Photon Production at NLO in Hot QCD

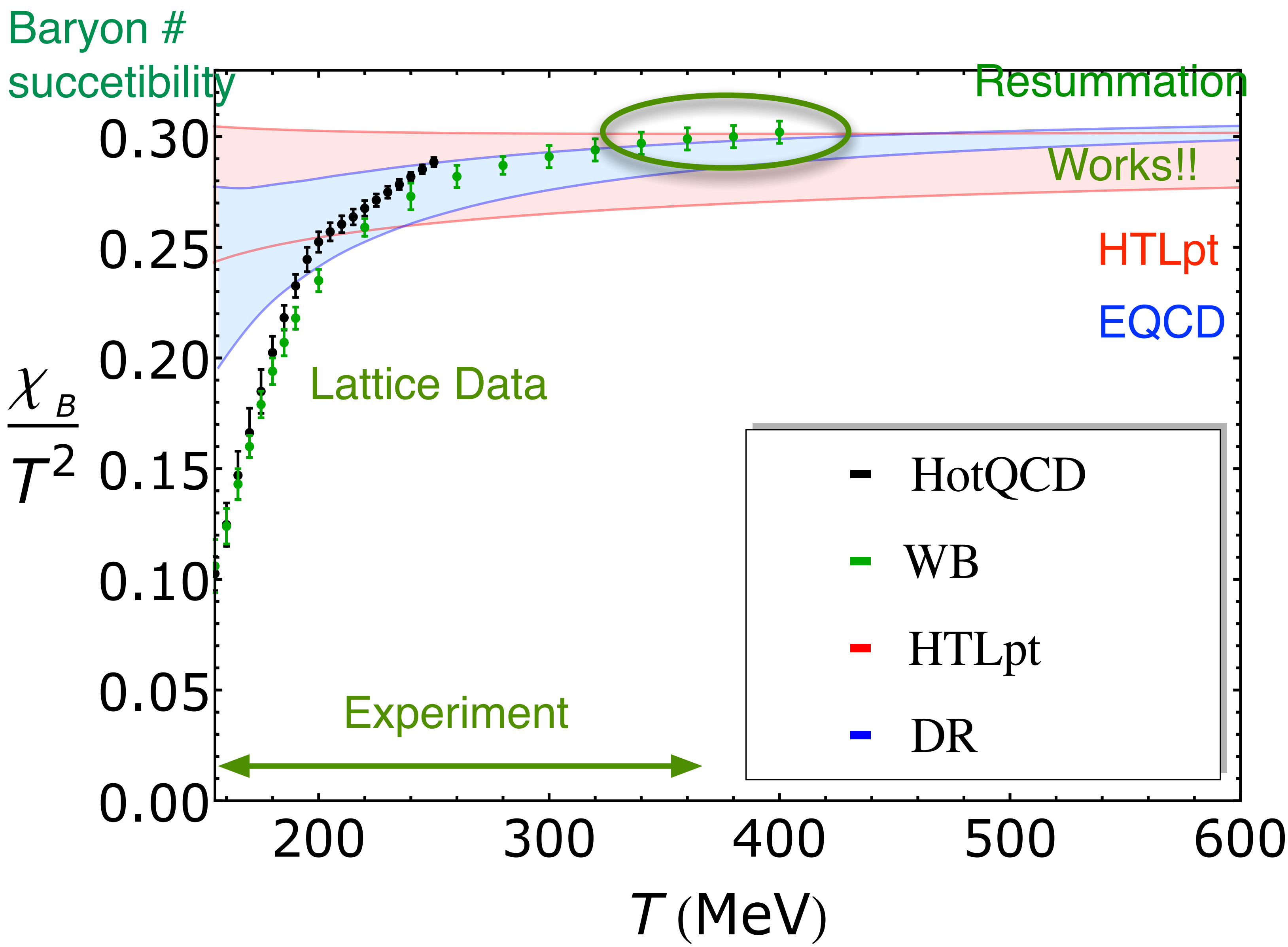
Derek Teaney

SUNY Stony Brook and RBRC Fellow

- Photons – In collaboration with Jacopo Ghiglieri, Juhee Hong, Aleksi Kurkela, Egang Lu, Guy Moore, arXiv:Almost.Done

Perturbation theory can work for thermodynamic quantities! Let's use it!

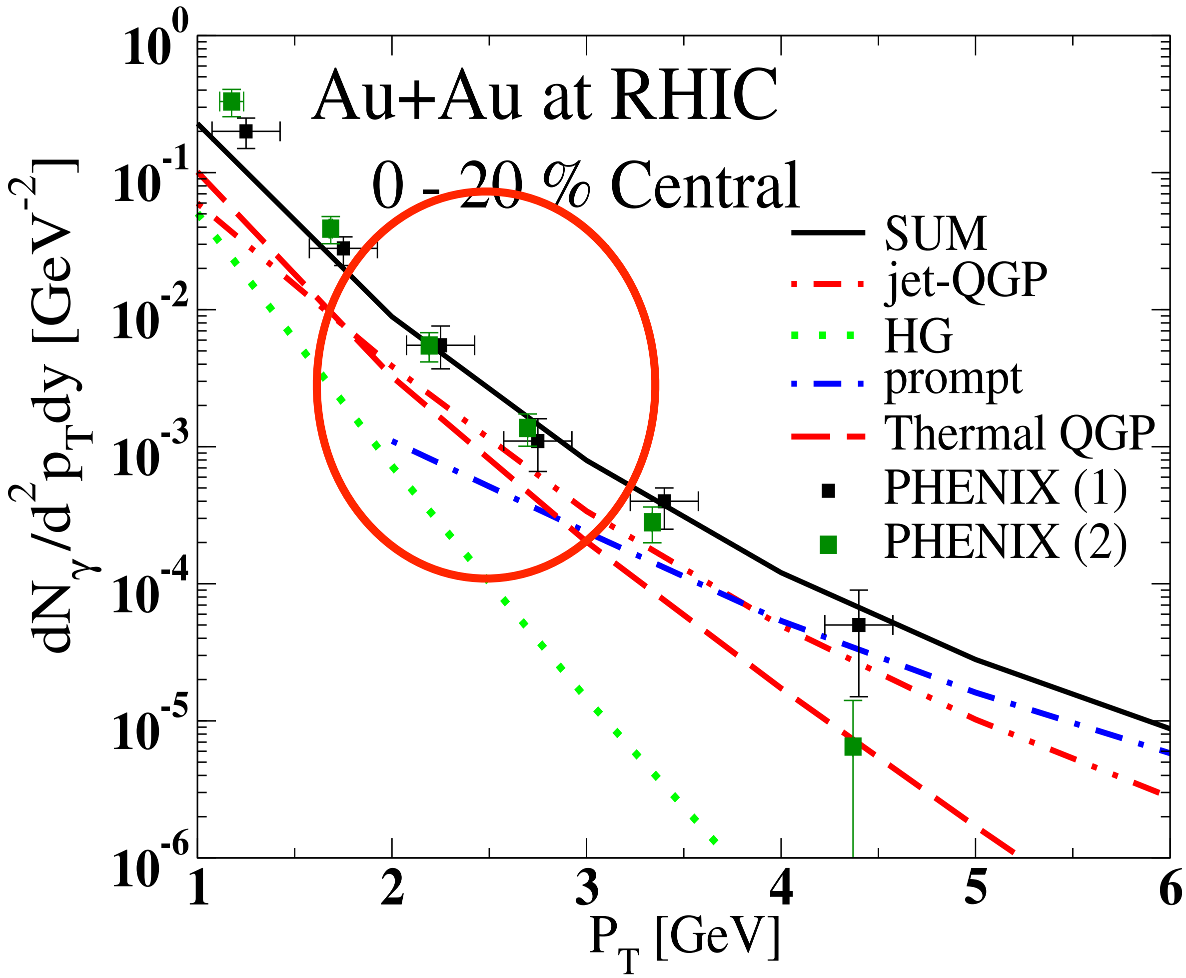
- HTLpt from Andersen, Su, Strickland. Dimensional Reduction/EQCD – the Finish Group



Want to compute transport with similar precision at high T

Motivation

- This calculation uses LO order photon production rates (Turbide, Rapp, Gale)



We want to compute this rate at NLO

Thermal rate is dominant for a certain momentum range

Direct photons are measured, but this is not my real motivation . . .

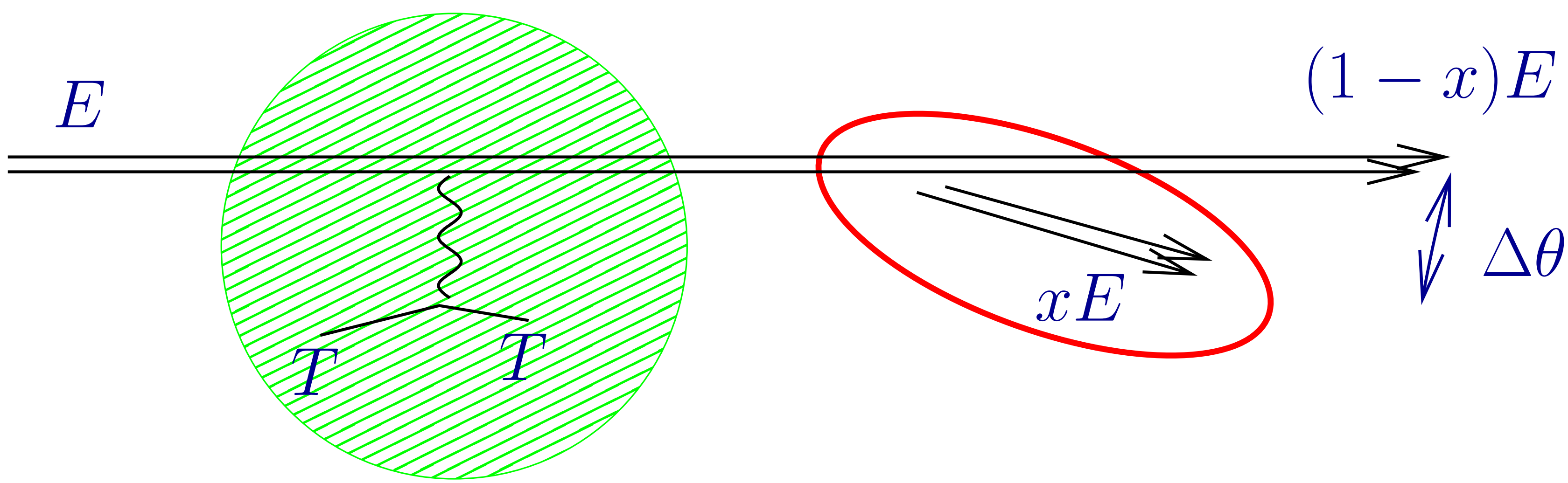
My real motivations:

1. Energy loss.

2. The shear viscosity.

My real motivation. Energy loss at sub-asymptotic energies is important:

1. Kinematic constraints limit the agreement between energy loss formalisms
   - See the report of the Jet Collaboration: arXiv:1106.1106
2. Finite energy leads to large angle emission outside of radiative loss formalism



As the bremmed energy gets lower and lower, the angle $\Delta\theta$ gets larger and larger, limiting the agreement

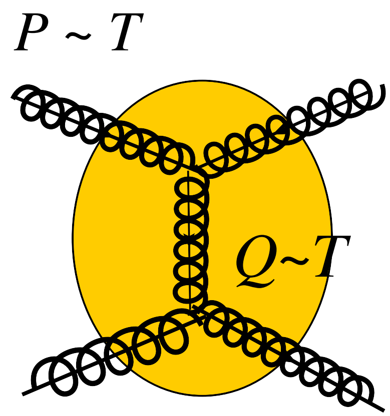
My real motivations:

✓ Energy loss

2. The shear viscosity

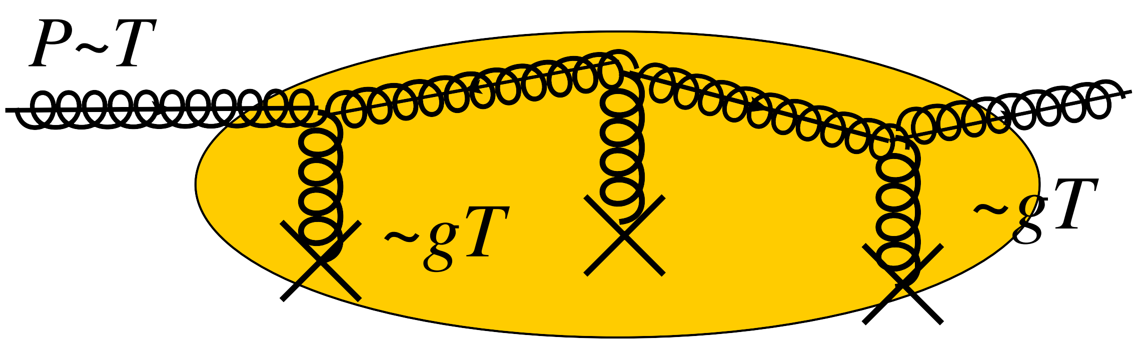
My real motivation. Shear viscosity and the kinetics of weakly coupled QGP

1. Hard Collisions: $2 \leftrightarrow 2$



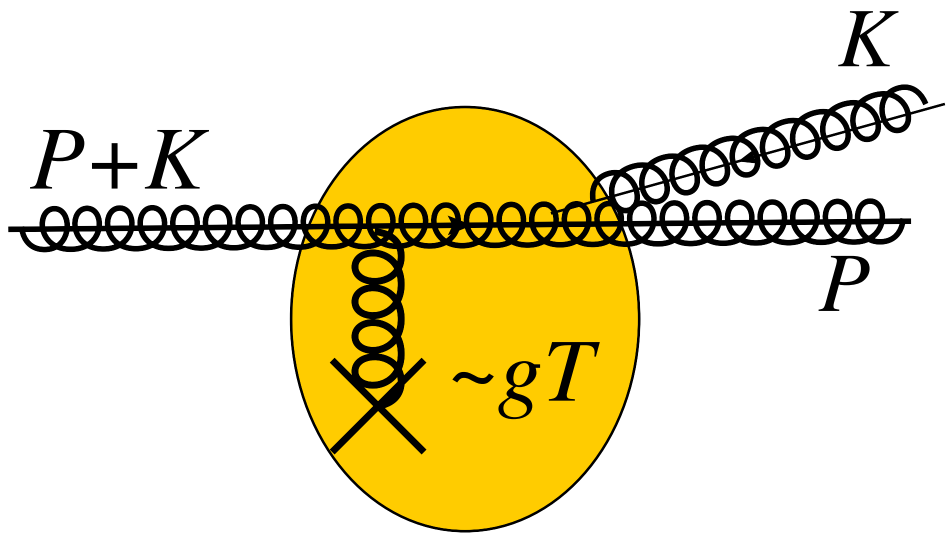
2. Diffusion: collisions with soft random classical field

soft fields have $p \sim gT$ and *large occupation numbers* $n_B \sim \frac{T}{p} \sim \frac{1}{g}$



3. Brem: $1 \leftrightarrow 2$

- random walk induces collinear bremsstrhalung



NLO involves corrections to these processes and the relation between them

But shear viscosity is too hard . . .

My real motivations:

- ✓ Energy loss
- ✓ The shear viscosity

Photon production at NLO is a good warm-up calculation.

Lets do it!

$$2k(2\pi)^3 \frac{d\Gamma}{d^3k} = \text{Photon emission rate per phase-space}$$

The photon emission rate at weak coupling:

- The rate is function of the coupling coupling constant and k/T:

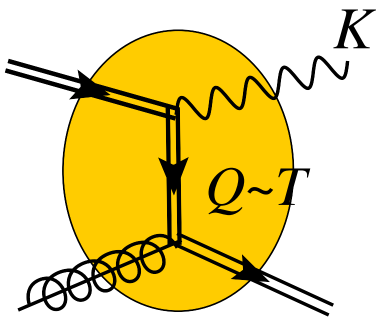
$$2k(2\pi)^3 \frac{d\Gamma}{d^3k} \propto e^2 T^2 \Big[\underbrace{O(g^2 \log) + O(g^2)}_{\text{LO AMY}} + \underbrace{O(g^3 \log) + O(g^3)}_{\text{From soft } gT \text{ gluons, } n_B \simeq \frac{T}{\omega} \simeq \frac{1}{g}} + \ldots$$

$O(g^3)$ is closely related to open issues in energy loss:

- At NLO must include drag, collisions, bremsstrhalung, and kinematic limits

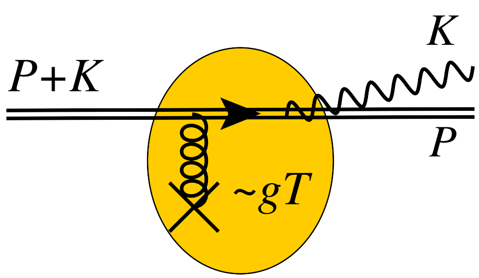
Three rates for photon production at Leading Order — Baier, Kapusta, AMY

1. Hard Collisions – a $2 \leftrightarrow 2$ processes



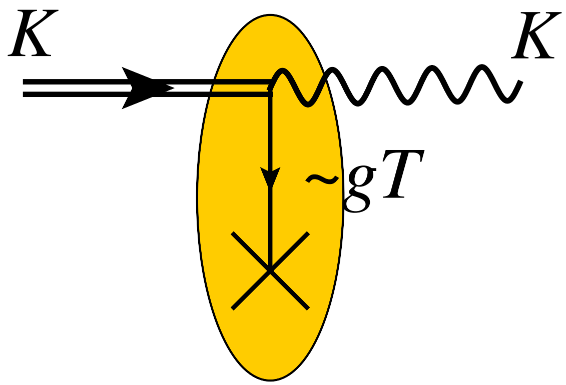
$$\sim e^2 \underbrace{m_\infty^2}_{g^2 C_F T^2/4} \times \underbrace{n_F(k)}_{\text{fermi dist.}} \times \left[\log\left(T/\mu\right) + C_{2\mathrm{to}2}(k) \right]$$

2. Collinear Bremsstrhalung – a $1 \leftrightarrow 2$ processes

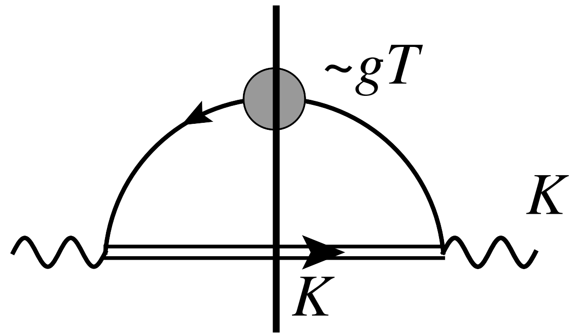


$$\sim e^2 \, m_\infty^2 n_F \left[\underbrace{C_{\mathrm{brem}}(k)}_{\text{LPM + AMY and all that stuff!}} \right]$$

3. Quark Conversions – $1 \leftrightarrow 1$ processes (analogous to drag)



or



$$= \sim e^2 m_\infty^2 n_F \left[\log(\mu_\perp/m_\infty) + C_{\rm cnvrt}\right]$$

Full LO Rate is independent of scale $\mu_\perp$:

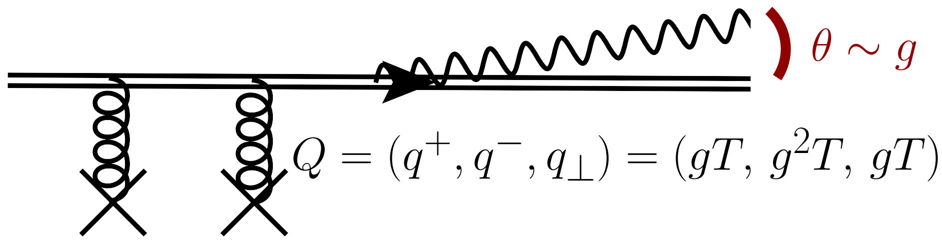
$$2k\frac{d\Gamma}{d^3k} \propto e^2 m_\infty^2 n_F \Big[\log\left(T/m_\infty\right) + \underbrace{C_{\rm cnvrt} + C_{\rm brem}(k) + C_{\rm 2to2}(k)}_{\equiv C_{LO}(k)} \Big]$$

$O(g)$ Corrections to Hard Collisions, Brem, Conversions:

1. No corrections to Hard Collisions:

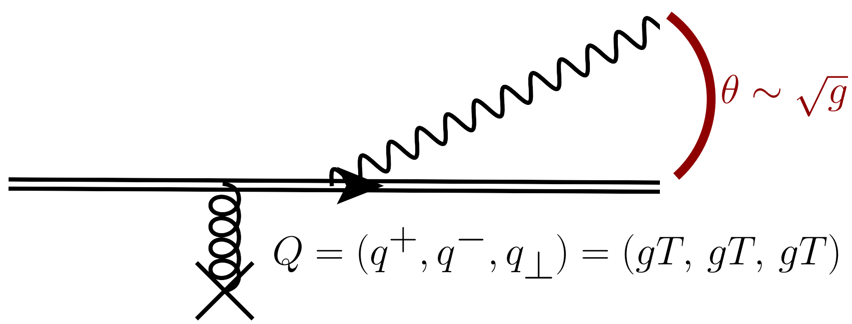
2. Corrections to Brem:

   (a) Small angle brem. Corrections to AMY coll. kernel. (Caron-Huot)

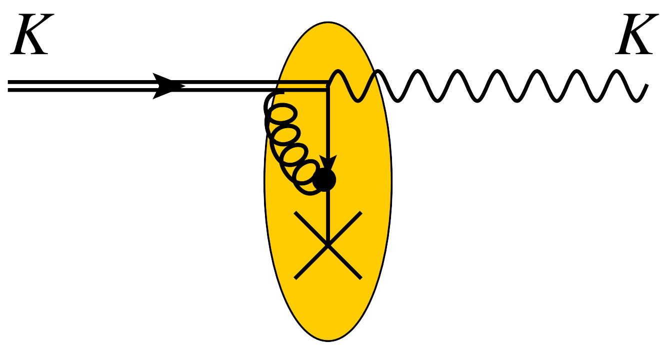


$$C_{LO}[q_\perp] = \frac{T g^2 m_D^2}{q_\perp^2 (q_\perp^2 + m_D^2)} \rightarrow \text{A complicated but analytic formula}$$

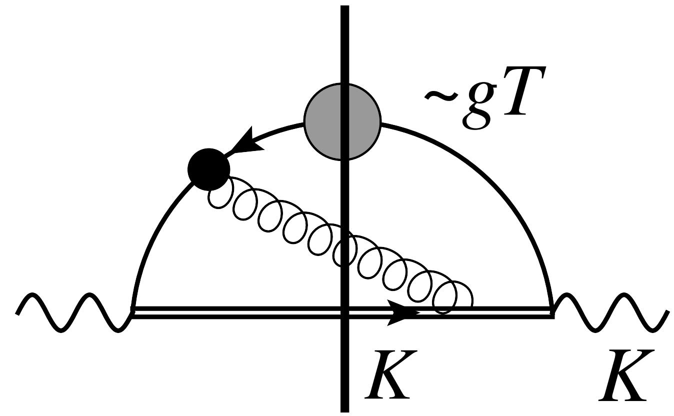
   (b) Larger angle brem. Include collisions with energy exchange, $q^- \sim gT$.



3. Corrections to Conversions:



or



- Doable because of HTL sum rules (light cone causality)

Simon Caron-Huot

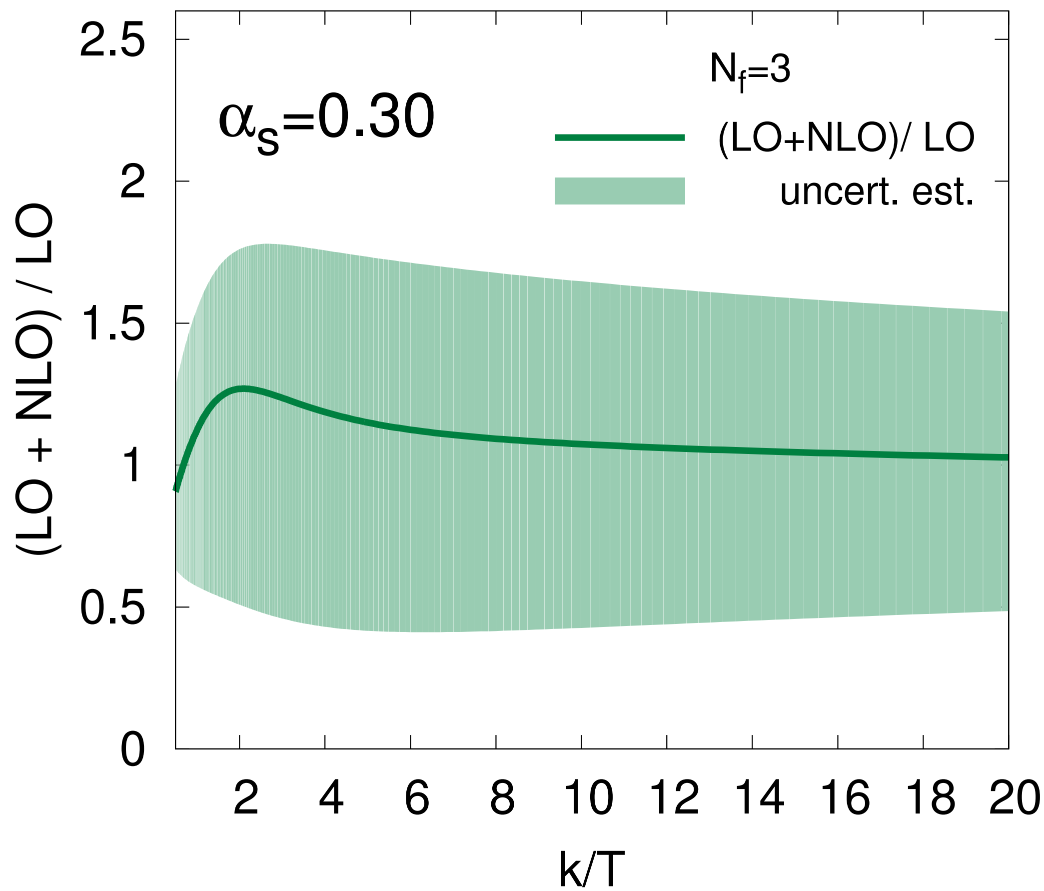
- Gives a numerically small and momentum indep. contribution to the NLO rate

Full results depend on all these corrections.

These rates smoothly match onto each other as the kinematics change.

NLO Results: $\Gamma_{LO+NLO} \sim \text{LO} + g^3 \log(1/g) + g^3$

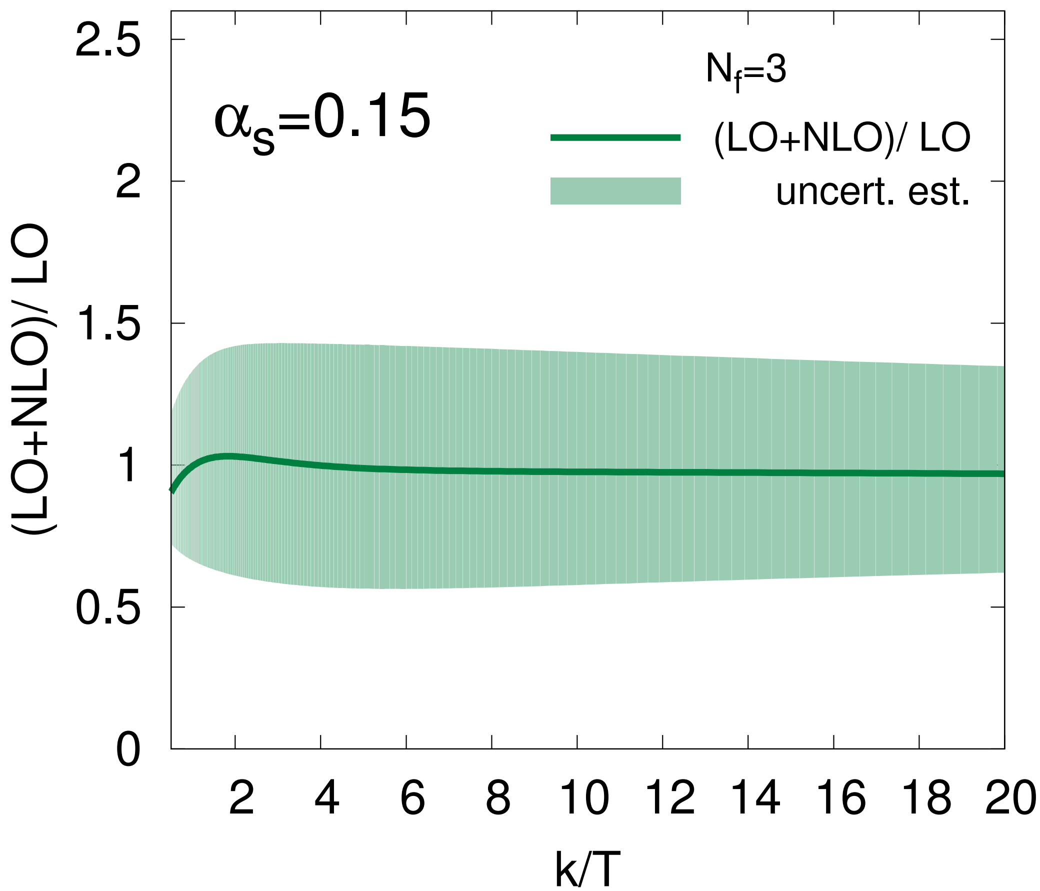
$$2k\frac{\mathrm{d}\Delta\Gamma_{NLO}}{\mathrm{d}^3k} \propto e^2 m_\infty^2 n_F(k)\Big[\overbrace{\frac{\delta m_\infty^2}{m_\infty^2}\log\left(\frac{\sqrt{2Tm_D}}{m_\infty}\right)}^{\text{conversions}} + \overbrace{\frac{\delta m_\infty^2}{m_\infty^2} C_{\text{large}-\theta}(k)}^{\text{large-}\theta\text{-brem}} + \overbrace{\frac{g^2 C_A T}{m_D} C_{\text{small}-\theta}(k)}^{\text{small-}\theta\text{-brem}}\Big]$$



Corrections are small and k independent

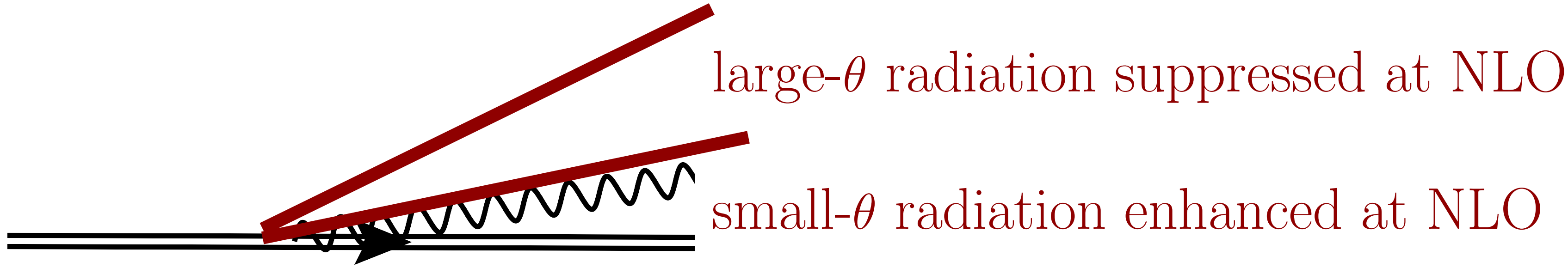
NLO Results: $\Gamma_{LO+NLO} \sim \text{LO} + g^3 \log(1/g) + g^3$

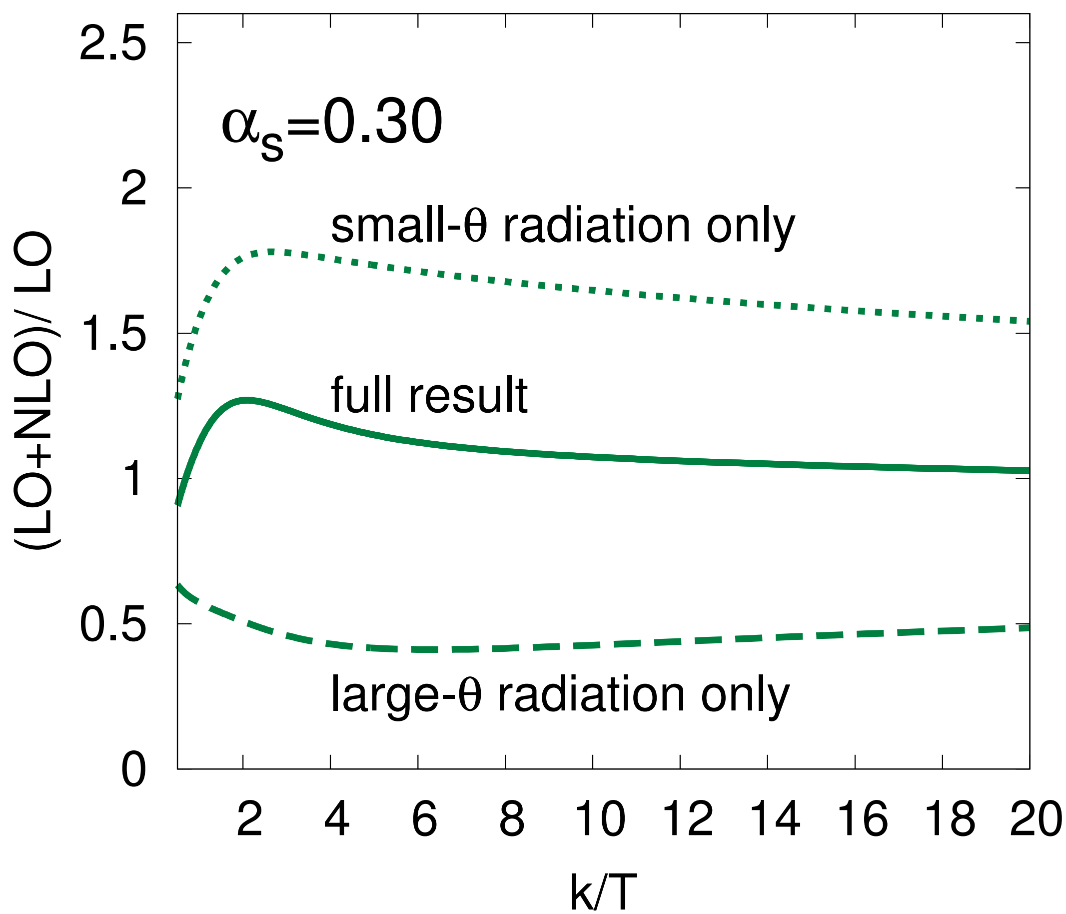
$$2k\frac{\mathrm{d}\Delta\Gamma_{NLO}}{\mathrm{d}^3k} \propto e^2 m_\infty^2 n_F(k)\Big[\overbrace{\frac{\delta m_\infty^2}{m_\infty^2}\log\left(\frac{\sqrt{2Tm_D}}{m_\infty}\right)}^{\text{conversions}} + \overbrace{\frac{\delta m_\infty^2}{m_\infty^2}C_{\text{large}-\theta}(k)}^{\text{large-}\theta\text{-brem}} + \overbrace{\frac{g^2 C_A T}{m_D}C_{\text{small}-\theta}(k)}^{\text{small-}\theta\text{-brem}}\Big]$$



NLO Corrections are small and k independent

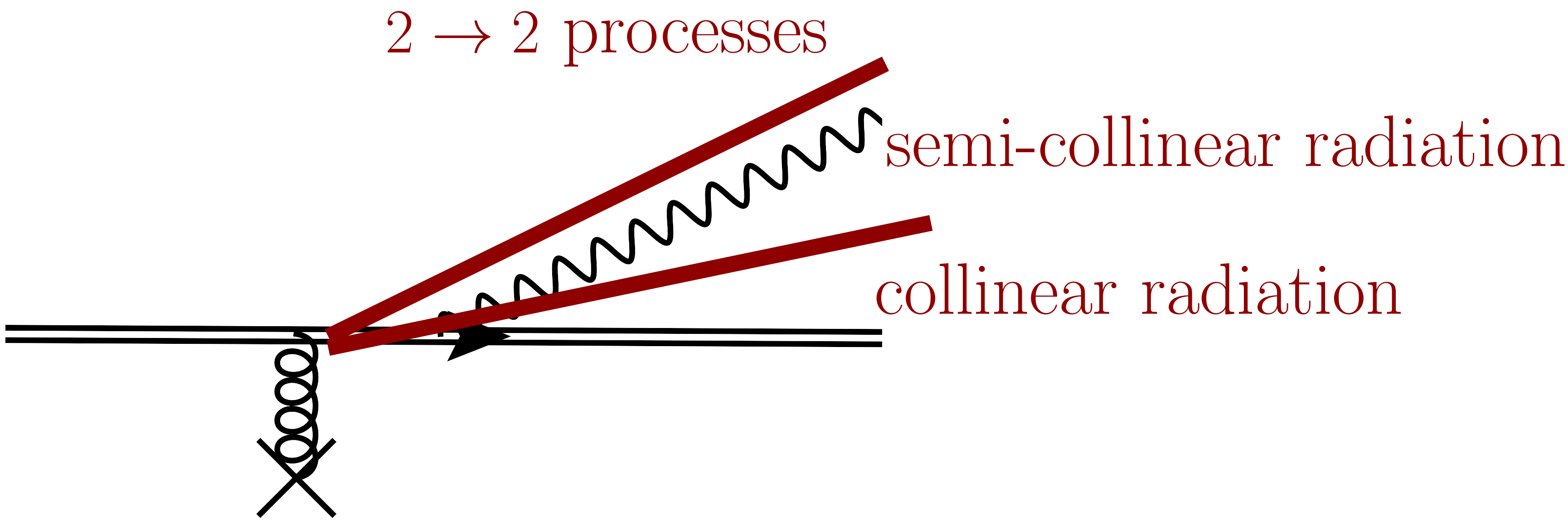
The different contributions at NLO (conversions are not numerically important)





The calculation

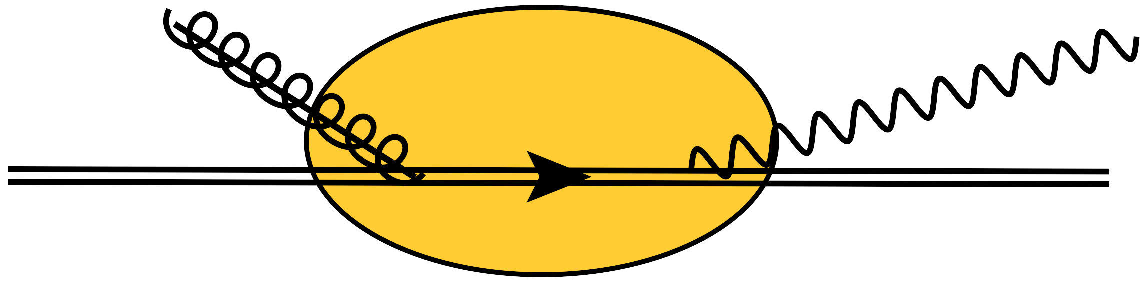
Semi-collinear radiation – a new kinematic window



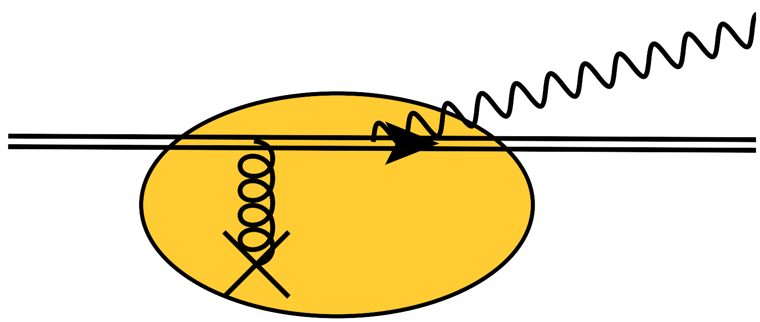
The semi-collinear regime interpolates between brem and collisions

Matching collisions to brem

- When the gluon is hard the $2 \leftrightarrow 2$ collision:

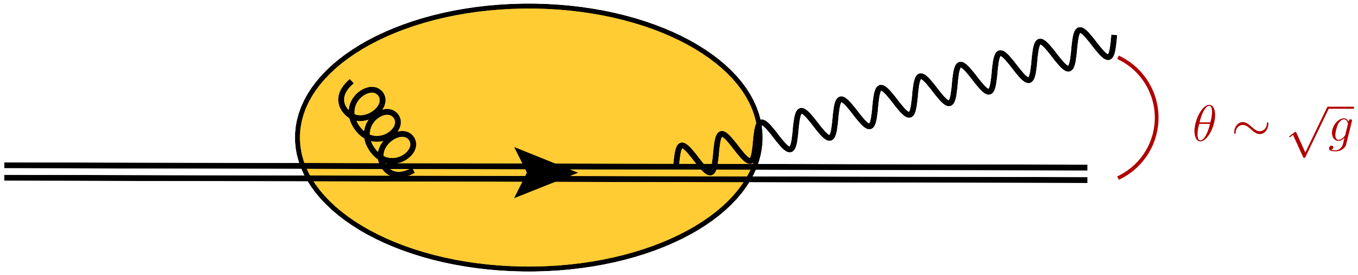


is physically distinct from the wide angle brem

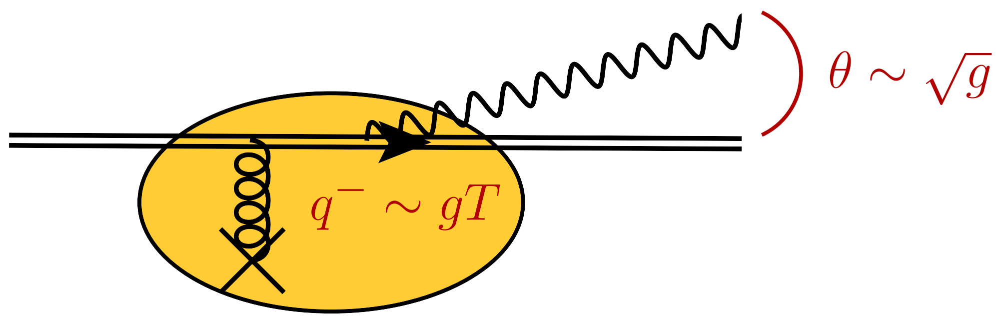


Matching collisions to brem

- When the gluon becomes soft (a plasmon), the $2 \leftrightarrow 2$ collision:



is not physically distinct from the wide angle brem

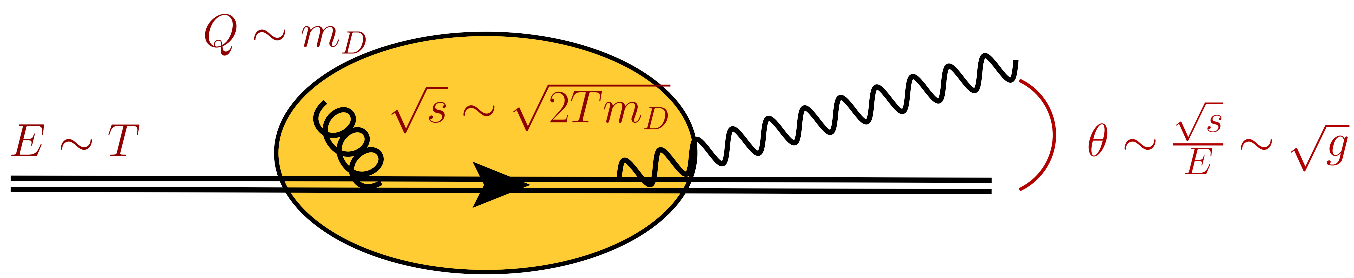


Need both processes

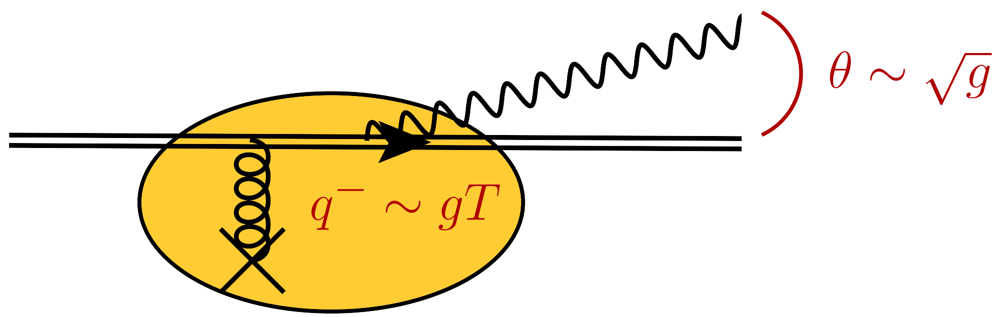
- For harder gluons, $q^- \to T$, this becomes a normal $2 \to 2$ process.
- For softer gluons, $q^- \to g^2T$, this smoothly matches onto AMY.

Matching collisions to brem

- When the gluon becomes soft (a plasmon), the $2 \leftrightarrow 2$ collision:



is not physically distinct from the wide angle brem



Need both processes

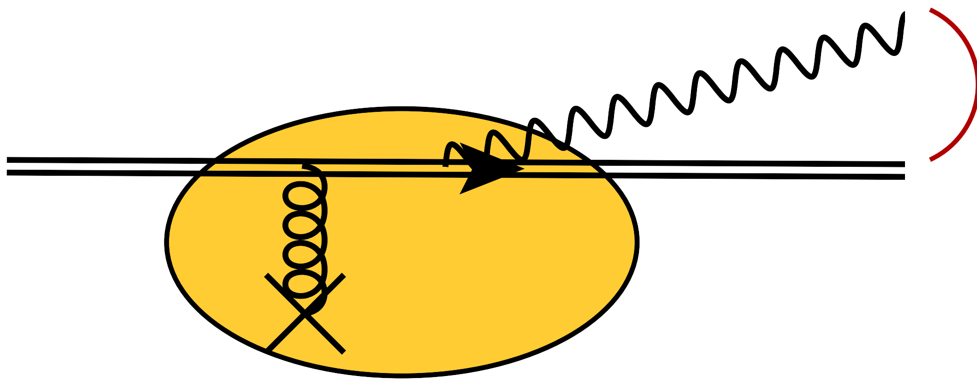
- For harder gluons, $q^- \to T$, this becomes a normal $2 \to 2$ process.
- For softer gluons, $q^- \to g^2T$, this smoothly matches onto AMY.

Brem and collisions at wider angles (but still small!)

- Photon emission rate

$$2k\frac{d\Gamma}{d^3k} \sim \int_{\text{phase}-\text{space}} n_{\mathbf{p}}(1-n_{\mathbf{p}+\mathbf{k}})\,|\mathcal{M}|^2\,(2\pi)^4\delta^4(P_{\text{tot}})$$

- The matrix element is



P^+_{in}

$\theta \sim \sqrt{g}$

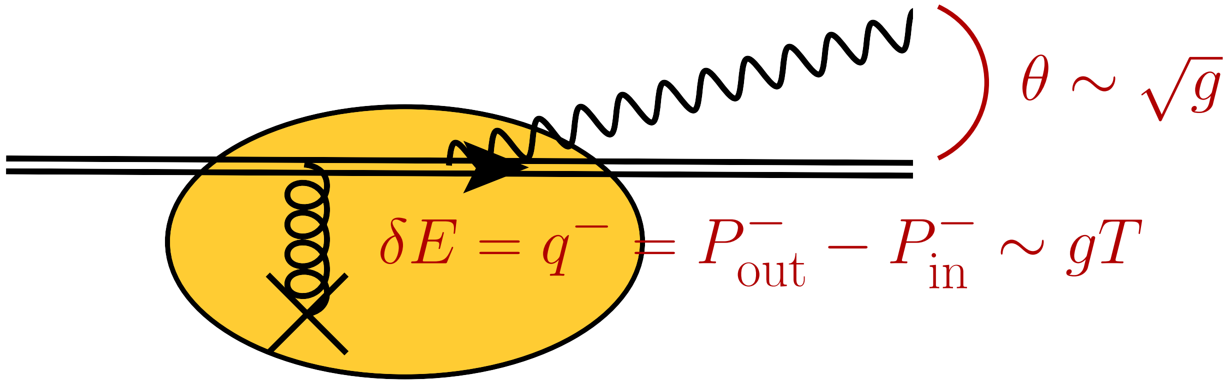
$P^+_{\text{out}} \equiv z\,P^+_{\text{in}}$

$q^- = \delta E \equiv P^-_{\text{out}} - P^-_{\text{in}} \sim gT$

$$|\mathcal{M}|^2\,(2\pi)^4\delta^4(P_{\text{tot}}) \propto \int_Q \underbrace{\frac{1+z^2}{z}}_{\text{QCD splitting fcn}} \;\frac{1}{(q^-)^2}\; \underbrace{\langle F_{i+}\,F_{i+}(Q)\rangle}_{\text{scattering-center}} \; 2\pi\delta(q^- - \delta E)$$

All of the dynamics of the scattering center in a single matrix element $\langle F_{i+}F_{i+}(Q)\rangle$

Finite energy transfer sum-rule



- The AMY collision kernel $C[q_\perp]$ involves

Aurenche, Gelis, Zakarat

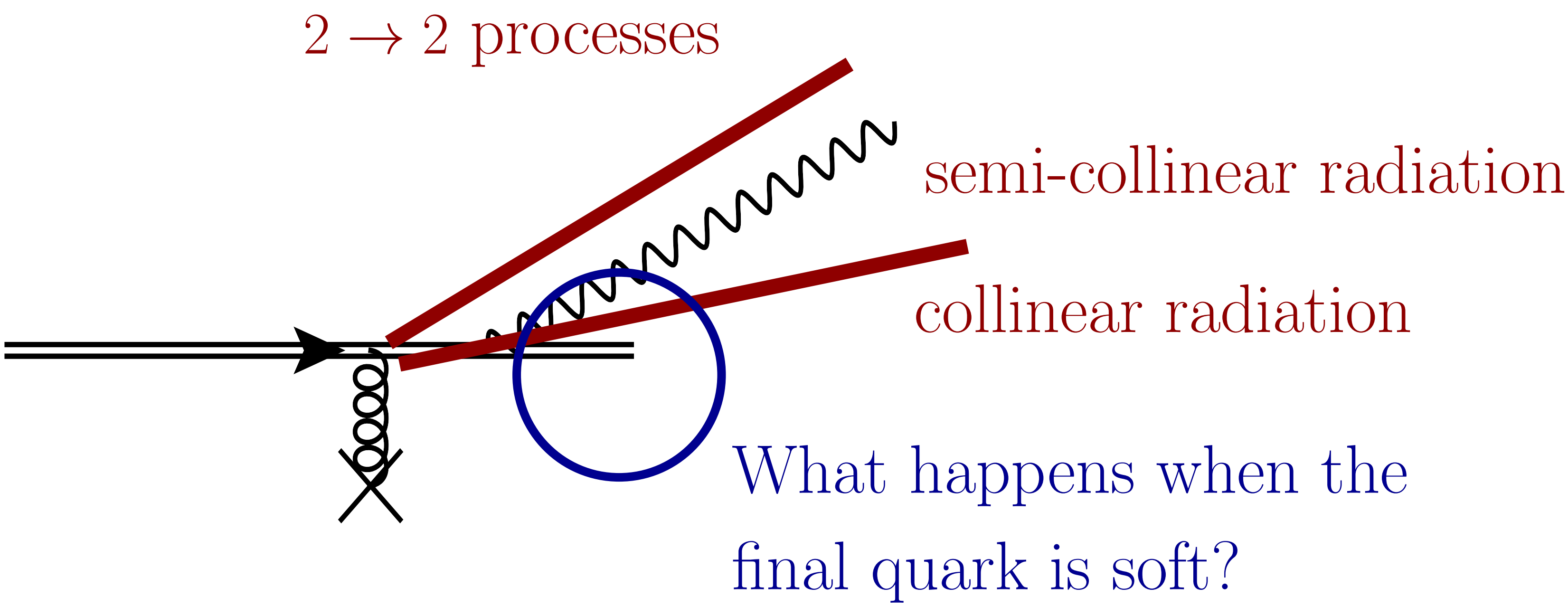
$$q_\perp^2 C[q_\perp] = \int_{-\infty}^{\infty} \frac{\mathrm{d}q^+}{2\pi} \, \langle F_{i+} F_{i+}(Q) \rangle |_{q^-=0} = \frac{T m_D^2}{q_T^2 + m_D^2}$$

- We need a finite $q^- = \delta E$ generalization of the sum rule

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}q^+}{2\pi} \, \langle F_{i+} F_{i+}(Q) \rangle |_{q^-=\delta E} = T \left[\frac{2(\delta E)^2(\delta E^2 + q_\perp^2 + m_D^2) + m_D^2 q_\perp^2}{(\delta E^2 + q_\perp^2 + m_D^2)(\delta E^2 + q_\perp^2)} \right]$$

Wider angle emissions can be included by a “simple” modified collision kernel

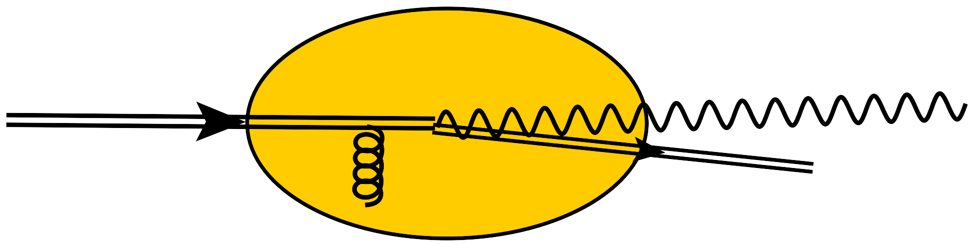
Matching between brem and conversions



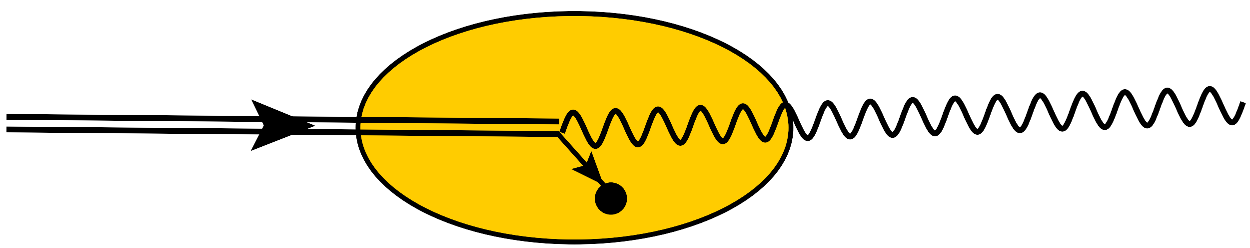
When the quark becomes soft need to worry about conversions.

Matching between brem and conversions

- When the final quark line is hard, the brem process :

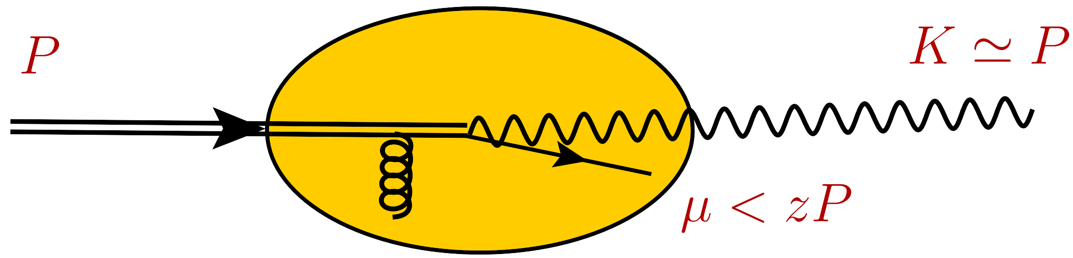


is physically distinct from the conversion process:

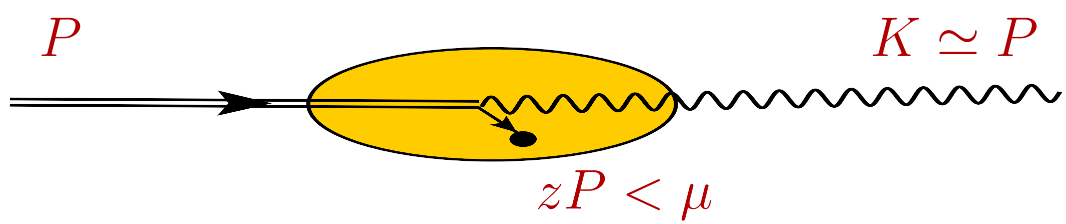


Matching between brem and conversions

- When the final quark line becomes soft, the brem process :



is not physically distinct from the conversion process



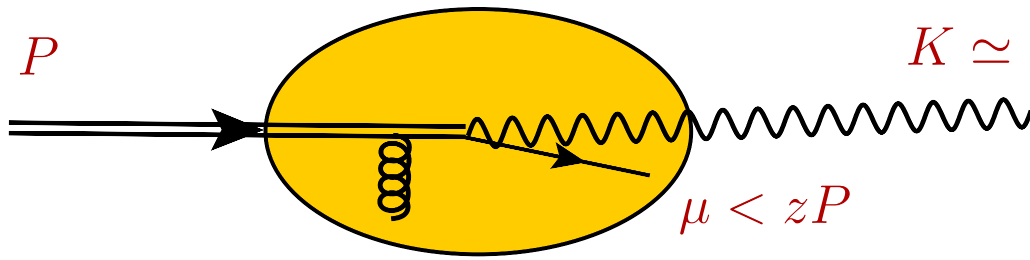
Separately both processes depend on the separation scale, $\mu \sim gT$, but . . .

the μ dep. cancels when both rates are included

- The LO small-θ and large-θ brem rates depend linearly and logarithmically on an infrared separation scale, μ.

The NLO conversion rate will depend on a UV cutoff μ and cancels this dependence

Brem rates with a soft quark



- Small angle brem

$$2k \frac{\mathrm{d}\Gamma}{d^3k}\bigg|_{zP>\mu} = \text{Leading Order Rate} + \text{Finite} - \underbrace{\#\, g^2\mu}_{\text{linear IR dependence } \mu}$$

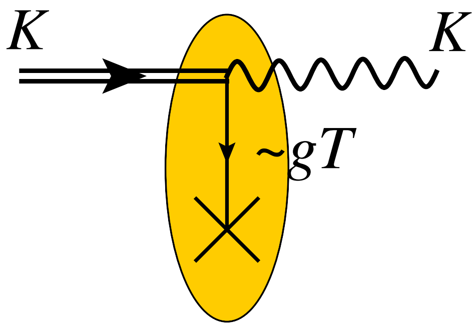
- Wide angle brem

$$2k \frac{\mathrm{d}\Gamma}{d^3k}\bigg|_{zP>\mu} \propto \underbrace{\frac{\delta m_\infty^2}{4\pi} \log \frac{\sqrt{2Tm_D}}{\mu}}_{\text{Log IR dependence on } \mu} + \text{Finite}$$

The conversion rate should cancel this dependence on μ

Computing the conversion rate with sum-rules (LO):

(see also Bodeker)



$$2k(2\pi)^3 \frac{\mathrm{d}\Gamma_{\mathrm{cnvrt}}}{\mathrm{d}^3k} \propto e^2 n_F(k)\ \hat{q}_{\mathrm{cnvrt}}(\mu)$$

- $\hat{q}_{\mathrm{cnvrt}}$ is the quark version of $\hat{q}$

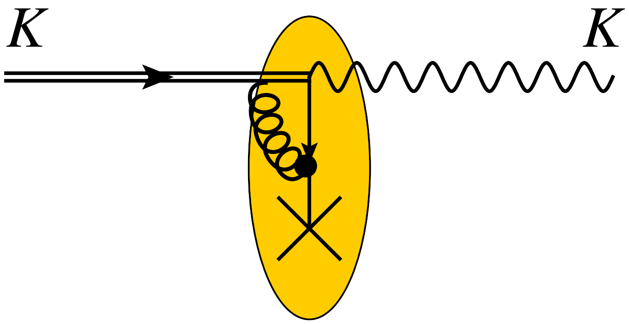
$$\hat{q}_{\mathrm{cnvrt}}(\mu_\perp) = \int^{\sim\mu} \frac{\mathrm{d}^2\mathbf{p}_T}{(2\pi)^2} \underbrace{\int_{-\mu}^{\mu} \frac{\mathrm{d}p^z}{2\pi} \mathrm{Tr}\left[\gamma_+ S^<(\omega, \mathbf{p})\right]_{\omega=p^z}}_{\text{evaluate with sum rule}}$$

$$= \int^{\mu} \frac{d^2\mathbf{p}_T}{(2\pi)^2} \frac{m_\infty^2}{p_T^2 + m_\infty^2}$$

where

$$S_R(X) = \left\langle \psi(X) e^{ig\int_0^X dx^\mu A_\mu} \bar{\psi}(0) \right\rangle$$

Computing the conversion rate at NLO with sum-rules:



$$2k(2\pi)^3 \frac{\mathrm{d}\Gamma_{\rm cnvrt}}{\mathrm{d}^3k} \propto e^2 \, n_F(k) \, \hat{q}_{\rm cnvrt}(\mu)$$

- At NLO we have only to replace $m_\infty^2 \to m_\infty^2 + \delta m_\infty^2$

$$\hat{q}_{\rm cnvrt} = \underbrace{\int^{\mu} \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} \frac{m_\infty^2 + \delta m_\infty^2}{p_T^2 + m_\infty^2 + \delta m_\infty^2}}_{\text{finite + UV logarithmic divergence in } \mu} + \underbrace{\# g^2 \mu}_{\text{linear UV divergence in } \mu}$$

The UV divergences of conversion rate match with the IR divergences of large and small angle brem giving a finite answer

Summary of the matching calculations at NLO

$$2k(2\pi)^3 \frac{\mathrm{d}\Delta\Gamma_{NLO}}{\mathrm{d}^3k} \propto \underbrace{\text{finite} - C_1 g^2 \mu}_{\text{collinear contribution}}$$

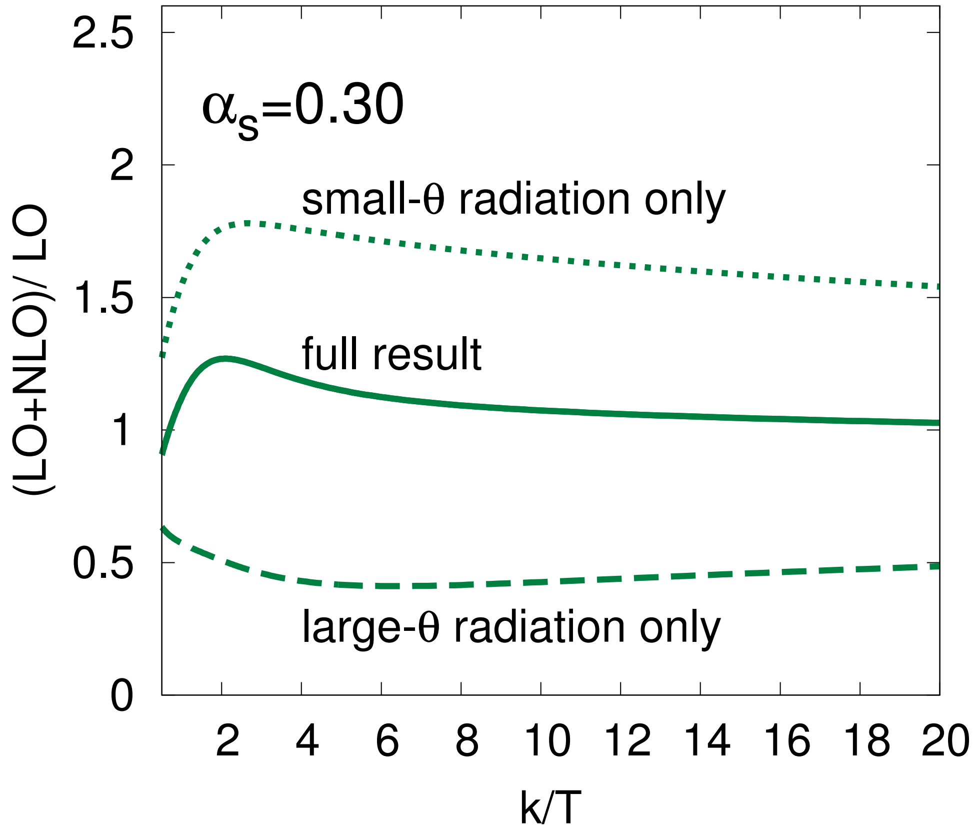
$$+ \underbrace{\text{finite} + C_2 \frac{\delta m_\infty^2}{4\pi} \log \frac{\sqrt{2Tm_D}}{\mu}}_{\text{semi-collinear contribution}}$$

$$+ \underbrace{\text{finite} + C_1 g^2 \mu + C_2 \frac{\delta m_\infty^2}{4\pi} \log \frac{\mu}{m_D}}_{\text{conversions}}$$

The μ dependence cancels between the different contributions

Conclusion

- The result again



- All of the soft sector buried into a few coefficients, δm_∞^2 and $\hat{q}_{\rm cnvrt}$
  - Can we compute these non-perturbatively ?

Many things can be computed next (e.g. shear viscosity and e-loss)