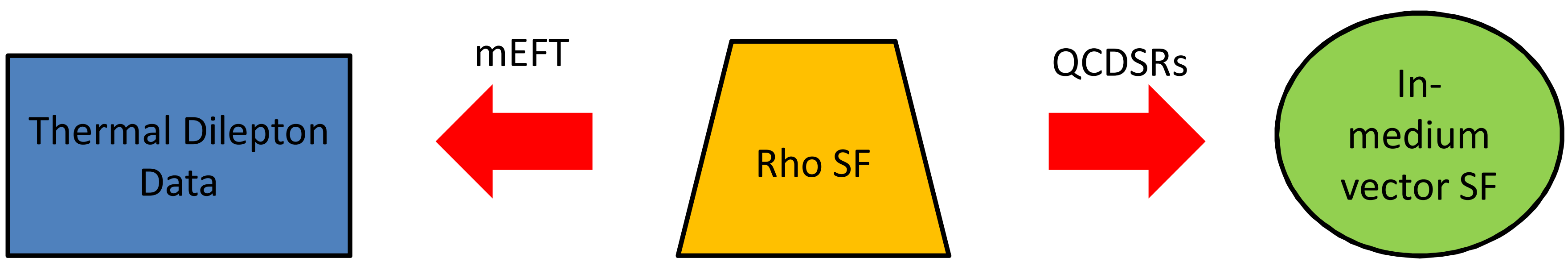


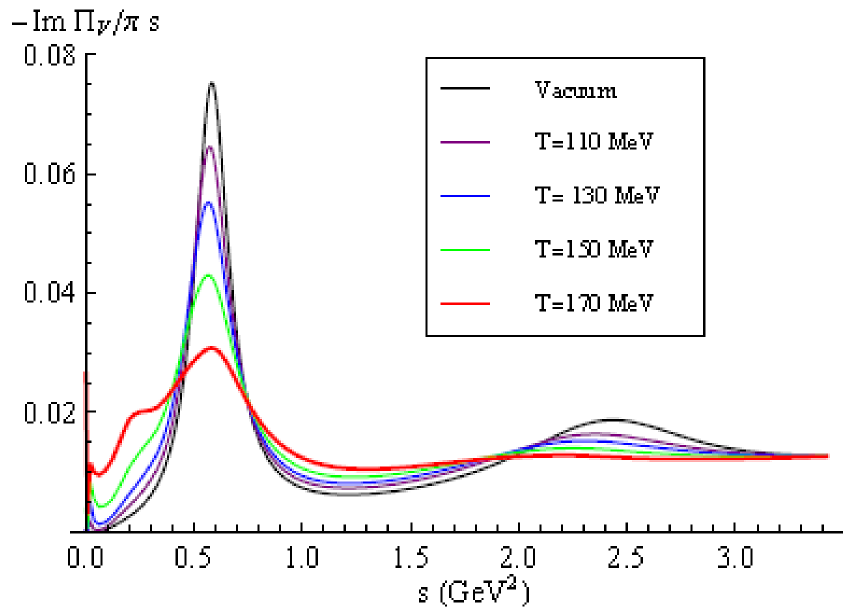
4-quark condensate

"modified HRG" = HRG + T^{10} term

Reduction in condensates induce changes in SFs

- Vector Channel
  - ρ spectral function. Rapp and Wambach, 1999
    - Provides a handle to base the rest of the study.
    - THESE SFS ARE CONSISTENT WITH DILEPTON MEASUREMENTS.
    - Spectral strength is allowed to vary slightly (deviations to VMD)
  - ρ’ spectral function
    - Adjust mass, width, spectral strength
  - Continuum has no temperature dependence.
  - QCDSR determine temperature dependence





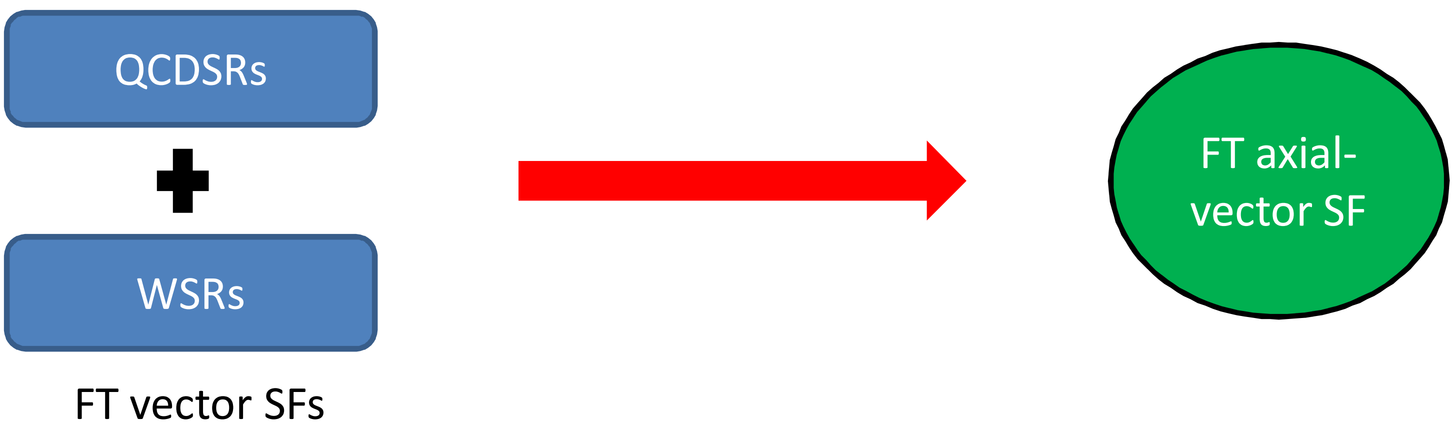
ρ peak develops a low energy shoulder and a reduction in strength

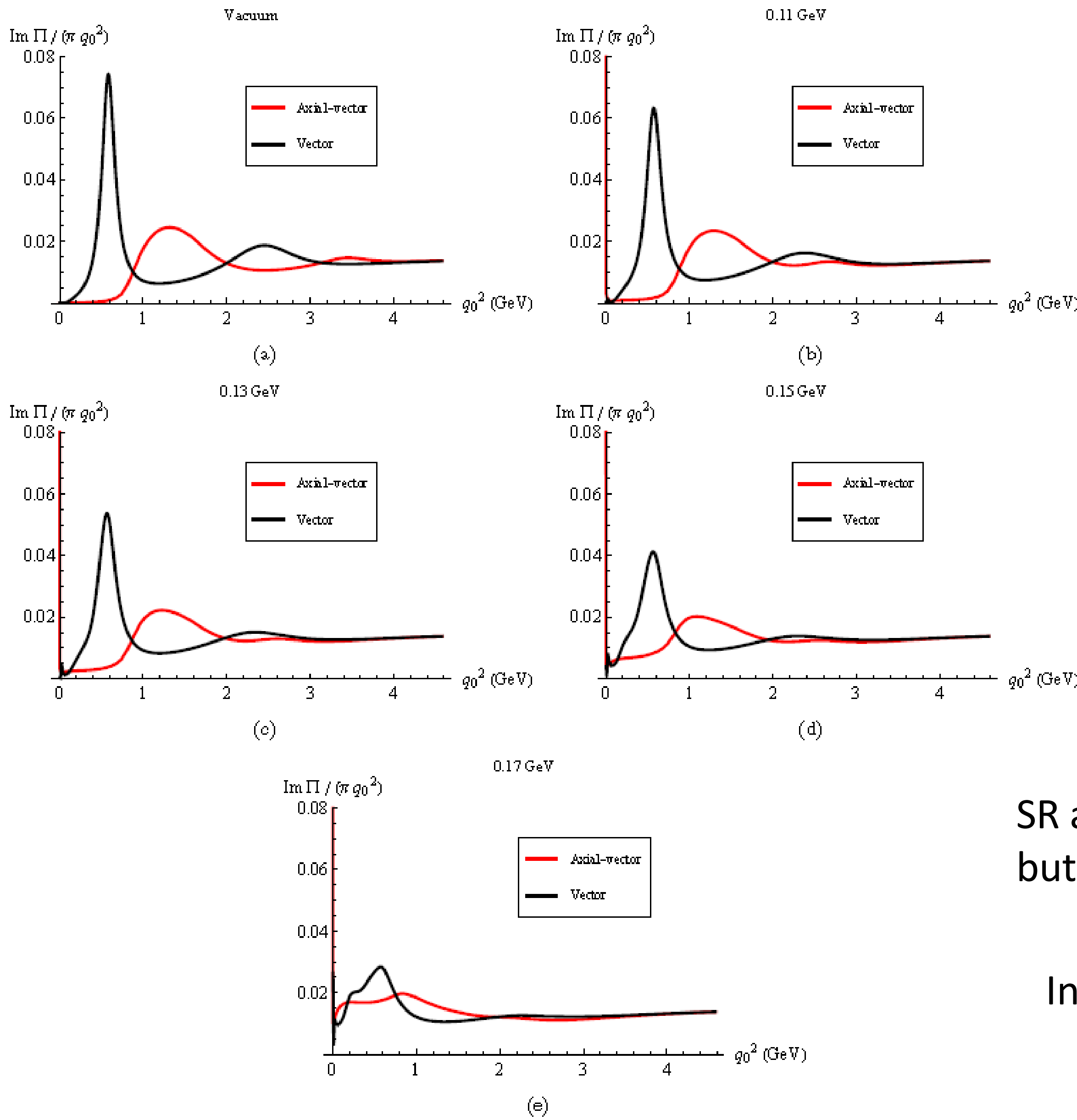
ρ' peak is reduced and flattens out.

Correction to VMD needed range from <1% to 6%.

QCDSR are satisfied with deviations ranging from 0.44% to 0.75%.

- Axial-Vector Channel
  - a1' peak:
    - Adjust the mass, width, and spectral strength.
  - a1 peak:
    - Adjust mass, width, and spectral strength
    - Additional width component at low energies. (Needed for axial-vector "conductivity" and broadening below threshhold.)
    - Additional low energy peak – "a-sobar"
  - Pion pole:
    - Assume that pion does not develop a width.
    - Temperature dependence of pion mass is chosen from XPT.
    - Temperature dependence of f_π taken from quark condensate and GOR.





- Peaks shift and broaden
- New low energy structure
- Trend towards agreement

- WSR 1-2 satisfied <1%
- QCDSR satisified 0.45% -0.95%

SR analysis can give precise SF, but some ambiguity remains.

Info beyond SRs is needed

Step 4: Hadronic effective field theory

- Massive Yang-Mills

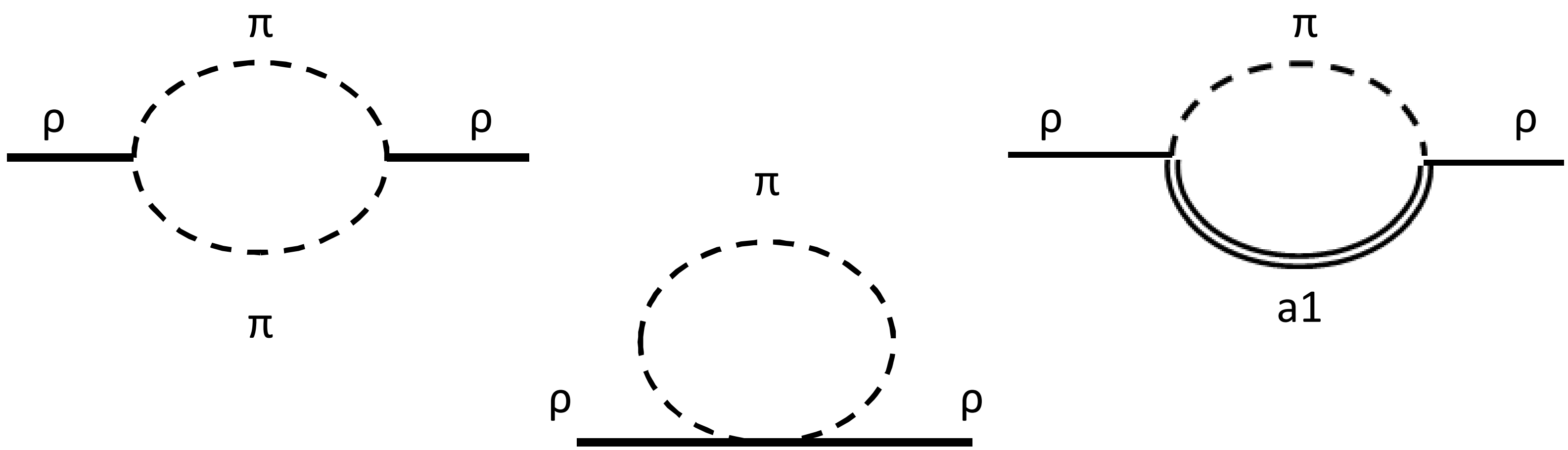
Gomm, Kaymakcalan, and Schecter, 1984; Ko and Rudaz, 1994, etc.

  - Vector and axial-vector SFs and quark condensate can all be calculated simultaneously within this framework.

  - Pions are implemented by a non-linear sigma model
  - Gauge theory with two local chiral gauge symmetries. $SU_L(2) \times SU_R(2)$
    - Perserves chiral symmetry
  - Vector and axial-vector mesons are represented by the corresponding gauge bosons.
  - Gauge symmetry is broken by an explicit mass term for the mesons.

  - Lagrangian has 4 free parameters: m0, g, σ, and ξ.
    - Will use m_ρ, m_a, $g_{\rho\pi\pi 1}$, $g_{\rho\pi\pi 3}$

Vector self energy diagrams

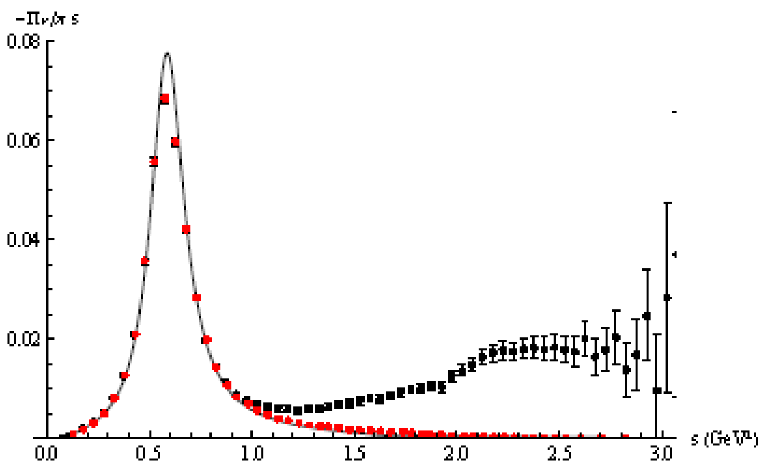


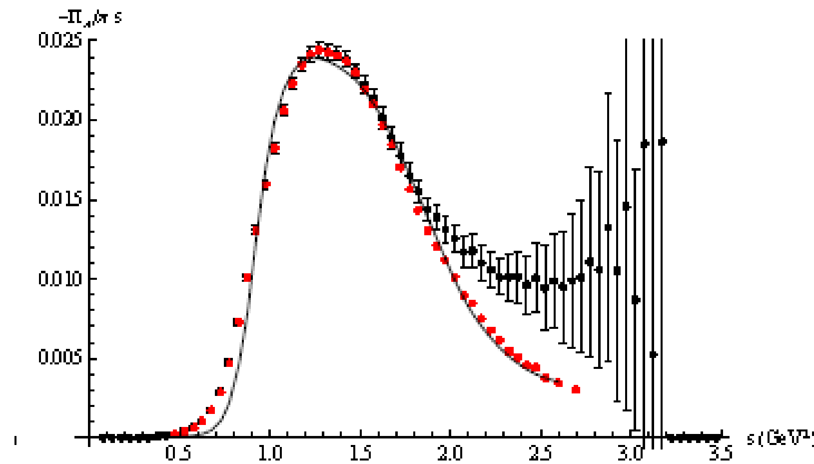
Axial-vector self energy diagrams

Calculate 1-loop diagrams and then resum.

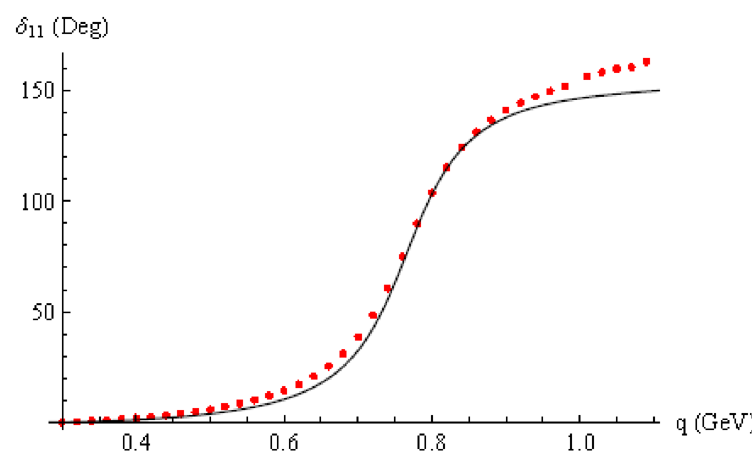
Vacuum Spectral functions

Data from ALEPH (Barate et al. 1998)





	Calc.	Expt.
$\Gamma(a_1 \to \gamma\pi)$	828 keV	640±246 keV
D/S	-0.084	-0.09±.03



Data from Froggatt and Petersen, 1977

In-medium calculation on the way.

Summary

- Explored the connection between vector and axial-vector SFs in order to probe chiral symmetry restoration.

- Vacuum
  - Spectral functions were constructed which agreed with SRs
  - Sum rules indicate a need for excited axial-vector state
- Low temperatures
  - SF constructed in a rigorous low temperature prescription
  - QCDSR along with finite pion mass indicate that analysis is limited in T
- Beyond low temperatures
  - Rho SF used from microscopic model which agrees with dilepton data
  - Constructed axial-vector SF from sum rule analysis
  - SF exhibit a shift of spectral strength to lower energies
  - SRs give precise SFs but there remains some ambiguity
- Hadronic effective theory
  - Constructed vacuum fits
  - More work is need for in-medium study.