Viscosity and thermal photon production

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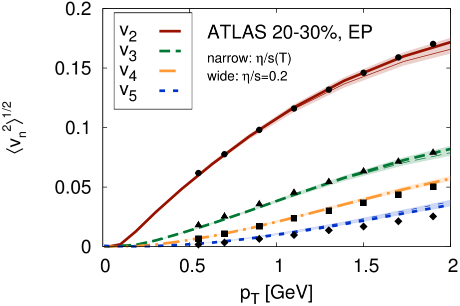
- Reminder on shear viscosity and photons
- Bulk viscosity and hadronic observables
- Bulk viscosity and electromagnetic observables

Elliptic flow from dissipative hydrodynamics

State of the art fits suggest $\eta/s \sim 0.2$

Gale, Jeon, Schenke, Tribedy, Venugopalan (2012)



Still a number of uncertainties (two that will be addressed here)

- Need for kinetics
- Bulk viscosity

Elliptic flow from dissipative hydrodynamics

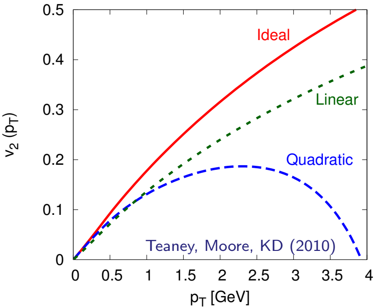
Spectra computed with Cooper-Frye:

$$E_{\mathbf{p}} \frac{dN}{d^3p} = \frac{1}{(2\pi)^3} \int_\sigma f(E_{\mathbf{p}})\, p^\mu d\sigma_\mu$$

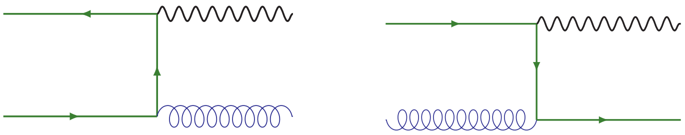
Expand $f = f_o + \delta f$ with constraint:

$$\delta T^{\mu\nu} = \int \frac{d^3\mathbf{p}}{(2\pi)^3 E_{\mathbf{p}}}\, p^\mu p^\nu \delta f(E_{\mathbf{p}})$$

Only moments of δf fixed by hydro;
leaving a need for kinetic models.

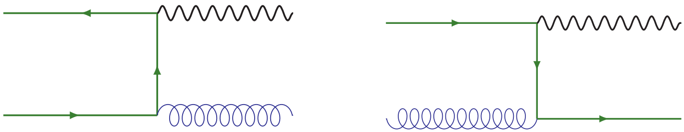


QGP distribution functions



- Photons are completely out of equilibrium (contrast to early universe)
- Photon spectra only appears thermal because quarks / gluons creating photons are thermal
- This is very clear at leading log where $P^{\mu}_{\rm quark} \approx Q^{\mu}_{\rm photon}$

QGP distribution functions



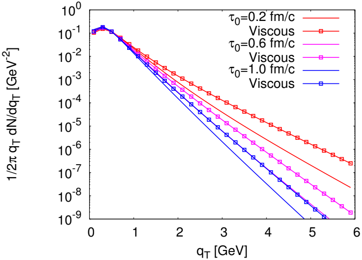
At leading log:

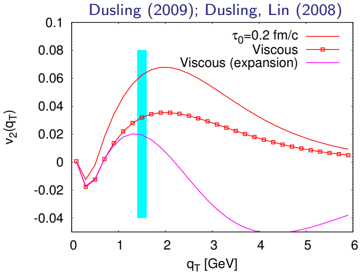
$$E\frac{dN_\gamma}{d^3q} \sim \alpha_{\rm em}\alpha_S f_{\rm quark}(Q_\gamma)T^2\log\left(\#\frac{E_\gamma}{g^2T}\right)$$

Out of equilbrium:

$$f_{\rm quark} = f_0\left(1 - \chi^{\rm shear}_{\rm quark}(q)\cdot q^i q^j \partial_{\langle i}u_{j\rangle} - \chi^{\rm bulk}_{\rm quark}(q)\cdot \partial_i u^i\right)$$

QGP photons



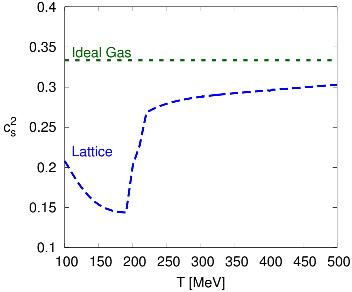


Above calculation shows the kinetic δf correction.

See the more sophisticated work by the McGill group.

Why bulk viscosity?

QCD is clearly not scale invariant and $\zeta \neq 0$.



So, do we need to understand bulk viscosity if we want to extract η/s?
Yes, but bulk viscosity is interesting in its own right.
Has implications for cosmology (*e.g.* relic abundances).

Relaxation Time Approximation

Approximate collision operator by single relaxation time:

$$\mathcal{C}[\delta f] \simeq -\frac{\delta f}{\tau_R(E_{\mathbf{p}})}$$

Bulk visocisity goes as 2^{nd} power of conformal breaking

$$\zeta \sim \eta \left(\frac{1}{3} - c_s^2\right)^2$$

Weinberg (1972)

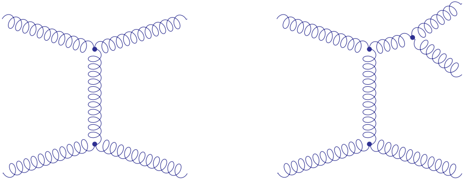
while distribution function goes as 1^{st} power

$$\frac{\delta f}{f_o} \sim p_T^2 \left(\frac{1}{3} - c_s^2\right)(\partial \cdot u)$$

Bulk viscous correction dominated by δf

QGP distribution functions

QCD: Elastic vs. Inelastic



Arnold, Dogan, Moore (2006)

For elastic $2 \leftrightarrow 2$ recast as Fokker-Planck

$$(\partial \cdot u)\left(\frac{p^2}{3} - c_s^2 E_{\mathbf{p}} \frac{\partial\left(\beta E_{\mathbf{p}}\right)}{\partial \beta}\right) = \frac{T\mu_A}{n_{\mathbf{p}}} \frac{\partial}{\partial p^i}\left(n_{\mathbf{p}} \frac{\partial}{\partial p^i}\left[\frac{\delta f_{\mathbf{p}}}{n_{\mathbf{p}}}\right]\right) + \cdots$$

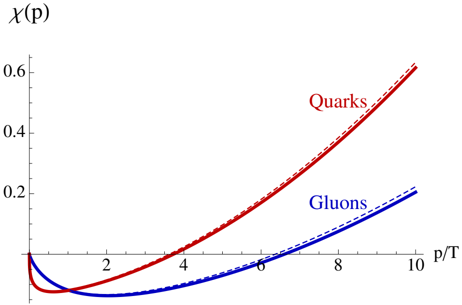
Drag coefficient:

$$\mu_A = \frac{g^2 C_A m_D^2}{8\pi} \ln\left(\frac{T}{m_D}\right)$$

QGP distribution functions

Result:

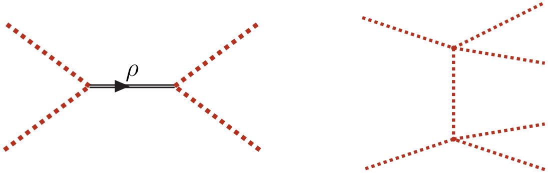
$$\delta f_{\rm bulk} \sim \left(\frac{1}{3} - c_s^2\right) \delta f_{\rm shear} \qquad \zeta \sim 50 \left(\frac{1}{3} - c_s^2\right)^2 \eta$$



where $\delta f = -f_o \, (\partial \cdot u) \, \chi(p)$

Pion Gas:

Pion Gas: Elastic vs. Inelastic



Bulk viscosity governed by chemical non-equilibrium

δf takes form of zero mode which dominates $\mathcal{C}^{-1}$

$$\begin{aligned} \delta f &= -f_o\left(\chi_0 - \chi_1 E_{\mathbf{p}}\right)(\partial \cdot u) \\ &= f_o\left(\frac{\delta\mu}{T} + \frac{\delta T}{T^2} E_{\mathbf{p}}\right) \end{aligned}$$

Lu, Moore (2011); Jeon, Yaffe (1995)

Pion Gas:

Bulk viscosity governed by chemical non-equilibrium

$$\delta f = -f_o \, (\partial \cdot u) \, (\chi_0 - \chi_1 E_{\mathbf{p}})$$

Chemical equilibration rate determines χ_0, energy conservation fixed χ_1

$$\chi_0 = \frac{\beta \mathcal{F}}{4\Gamma_{2\pi\to 4\pi}}, \qquad \zeta = \frac{\mathcal{F}^2}{4\Gamma_{2\pi\to 4\pi}}$$

where $\mathcal{F}$ characterizes conformal breaking

$$\mathcal{F} \equiv \int \frac{d^3\mathbf{p}}{(2\pi)^3 E_{\mathbf{p}}} \left(\frac{p^2}{3} - c_s^2 E_{\mathbf{p}}^2 \right) f_o$$

Hadron Resonance Gas: A model

Assume slowest process is chemical relaxation:

$$\delta f^a = -f_o\,(\partial \cdot u)\,(\chi_0^a - \chi_1 E_{\mathbf{p}})$$

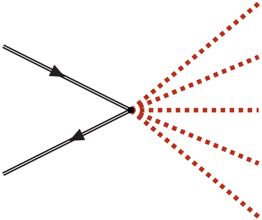
where $a = \pi, K, \rho, K^*, p, n, \Delta, \cdots$

Slowest rate determines ζ, other rates fix the relative δf^a:

$$\chi_0^a \simeq \chi_0^\pi \times \begin{cases} 2 & \text{Mesons} \\ 2.5 & \text{Baryons} \end{cases}$$

Motivated by $\mu_\rho = 2\mu_\pi$ and $2\mu_N = 5\mu_\pi$:

Goity (1993); Pratt, Haglin (1999)



Hadron Resonance Gas: A model

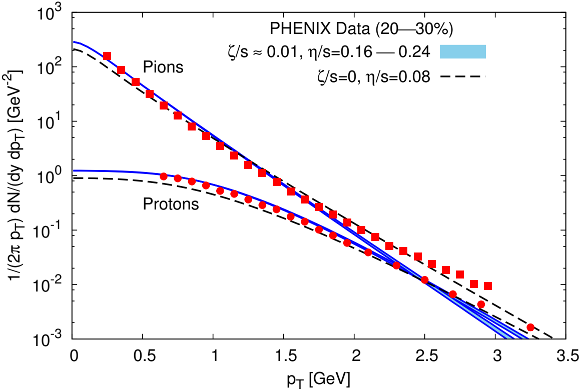
Hadron Gas: ζ determined by chemical non-equilibration

Bottom line for δf:

- $\delta\mu \sim \frac{\zeta}{s}\,(\partial \cdot u)$
- temperature shift $\delta T \approx 0.25\delta\mu$ to conserve energy

A new (dynamical) way to look at fugacity factors

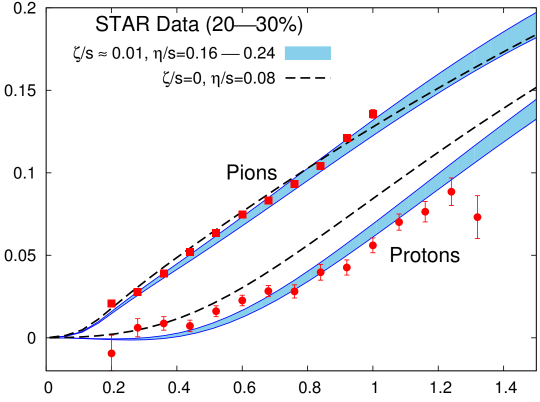
Pion / Proton p_T spectra



Data: PHENIX nucl-ex/0307022.

LHC: Bozek & Wyskiel arxiv:1203.6513

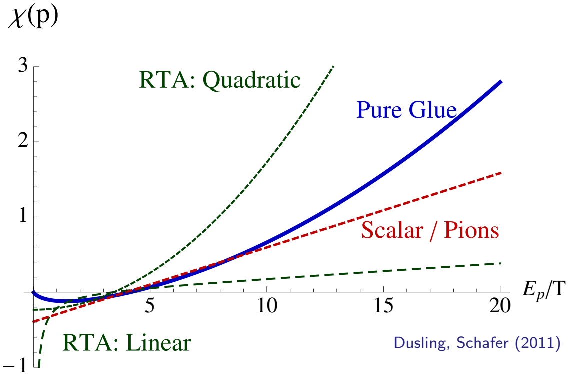
Pion / Proton differential $v_2(p_T)$ spectra



Data: STAR, nucl-ex/0409033.

Bulk viscosity and EM probes

As for shear viscosity, bulk δf modifies photon rates



Dusling, Schafer (2011)

Bulk viscosity and EM probes

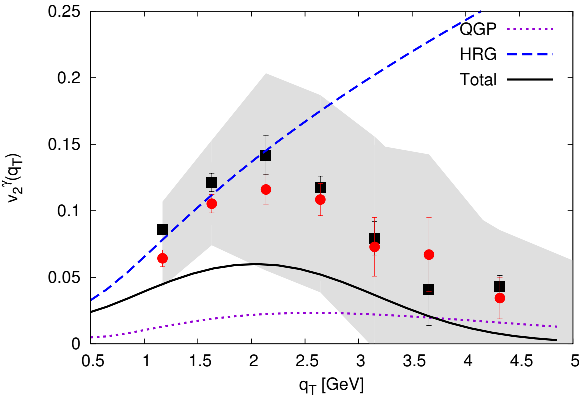
First shot:

- Using AMY rates with $f \to f + \delta f$ and pQCD δf

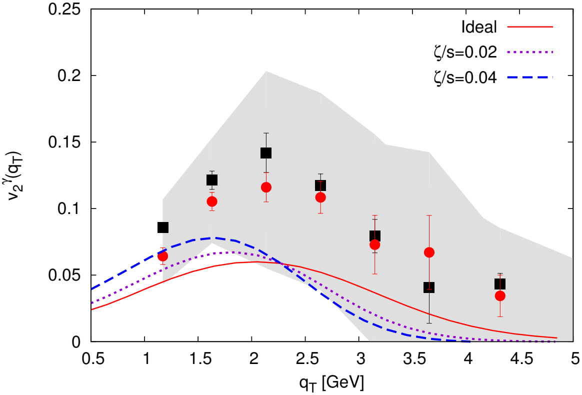
- Using χRF (Steele, Yamagishi, Zahed) with
  - z_π^3 enhancement with $\mu_\pi \sim \zeta/s\,(\partial \cdot u)$
  - $T \to T - \delta T$

- Results preliminary: Need to
  - include shear viscosity
  - fine-tune initial condition
  - worry about pQCD at high q_T
  - $\cdots$

Ideal photon $v_2(q_T)$



Viscous photon $v_2(q_T)$



Only including bulk viscosity – no fine-tuning.

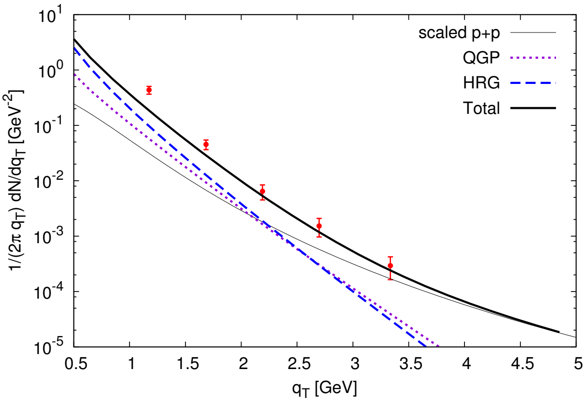
Summary

Bulk viscosity is not zero:

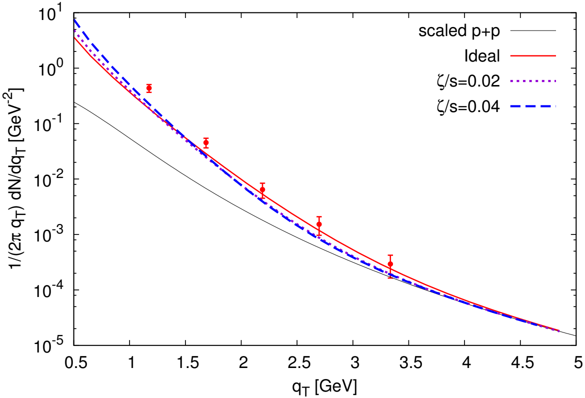
- fine structure of spectra improves
- may help with photon v_2
- dynamical mechanism for fugacity factors

Backup Slides

Ideal photon q_T spectra



Viscous photon q_T spectra



Only including bulk viscosity – no fine-tuning.

Thermal photon production

At leading order in $\alpha_{\rm em}$ but all orders in $\alpha_{\rm s}$

$$\frac{dN}{d^4Q} = \frac{\alpha_{\rm em}^2}{6\pi^3}\frac{1}{Q^4}\left(Q^\mu Q^\nu - Q^2 g^{\mu\nu}\right) W_{\mu\nu}(Q)$$

where

$$W_{\mu\nu}(Q) \equiv \int d^4x \; e^{-iQ\cdot X} \langle J_\mu^{\rm em}(X) J_\nu^{\dagger,{\rm em}}(0)\rangle_\beta$$

Evaluate $W_{\mu\nu}$ in two different ways

- Vacuum spectral functions
- Kinetic Theory

Thermal photon production

Kinetic theory (all processes $I \to F + l^+ l^-$):

$$W_{\mu\nu}(Q) = \sum_F \sum_I \int d^4x \; e^{-iQ\cdot X} \langle I | J_\mu^{\mathrm{em}}(X) | F \rangle \langle F | J_\nu^{\dagger,\mathrm{em}}(0) | F \rangle \frac{e^{-\beta E_I}}{\mathcal{Z}}$$

using $E_I = E_F + Q_0$ and $\sum_I |I\rangle\langle I| = 1$

$$W_{\mu\nu}(Q) = e^{-\beta Q_0} \sum_F \int d^4x \; e^{+iQ\cdot X} \langle F | J_\mu^{\dagger,\mathrm{em}}(X) J_\nu^{\mathrm{em}}(0) | F \rangle \frac{e^{-\beta E_F}}{\mathcal{Z}}$$

McLerran & Toimela, 1985