# Initial State Fluctuations and Hydrodynamics

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# Introduction - Flow and Fluctuations

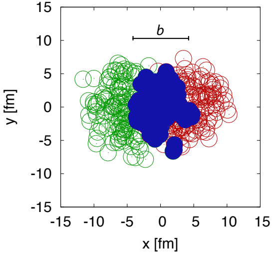
- Large elliptic flow has indicated fluid behavior of matter created at RHIC in early 2000's BNL announces "perfect liquid" in 2005 press release
- The importance of fluctuations was realized later and analysis of odd flow harmonics began in 2010
  since B. Alver, G. Roland, Phys.Rev. C81, 054905
- The knowledge of the (fluid dynamic) evolution of the system is necessary for the calculation of thermal photon and dilepton production
- I will
  - give an overview of recent developments
  - concentrate on a new QCD based model for the initial state including geometric and color charge fluctuations
  - present comparisons of theory and experimental data
  - present first photon flow calculations with the new initial state model

# Initial state fluctuations: MC-Glauber model

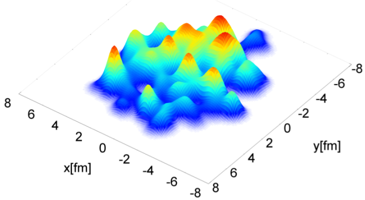
Positions of nucleons fluctuate; so does energy density in every event

Simple way to implement this: Monte-Carlo(MC)-Glauber model:

- Sample Woods-Saxon distributions to determine all nucleon positions (green and red circles)
- Sample impact parameter $b$ and overlap nuclei




- Nucleon-nucleon collision occurs if distance is $< \sqrt{\sigma_{NN}/\pi}$
- At position of collision (blue blobs) add 2D-Gaussian energy density distribution with width $\sigma_0$
  $\sigma_0$ (e.g. $0.4\,\mathrm{fm}$) is a model parameter

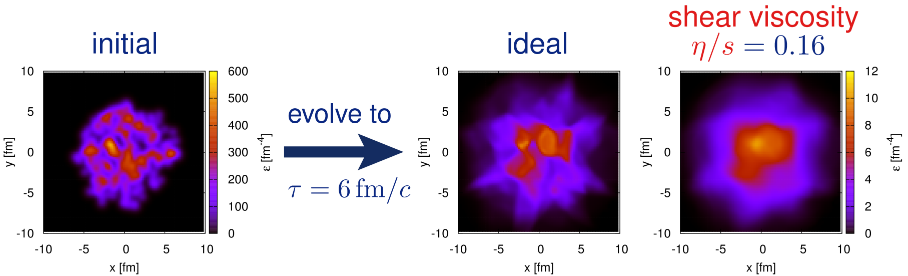


# Nearly perfect fluid $\leftrightarrow$ Hydrodynamic evolution

The system evolves from the initial energy density distribution according to energy and momentum conservation:

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - P g^{\mu\nu} + \pi^{\mu\nu}$$




MUSIC B. Schenke, S. Jeon, C. Gale, Phys. Rev. C82, 014903 (2010); Phys.Rev.Lett.106, 042301 (2011)

3+1D event-by-event relativistic viscous hydrodynamic simulation

# Flow analysis

System expands, becomes dilute, freezes out

Spatial anisotropy is transformed into momentum anisotropy

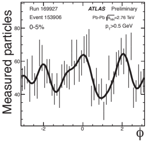
Compute particle spectra at freeze-out using the Cooper-Frye formula:

$$E\frac{dN}{d^3p} = \int_\Sigma d\Sigma_\mu p^\mu f(T, p_\mu u^\mu, \pi^{\mu\nu})$$

$\Sigma$ = freeze-out surface, $f$ = particle distribution Cooper and Frye, Phys.Rev.D10, 186 (1974)

Analyze azimuthal distribution of particles in terms of Fourier series:

$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + \sum_n (2v_n \cos(n(\phi - \psi_n)))\right)$$




ATLAS

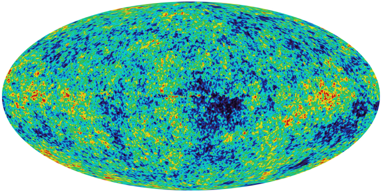
so

$$v_n = \langle \cos[n(\phi - \psi_n)] \rangle$$

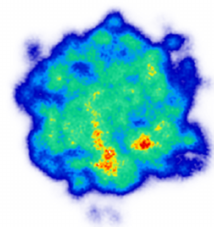
with the event-plane angle $\psi_n = \frac{1}{n}\arctan\frac{\langle \sin(n\phi)\rangle}{\langle \cos(n\phi)\rangle}$

# Why the study of fluctuations is so powerful

Analysis analogous to that of the cosmic microwave background (CMB)



CMB Credit: WMAP Science Team, NASA



Heavy-Ion Collision

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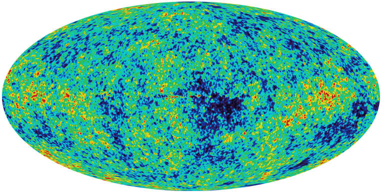

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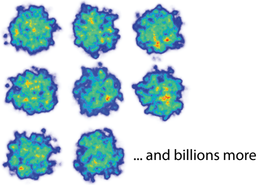


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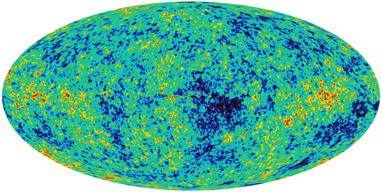

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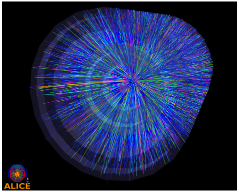


ALICE

Heavy-Ion Collision

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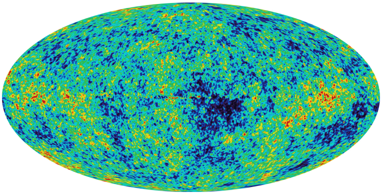

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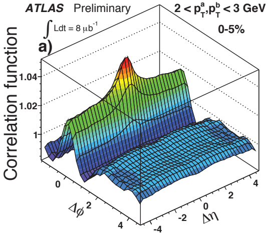


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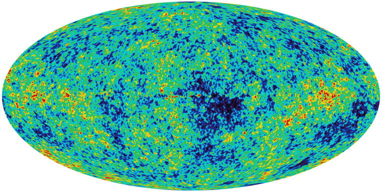

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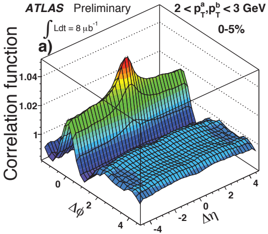
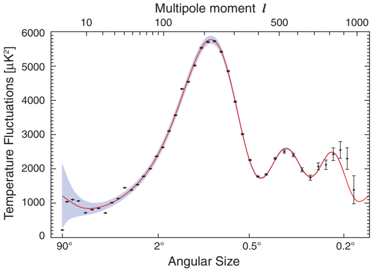
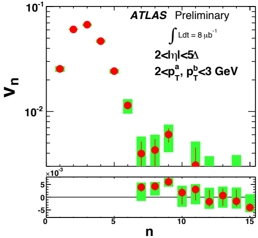



Heavy-Ion Collision

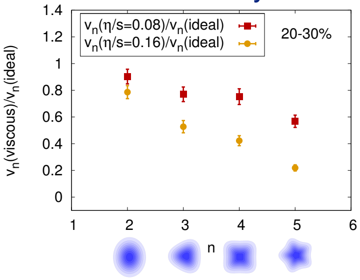


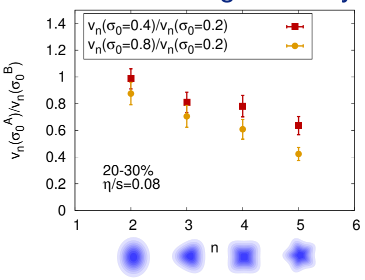
# Flow analysis B. Schenke, S. Jeon, C. Gale, Phys. Rev. C85, 024901 (2012)

## Sensitivity of event averaged $v_n$ on

viscosity




initial state granularity




Shown are ratios of

viscous to ideal results smoother to more granular results

Sensitivity to viscosity and initial state structure increases with $n$

# New model for the initial state

To make use of $v_n$ measurements we need a more rigorous understanding of the initial state and its fluctuations

Simple billard balls with a geometric cross section: not realistic

**QCD** should tell us what the incoming nuclei look like and what the fluctuations are

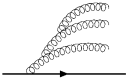
Color Glass Condensate (CGC) effective theory of QCD and Yang-Mills calculation of two colliding CGCs

# Gluon saturation

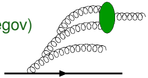
$x$ = longitudinal momentum fraction of partons in a hadron or nucleus

as we go to higher energy / smaller $x$, gluons split, number increases:

BFKL (Balitsky,Fadin,Kuraev,Lipatov) equation describes $x$-evolution but violates unitarity: cross-sections grow without bound



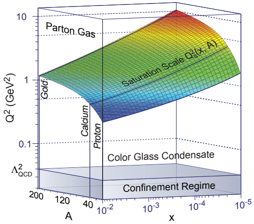
JIMWLK (Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner) and BK (Balitsky, Kovchegov) equations include saturation



small $x$: the hadron/nucleus wavefunction is characterized by the saturation scale $Q_s \gg \Lambda_{\rm QCD}$

$\mathbf{p}_T \lesssim Q_s$: strong fields $A_\mu \sim 1/g$:

- occupation numbers $\sim 1/\alpha_s$
- classical field approximation
- small $\alpha_s$, but non-perturbative

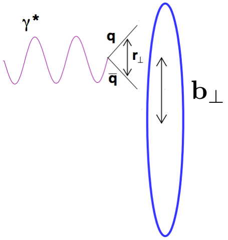


# Modeling $x$ and $b$ dependence: IP-Sat model

Want to determine color charge distribution in a nucleus. Proton first:
We use the IP-Sat model to parametrize

- $x$-dependence
- Impact parameter dependence (IP)

  Kowalski, Teaney, Phys.Rev. D68 (2003) 114005




$x$-evolution can be computed using JIMWLK, but parametrization is easier for now

Proton dipole cross section in DIS:

$$\frac{\mathrm{d}\sigma^{\mathrm{p}}_{\mathrm{dip}}}{\mathrm{d}^2\mathbf{b}_\perp}(\mathbf{r}_\perp, x, \mathbf{b}_\perp) = 2\mathcal{N}(\mathbf{r}_\perp, x, \mathbf{b}_\perp) = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c}{\mathbf{r}_\perp}^2\alpha_s(\tilde{\mu}^2)xg(x,\tilde{\mu}^2)T_p(\mathbf{b}_\perp)\right)\right]$$

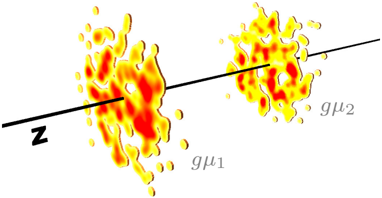
with $\tilde{\mu}^2$ an energy scale related to the dipole radius $\mathbf{r}_\perp$ and $xg(x,\tilde{\mu}^2)$ is the gluon density
$T_p$ is a Gaussian with width fit to HERA diffractive data

$Q_s$ is defined by the value of $r$ at which $\mathcal{N}$ reaches the saturated regime

$\mathcal{N}(R_s, x, \mathbf{b}_\perp) = 1 - e^{-1/2}$, with $Q_s^2 = \frac{2}{R_s^2}$

# Color charge densities of incoming nuclei

- Sample nucleon positions from Woods-Saxon distributions.
- Use IP-Sat model fit to HERA data to get $Q_s^2(x, \mathbf{b}_\perp)$ for each nucleon. The color charge density squared $g^2\mu^2$ is proportional to $Q_s^2$.
- Add all $g^2\mu^2(\mathbf{x}_\perp)$ in each nucleus to obtain $g^2\mu_1^2(\mathbf{x}_\perp)$ and $g^2\mu_2^2(\mathbf{x}_\perp)$.



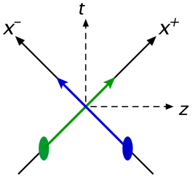

- Sample $\rho^a$ from local Gaussian distribution for each nucleus

$$\langle \rho^a(\mathbf{x}_\perp)\rho^b(\mathbf{y}_\perp)\rangle = \delta^{ab}\delta^2(\mathbf{x}_\perp - \mathbf{y}_\perp)g^2\mu^2(\mathbf{x}_\perp)$$

# Gauge fields before the collision

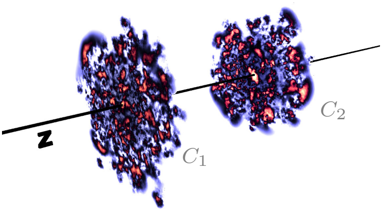
Color currents:

$$J_1^\nu = \delta^{\mu +}\rho_1(x^-, \mathbf{x}_\perp)$$
$$[D_\mu, F^{\mu\nu}] = J_1^\nu$$




$$J_2^\nu = \delta^{\mu -}\rho_2(x^+, \mathbf{x}_\perp)$$
$$[D_\mu, F^{\mu\nu}] = J_2^\nu$$

Correlations and fluctuations in the gluon fields:



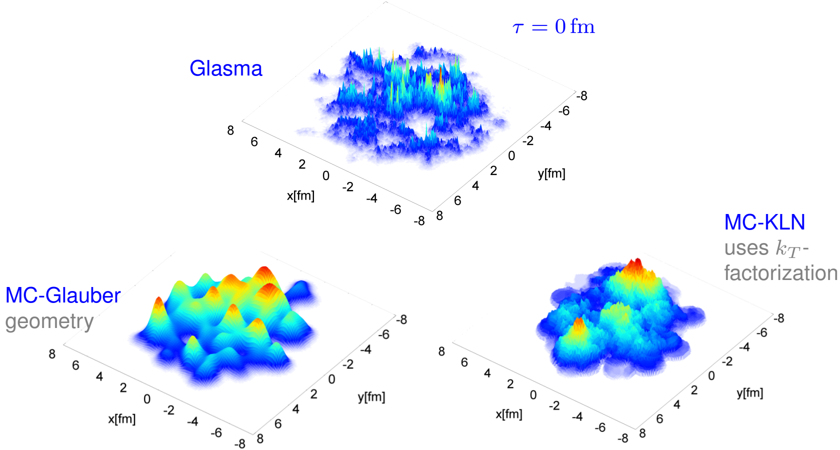

Shown is the correlator of the Wilson lines

$$C_{(1,2)}(\mathbf{x}_\perp) = \frac{1}{N_c}\mathrm{Re}[\mathrm{tr}(V(1,2)^\dagger(0,0)V(1,2)(x,y))]$$

The length scale of fluctuations is $1/Q_s$ - not the nucleon size

# Energy density B.Schenke, P.Tribedy, R.Venugopalan, Phys.Rev.Lett. 108, 252301 (2012)

Solve for gauge fields after the collision in the forward lightcone
Compute energy density in the fields at $\tau = 0$ and later times with CYM evolution
Lattice: Krasnitz, Venugopalan, Nucl.Phys. B557 (1999) 237




Very different initial energy density distributions in the models

MC-KLN: Drescher, Nara, nucl-th/0611017
mckln-3.52 from `http://physics.baruch.cuny.edu/files/CGC/CGC_IC.html` with defaults, energy density scaling

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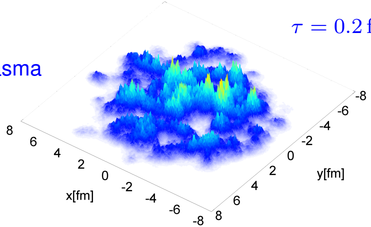
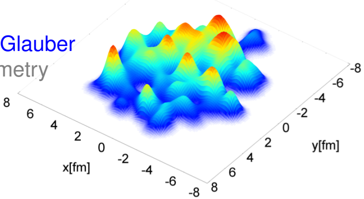
$\tau = 0.2\,\mathrm{fm}$

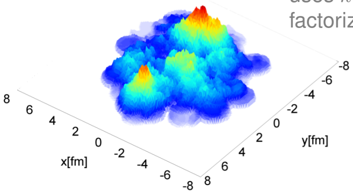
Glasma




MC-Glauber
geometry




MC-KLN
uses $k_T$-factorization

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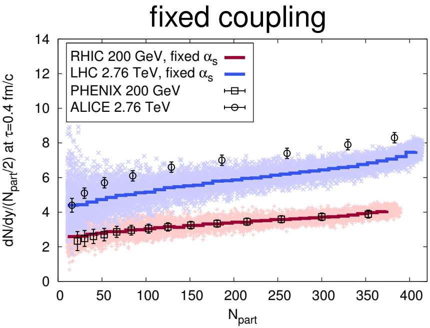
# Multiplicity B.Schenke, P.Tribedy, R.Venugopalan, Phys. Rev. C86, 034908 (2012)

$dN_{\rm g}/dy$ in transverse Coulomb gauge $\partial_i A^i = 0$
$N_{\rm part}$ from MC-Glauber with $\sigma_{NN} = 42\,{\rm mb}$ and $64\,{\rm mb}$ respectively




Experimental data: PHENIX, Phys.Rev.C71 034908 (2004) and ALICE, Phys.Rev.Lett. 106, 032301 (2011)

Scaled by $2/3$ to compare to charged particles.
Some freedom in normalization - will need to account for entropy production.

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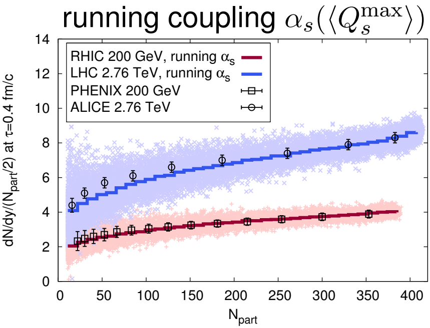


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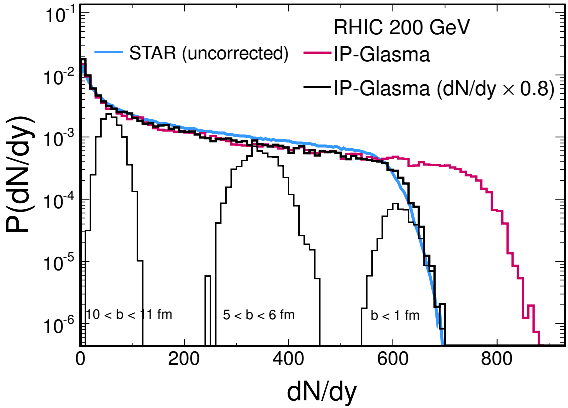
$P(dN_\mathrm{g}/dy)$ at time $\tau = 0.4\,\mathrm{fm}$ with $P(b)$ from a Glauber model

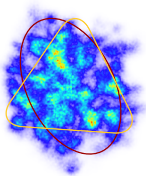
Experimental data: STAR, Phys. Rev. C79, 034909 (2009)




Glasma model gives a convolution of negative binomial distributions

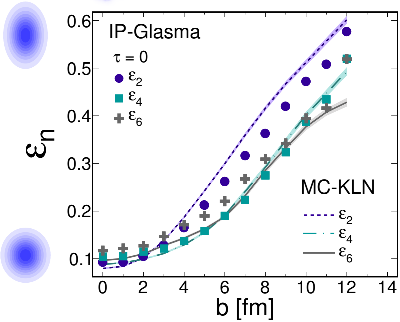
No need to put them in by hand

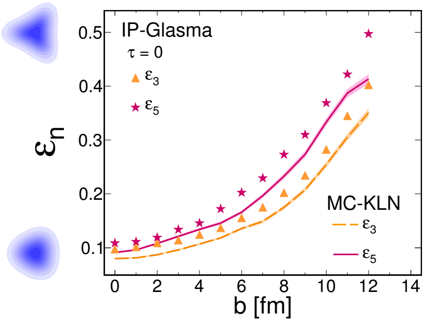
# Eccentricities B.Schenke, P.Tribedy, R.Venugopalan, Phys. Rev. C86, 034908 (2012)



Characterize the initial distribution by its ellipticity, triangularity, etc...

$$\varepsilon_n = \sqrt{\langle r^n \cos(n\phi)\rangle^2 + \langle r^n \sin(n\phi)\rangle^2}/\langle r^n\rangle$$

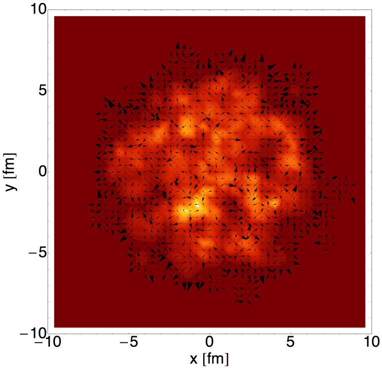



- $\varepsilon_n$ larger in Glasma model for odd $n$
- $\varepsilon_n$ smaller in Glasma model for $n = 2$ (for $b > 3\,\mathrm{fm}$)
  about equal for $n = 4$, larger for $n = 6$

# Matching to hydro: $T^{\mu\nu}$ and flow velocities

Compute all components of $T^{\mu\nu}$
Determine energy density and $(u^x, u^y)$ at $\tau > 0\,\mathrm{fm}$ from $u_\mu T^{\mu\nu} = \varepsilon u^\nu$ as input for hydrodynamic simulations

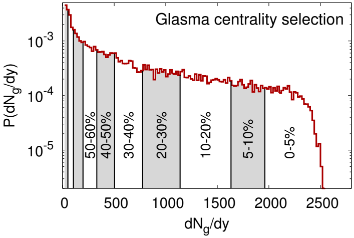



Energy density and $(u_x, u_y)$ at $\tau = 0.4\,\mathrm{fm}/c$

No instabilities (need full 3+1D Yang-Mills for that):
system is far from equilibrium - cannot yet match $\Pi^{\mu\nu}$

# Centrality selection and flow

C. Gale, S. Jeon, B.Schenke, P.Tribedy, R.Venugopalan, arXiv:1210.5144 (2012)



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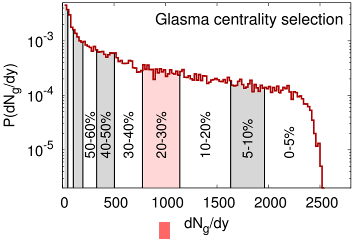


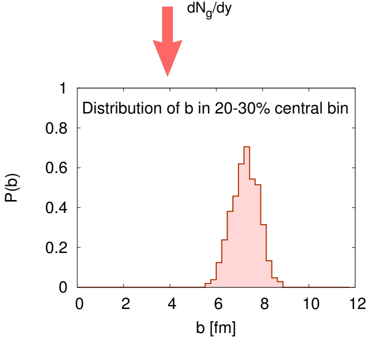

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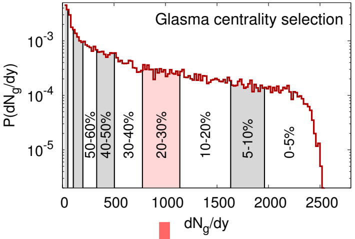


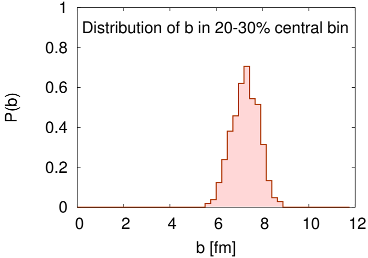


Hydro evolution

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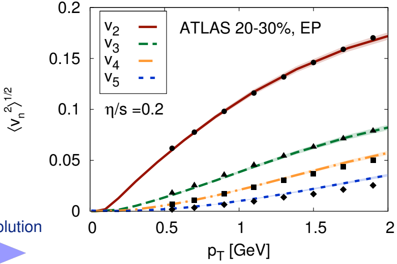

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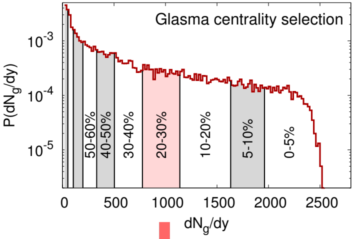
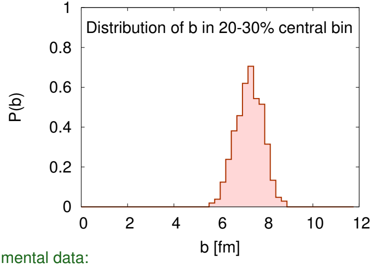
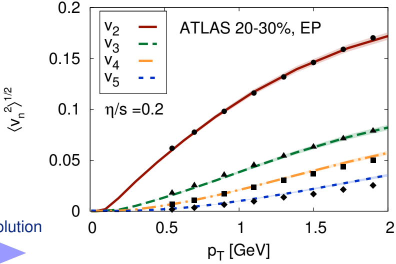
C. Gale, S. Jeon, B.Schenke, P.Tribedy, R.Venugopalan, arXiv:1210.5144 (2012)

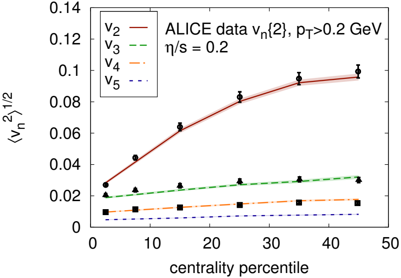



Hydro evolution

MUSIC



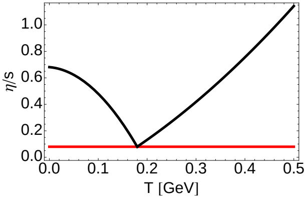

Experimental data:

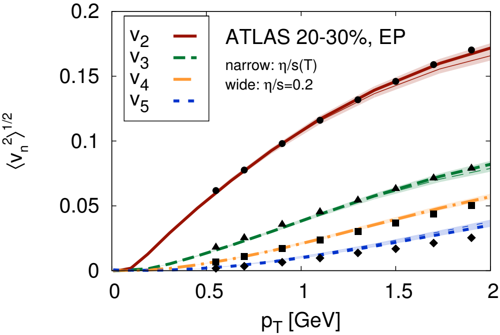
ATLAS collaboration, Phys. Rev. C 86, 014907 (2012)

ALICE collaboration, Phys. Rev. Lett. 107, 032301 (2011)

# Temperature dependent $\eta/s$

Use $\eta/s(T)$ as in Niemi et al., Phys.Rev.Lett. 106 (2011) 212302 and arXiv:1203.2452



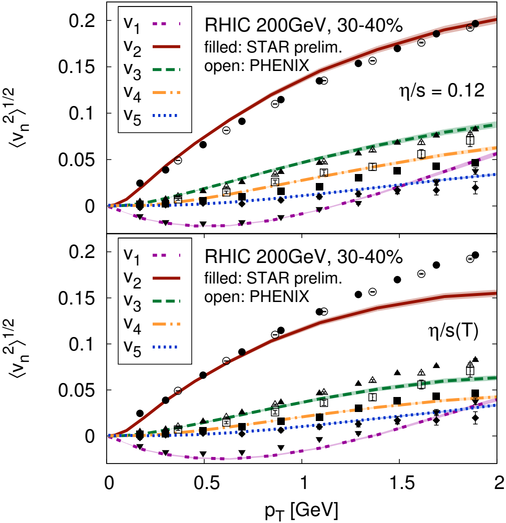




$v_n(p_T)$ for given $\eta/s(T)$ indistinguishable from constant $\eta/s = 0.2$

More detailed study needed - include different RHIC energies and LHC

Experimental data:
ATLAS collaboration, Phys. Rev. C 86, 014907 (2012)

# Comparison to RHIC data




Lower effective $\eta/s$ than at LHC needed to describe data

Hints at increasing $\eta/s$ at high temperature

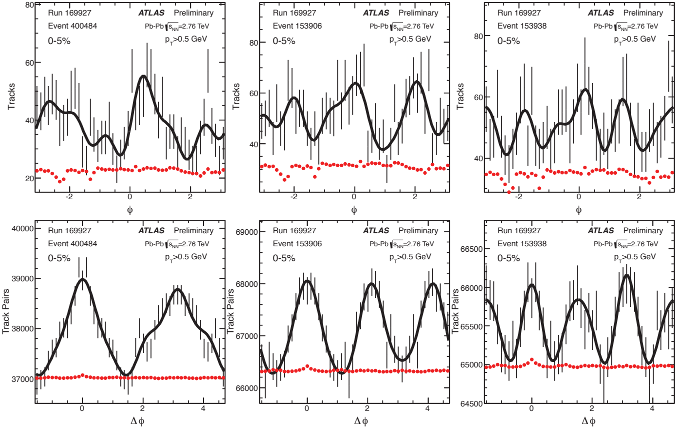
Can be used to gain information on $(\eta/s)(T)$


Experimental data:
A. Adare et al. (PHENIX Collaboration), Phys.Rev.Lett. 107, 252301 (2011)
Y. Pandit [for the STAR collaboration], Quark Matter 2012, (2012)

# New measurements of single event $v_n$

The ATLAS collaboration presented first measurements of $v_n$ in single events at Quark Matter 2012

https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CONFNOTES/ATLAS-CONF-2012-114/



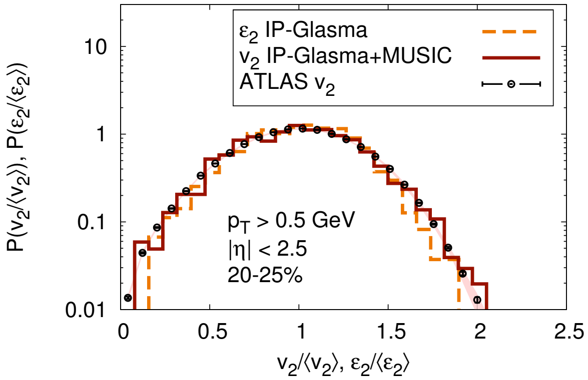

$\phi$-distribution of particles(upper) or pairs(lower).

Black lines: Fourier expansion, $v_n$ fit.

# Event-by-event distributions of $v_n$

So now we can compare event-by-event distributions of $v_n$.

https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CONFNOTES/ATLAS-CONF-2012-114/




C. Gale, S. Jeon, B.Schenke, P.Tribedy, R.Venugopalan, arXiv:1210.5144 (2012)

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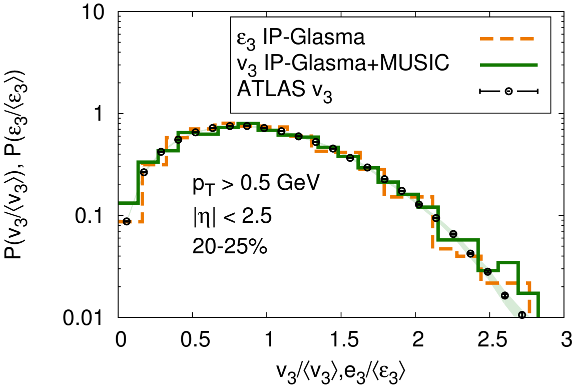


C. Gale, S. Jeon, B.Schenke, P.Tribedy, R.Venugopalan, arXiv:1210.5144 (2012)

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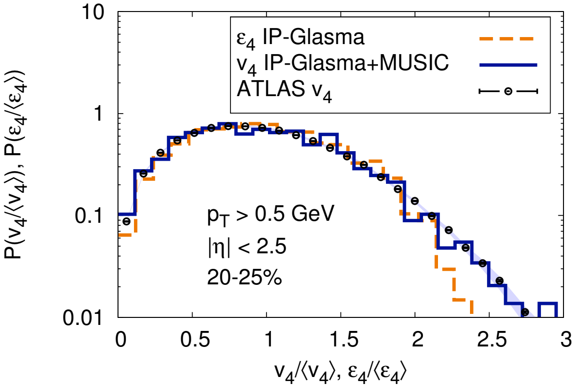
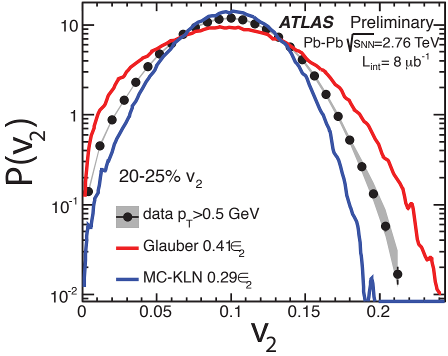
https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CONFNOTES/ATLAS-CONF-2012-114/

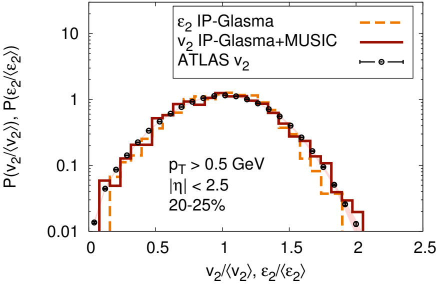



C. Gale, S. Jeon, B.Schenke, P.Tribedy, R.Venugopalan, arXiv:1210.5144 (2012)

# Event-by-event distributions of $v_n$ - other models

Showing eccentricity distributions (yellow on the right)

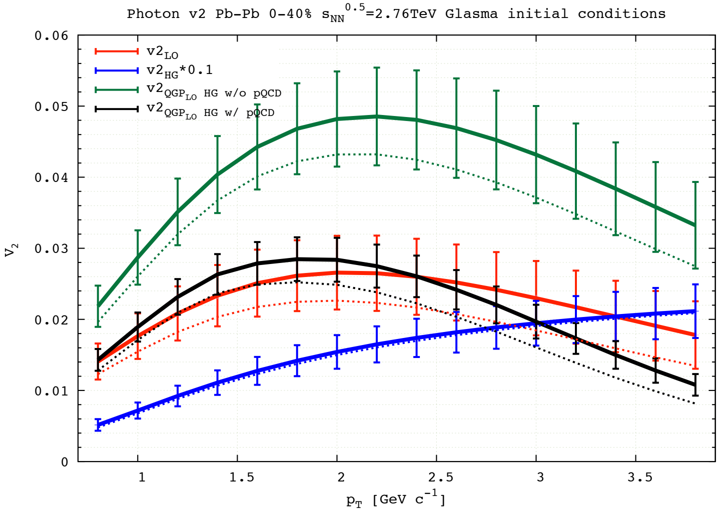



Event-by-event distributions can distinguish between different initial state models

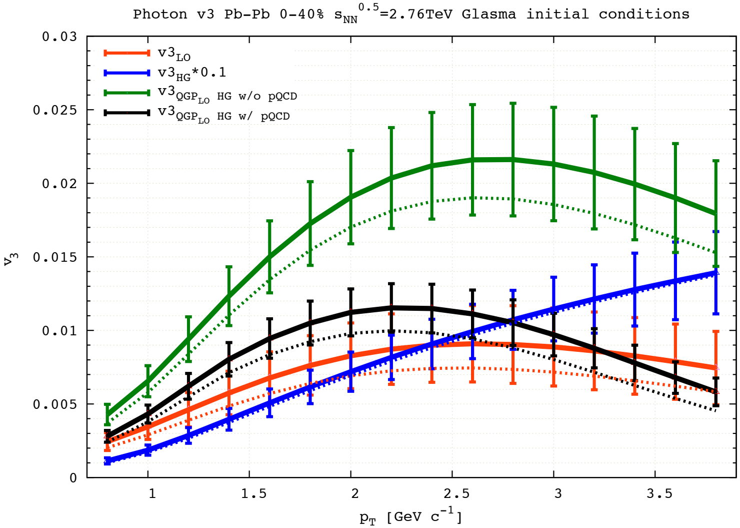
Experimental data: `https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CONFNOTES/ATLAS-CONF-2012-114/`

# Photon elliptic flow at LHC
Kozlov, Paquet, Schenke, Jeon, Gale, in preparation




Effect of initial flow (solid vs. dashed) not large enough to explain data

# Photon triangular flow at LHC Kozlov, Paquet, Schenke, Jeon, Gale, in preparation



# Summary and conclusions

- Knowledge of initial state and hydrodynamic evolution important for thermal radiation
- Different initial state models exist
  Yield rather different initial energy density distributions
- IP-Glasma model
  - includes geometric and color charge fluctuations
  - produces negative binomial fluctuations
  - has different eccentricities than previous CGC based models
  - provides initial flow profile from the non-equilibrium stage
  - describes flow coefficients up to at least $v_5$
  - describes $v_n$ distributions where others fail
  - does not (yet) include instabilities and (possible) isotropization
- Outlook: study different collision energies, learn about $(\eta/s)(T)$, more on em-probes, study instabilities/isotropization, combine IP-Glasma and anisotropic hydro

# BACKUP

# KLN, fKLN, MC-KLN, ...

- original KLN:
  - uses $k_T$-factorization
  - $(Q_s^A)^2(\mathbf{x}_\perp) \propto N_{\mathrm{part,A}}(\mathbf{x}_\perp)$.
  - Saturation scales are not universal: $N_{\mathrm{part,A}}(\mathbf{x}_\perp)$ depends on nucleus B.
  - The energy density ($\epsilon \propto Q_{s,\mathrm{larger}} Q^2_{s,\mathrm{smaller}}$) is suppressed in the edge region along the impact parameter direction $\rightarrow$ larger eccentricity.
- fKLN:
  - uses $k_T$-factorization
  - Different definition of unintegrated gluon distribution (correct limit: where there is one nulceon at the edge the uGDF is that of one nucleon - not so in KLN)
  - Universal saturation scales in nucleus A and B. (Important at the edges of the nuclei)
- MC-KLN: Monte-Carlo implementation of fKLN with fluctuating positions of the nucleons
- IP-Glasma (CYM):
  - Does not use $k_T$-factorization (because it is strictly not valid in A+A collisions - at least one source has to be dilute)
  - $Q_s(\mathbf{x}_\perp)$ universal and constrained by HERA data.
  - No utilization of the nucleon-nucleon cross section.
  - Takes into account non-linearities.
  - Includes fluctuations of color charges within a nucleon.

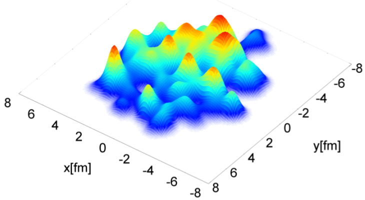
# Comparison with other models: “MC-Glauber”

- Sample nucleon positions in nucleus $A$ and $B$, then overlap the two distributions.
- An interaction happens when the distance $d$ between a nucleon from nucleus A and one from nucleus B fulfills:

$$d \leq \sqrt{\sigma_{\text{inel}}^{NN}/\pi}$$

- Add Gaussian energy density with width $\sigma_0$ for every wounded nucleon, binary collision, or combination.

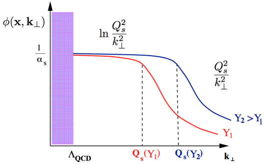
Result for $\sigma_0 = 0.4\,\text{fm}$



# Comparison with other models: "MC-KLN"

- Determine gluon production using $k_T$-factorization:

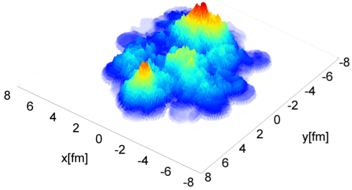
$$\frac{dN}{dr_\perp^2 dy} \sim \int \frac{d^2p_\perp}{p_\perp^2} \int d^2k_\perp \alpha_s \phi_A(x_1, k_\perp^2)\phi_B(x_2, (p_\perp - k_\perp)^2)$$




- Energy density analogously.
- Kharzeev-Levin-Nardi (KLN) model for $\phi$:

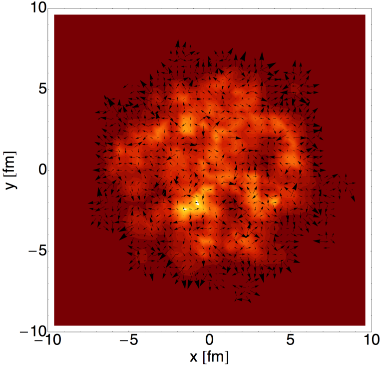
$$\phi_{A,B}(x, k_\perp^2, \mathbf{r}_\perp) \sim \frac{1}{\alpha_s(Q_s^2)} \frac{Q_s^2}{\max(Q_s^2, k_\perp^2)}$$

with $Q_{s,A}^2(x, \mathbf{r}_\perp) = 2\,\mathrm{GeV}^2 \frac{T_A(\mathbf{x}_\perp)}{1.53\,\mathrm{fm}^{-2}} \left(\frac{0.01}{x}\right)^\lambda, \lambda = 0.28$



# $T^{\mu\nu}$ and flow velocities

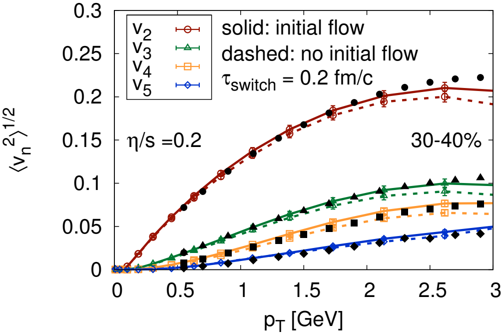
Compute all components of $T^{\mu\nu}$
Determine energy density and $(u^x, u^y)$ at $\tau > 0\,\mathrm{fm}$ from $u_\mu T^{\mu\nu} = \varepsilon u^\nu$
as input for hydrodynamic simulations




Energy density
and $(u_x, u_y)$
at $\tau = 0.4\,\mathrm{fm}/c$

No instabilities (need full 3+1D Yang-Mills for that):
system is far from equilibrium - cannot yet match $\Pi^{\mu\nu}$

# Effect of initial flow



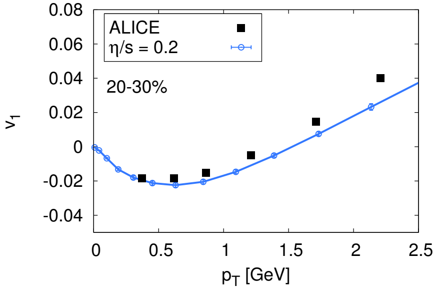

Weak effect of initial flow on hadron $v_n(p_T)$

Expect stronger effect for photon $v_n$:
Photons are mostly produced early at high temperatures

Effect of different switching time $0.4\,\mathrm{fm}/c$ is very weak

Experimental data:
ATLAS collaboration, Phys. Rev. C 86, 014907 (2012)

# Directed flow $v_1$




Experimental data:

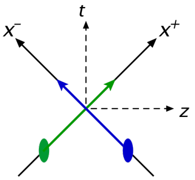
extracted in Retinskaya et al., Phys.Rev.Lett. 108 (2012) 252302

from ALICE data in K. Aamodt et al., Phys. Lett. B 708, 249 (2012)

# Gauge fields before the collision

Color currents:

$$J_1^\nu = \delta^{\mu +}\rho_1(x^-, \mathbf{x}_\perp)$$
$$[D_\mu, F^{\mu\nu}] = J_1^\nu$$




$$J_2^\nu = \delta^{\mu -}\rho_2(x^+, \mathbf{x}_\perp)$$
$$[D_\mu, F^{\mu\nu}] = J_2^\nu$$

Solution in covariant gauge:

$$A^+_{\mathrm{cov}(1,2)}(x^-, \mathbf{x}_\perp) = -\frac{g\rho_{(1,2)}(x^-, \mathbf{x}_\perp)}{\nabla_\perp^2 + m^2}$$

with infrared cutoff $m$ of order $\Lambda_{\mathrm{QCD}}$.

Solution in light cone gauge:

$$A^+_{(1,2)}(\mathbf{x}_\perp) = A^-_{(1,2)}(\mathbf{x}_\perp) = 0$$

$$A^i_{(1,2)}(\mathbf{x}_\perp) = \frac{i}{g} V_{(1,2)}(\mathbf{x}_\perp)\partial_i V^\dagger_{(1,2)}(\mathbf{x}_\perp)$$

$V$ is the path-ordered exponetial of $A^+_{\mathrm{cov}(1,2)}$

# Gauge fields before the collision

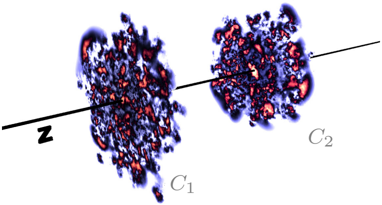
The correlator of the Wilson lines

$$C_{(1,2)}(\mathbf{x}_\perp) = \frac{1}{N_c}\mathrm{Re}[\mathrm{tr}(V(1,2)^\dagger(0,0)V(1,2)(x,y))]$$

with

$$V_{(1,2)}(\mathbf{x}_\perp) = P\exp\left(-ig\int dx^- \frac{\rho_{(1,2)}(x^-,\mathbf{x}_\perp)}{\nabla_\perp^2 + m^2}\right)$$

shows the degree of correlations and fluctuations in the gluon fields.




The length scale of fluctuations is $1/Q_s$. Not the nucleon size.

# Gauge fields after the collision (Glasma)

Initial condition on the lightcone: require that fields match smoothly on the lightcone.

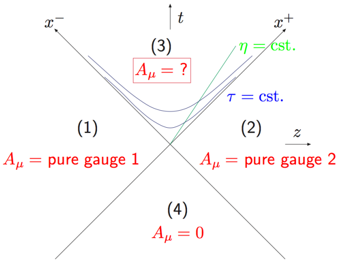



figure from Lappi, arXiv:1003.1852

Solution:

$$A^i_{(3)}|_{\tau=0} = A^i_{(1)} + A^i_{(2)}$$

$$A^\eta_{(3)}|_{\tau=0} = \frac{ig}{2}[A^i_{(1)}, A^i_{(2)}]$$

On the lattice the Wilson lines in the future lightcone are obtained from the condition:

$$\mathrm{tr}\left\{t^a\left[\left(U^i_{(1)} + U^i_{(2)}\right)(1 + U^{i\dagger}_{(3)}) - (1 + U^i_{(3)})\left(U^{i\dagger}_{(1)} + U^{i\dagger}_{(2)}\right)\right]\right\} = 0$$

where $t^a$ are the generators of $SU(N_c)$ in the fundamental representation. Solve iteratively.

Krasnitz, Venugopalan, Nucl.Phys. B557 (1999) 237

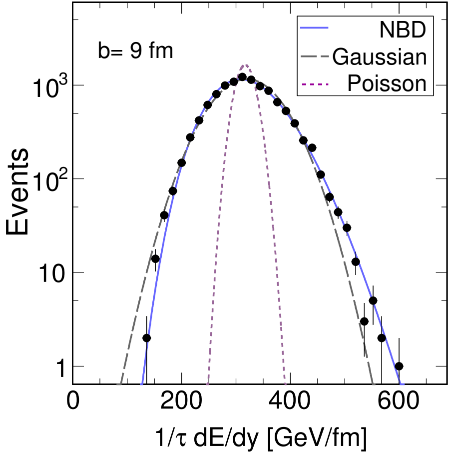
$$U^i_{(1,2),j} = V_{(1,2),j}V^\dagger_{(1,2),j+\hat{e_i}}$$

(gauge transform of $\mathbf{1}$: pure gauge)

# Negative binomial fluctuations

Fluctuations in the total energy per unit rapidity produce negative binomial distribution (NBD).

B.Schenke, P.Tribedy, R.Venugopalan, arXiv:1202.6646, Phys.Rev.Lett. 108, 252301 (2012)




$$P(n) = \frac{\Gamma(k+n)}{\Gamma(k)\Gamma(n+1)} \frac{\bar{n}^n k^k}{(\bar{n}+k)^{n+k}}$$

Good, since multiplicity in pp collisions can be described well with NBD.

In AA, convolution of NBDs at all impact parameters describes data well too.

P. Tribedy and R. Venugopalan
Nucl.Phys. A850 (2011) 136-156

MC-KLN does not do that - these fluctuations need to be put in by hand.

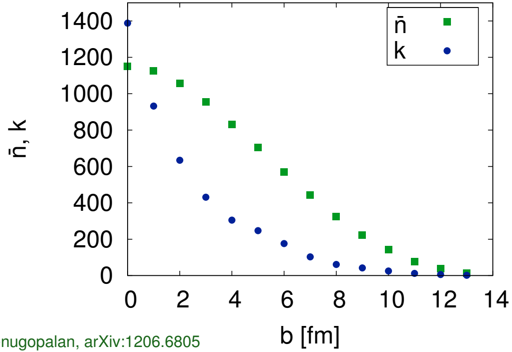
see Dumitru and Nara arXiv:1201.6382

# Negative binomial fluctuations

Extract $k$ and $\bar{n}$ using a fit with

$$P(n) = \frac{\Gamma(k+n)}{\Gamma(k)\Gamma(n+1)} \frac{\bar{n}^n k^k}{(\bar{n}+k)^{n+k}}$$

at fixed impact parameters.




B.Schenke, P.Tribedy, R.Venugopalan, arXiv:1206.6805

Ratio of $k/\bar{n}$ is $>1$ for small $b$ and becomes small $\sim 0.14$ for large $b$.
That is close to the value extracted for $p+p$ collisions: Dumitru and Nara arXiv:1201.6382

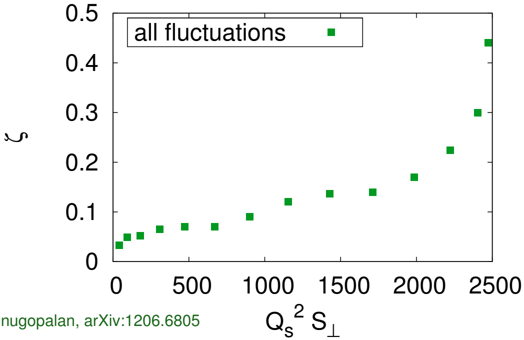
# NBDs and Glasma flux tubes

Glasma flux tube picture:

$$k = \zeta \frac{N_c^2 - 1}{2\pi} Q_s^2 S_\perp$$

Gelis, Lappi, Mclerran, arXiv:0905.3234.

Width of NBD is inversely proportional to the number of flux tubes $Q_s^2 S_\perp$.
$S_\perp =$ interaction area.




B.Schenke, P.Tribedy, R.Venugopalan, arXiv:1206.6805

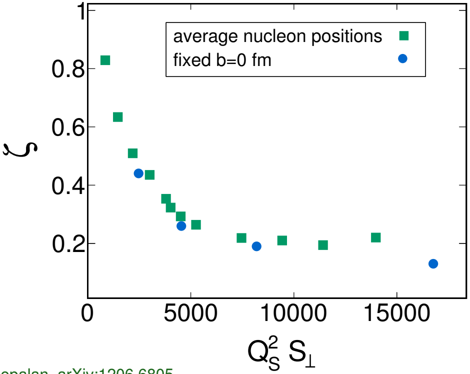
$\zeta$ should be close to constant in the flux tube picture.

# NBDs and Glasma flux tubes

$\zeta$ is not constant because geometric fluctuations are very important.
Were not considered in the derivation of

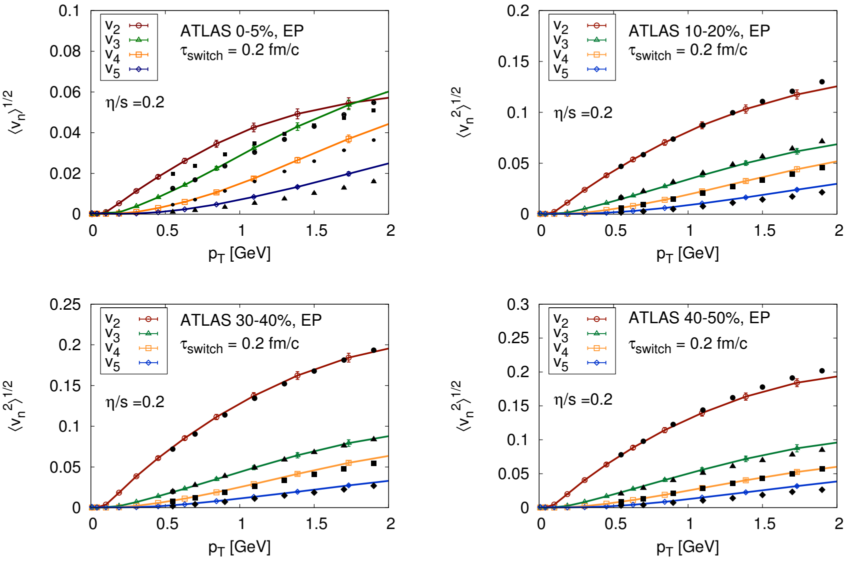
$$k = \zeta \frac{N_c^2 - 1}{2\pi} Q_s^2 S_\perp$$

Eliminate by using smooth nucleon distributions:

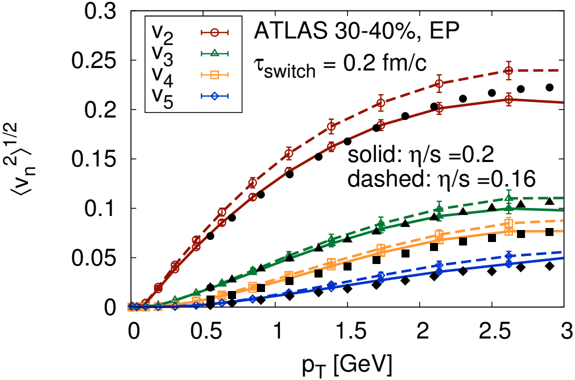



B.Schenke, P.Tribedy, R.Venugopalan, arXiv:1206.6805

# More centrality classes: IP-Glasma + MUSIC



# Smaller average $\eta/s$




Using $\eta/s = 0.16$ overestimates all $v_n$

Experimental data:
ATLAS collaboration, Phys. Rev. C 86, 014907 (2012)