Photons and di-leptons in strong magnetic field in heavy-ion collisions

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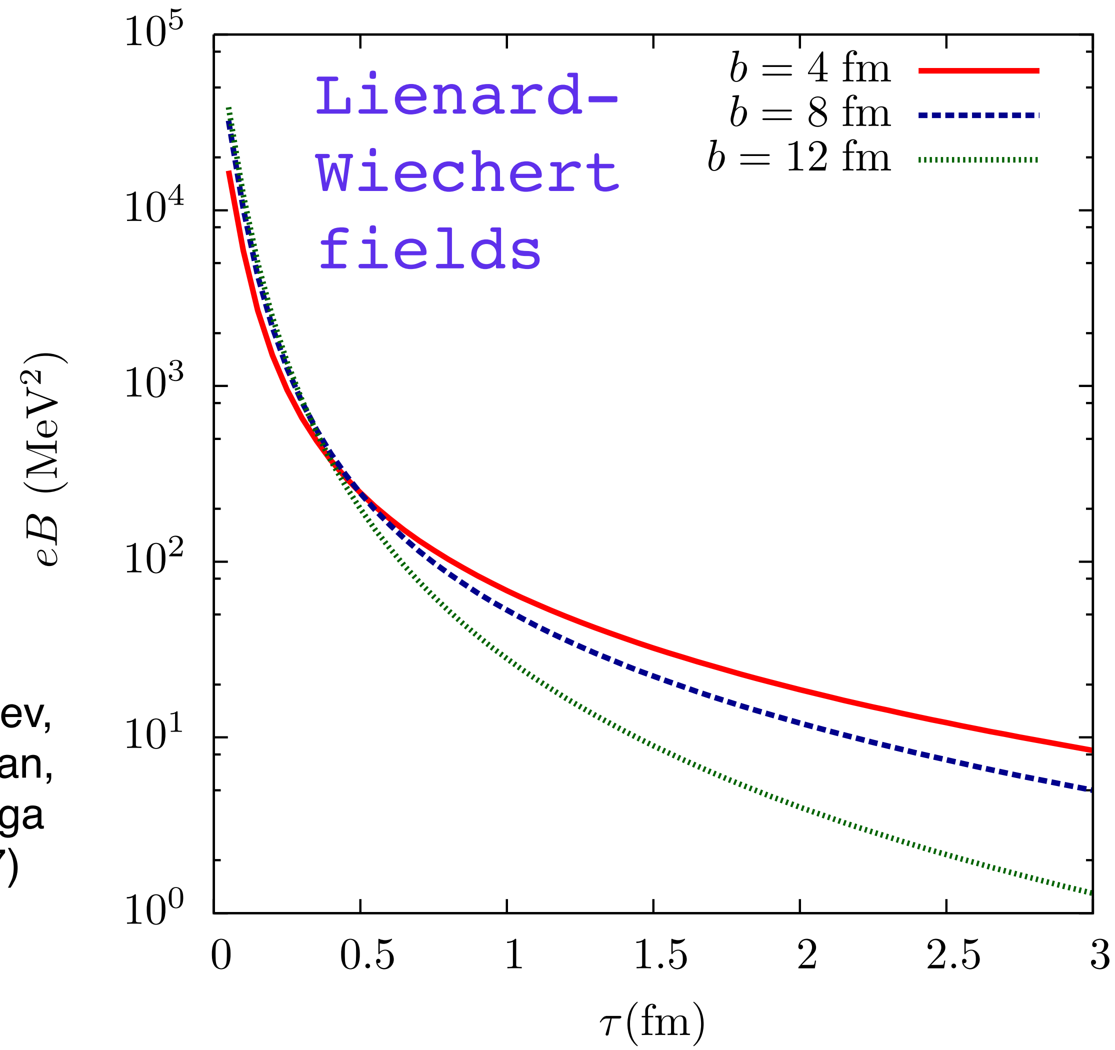
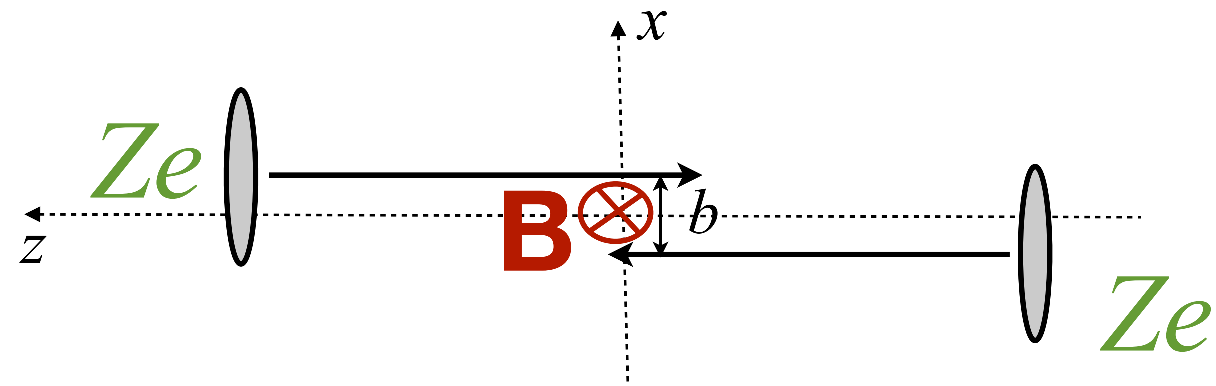
OUTLINE

1. Time-dependence of magnetic field.
2. Synchrotron radiation.
3. Energy loss and polarization in magnetic field.

MAGNETIC FIELD IN VACUUM



Kharzeev, McLerran, Warringa (2007)



$$B \sim Ze\frac{b}{R^3}\gamma$$

For Z=79, b=7 fm, γ=100 we get $eB = (200\ MeV)^2 \approx m_\pi^2$

Similar results:

UrQMD based calculation: Skokov, Illarionov, Toneev (2009)

Hadron String Dynamic transport code: Voronyuk, Toneev, Cassing, Bratkovskaya, Konchakovski, Voloshin (2011)

LEINARD-WIECHERT FILED IN DISPERSIVE MEDIUM

Medium is formed at a very early stage after a Heavy Ion Collision: Glasma (t~0.2 fm) gives way to Quark Gluon Plasma (t~0.5-2 fm). According to the state-of-the-art phenomenology it can be characterized by transport coefficients.

Magnetic field created by a single point charge *in medium:*

$$e\boldsymbol{B} = \frac{\alpha}{\pi}\,\hat{\boldsymbol{y}}\int_{-\infty}^{\infty} s(\omega)\,K_1(s(\omega)b)\,e^{i\omega(z/v-t)}\,d\omega \qquad s(\omega) = \omega\sqrt{\frac{1}{v^2}-\epsilon(\omega)}$$

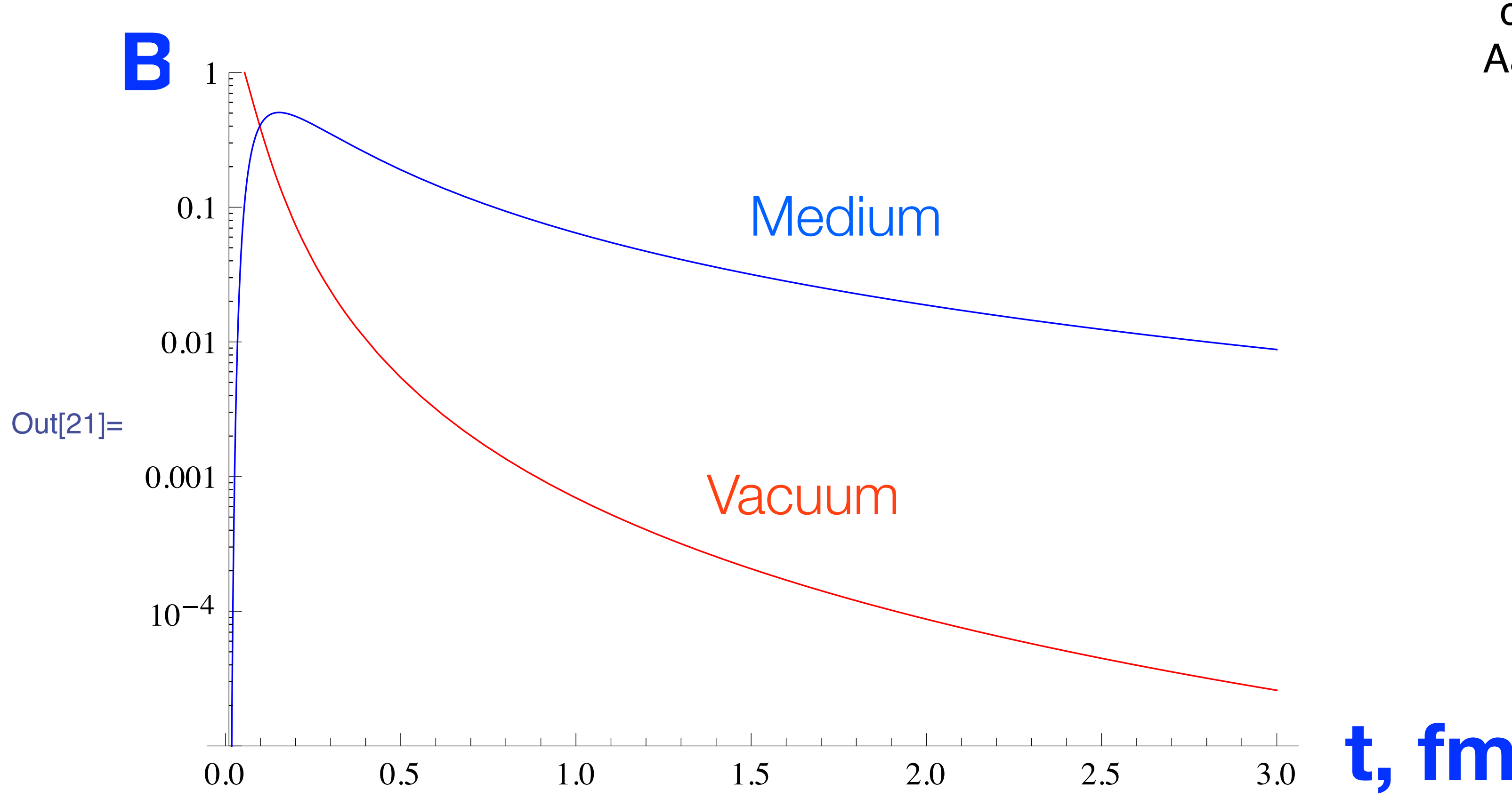
At low frequencies dielectric constant has a pole due to a finite electric conductivity:

$$\epsilon(\omega) = 1 + \frac{i\sigma}{\omega}$$

The corresponding magnetic field in medium: $eB = \hat{\boldsymbol{y}}\,\frac{\alpha_{\mathrm{em}} b\,\sigma}{2(t-z)^2}\,e^{-\frac{b^2\sigma}{4(t-z)}}$

Compared with the magnetic field in vacuum: $e\boldsymbol{B} = \hat{\boldsymbol{y}}\,\alpha_{\mathrm{em}}\,\frac{b\gamma}{(b^2+\gamma^2(t-z)^2)^{3/2}}$

MAGNETIC FIELD IN CONDUCTING PLASMA



conductivity of QGP on the lattice:
Aarts at al (2007), Ding at al (2010):
only gluon contribution!

$$\sigma = 5\,\mathrm{MeV}$$

- This is magnetic field due to outgoing valence charges in infinite medium.

MAGNETIC FIELD IN AN EXPANDING MEDIUM

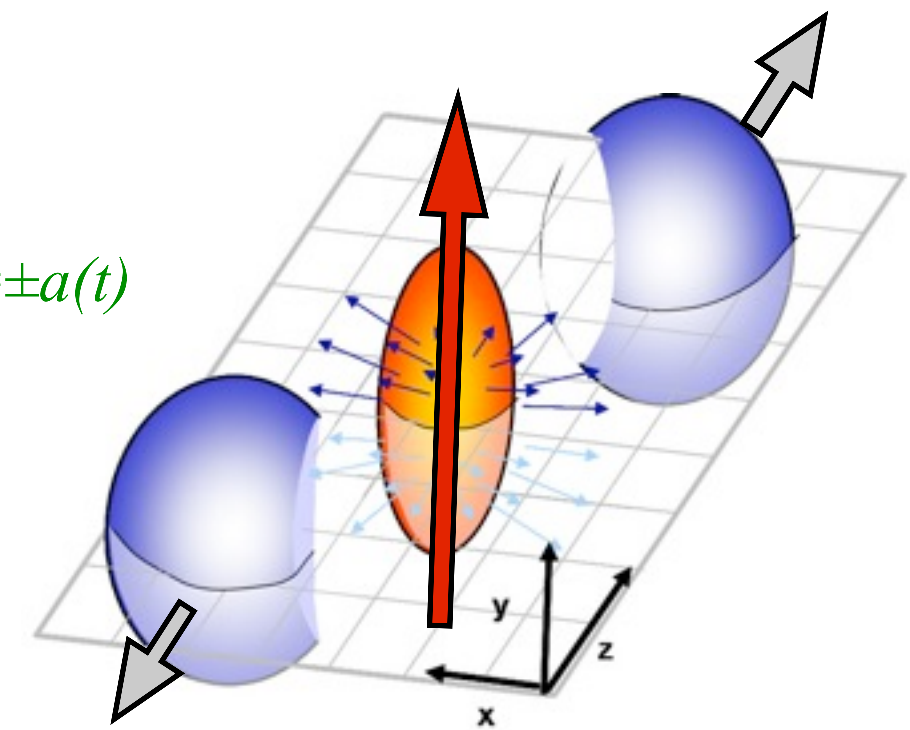
K.T. 2010

$$\nabla \times \mathbf{E} = -\dot{\mathbf{B}}$$
$$\nabla \times \mathbf{B} = \mathbf{j} = \sigma \mathbf{E}$$

Lenz's law: induced B is parallel to the original B.

j: Foucault currents

$$\nabla^2 \boldsymbol{B} = \sigma \dot{\boldsymbol{B}}$$



Problem: the boundary is time-dependent. It moves along $z=\pm a(t)$

Introduce rapidity $$\eta = \frac{1}{2}\ln\frac{a(t)+z}{a(t)-z}$$

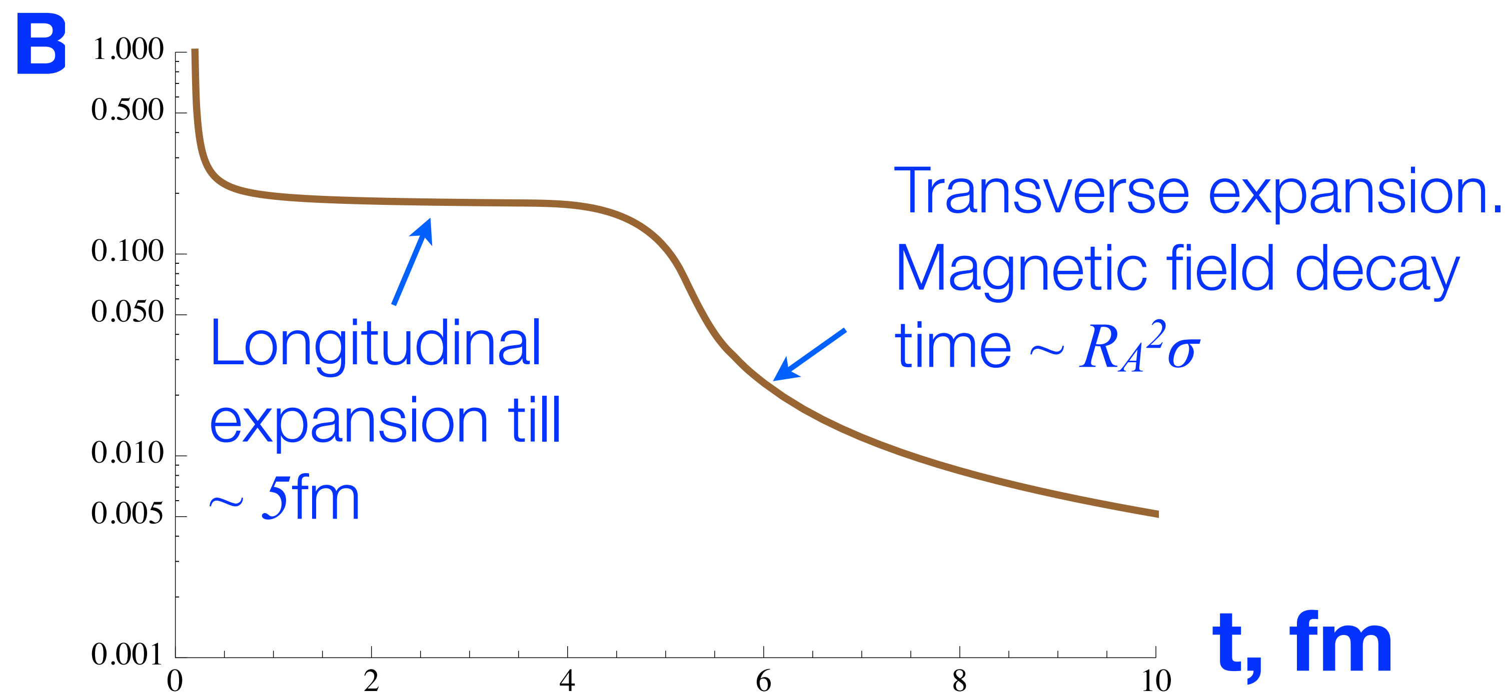
Now the boundary is time-independent: **η**=±∞

At **η**≪1 $$\nabla_\perp^2 \boldsymbol{B} + \frac{1}{a^2(t)}\frac{\partial^2 B}{\partial \eta^2} = \sigma \frac{\partial \boldsymbol{B}}{\partial t}$$

At early times $a(t) \ll R_A$, the transverse derivatives can be dropped, while at later times we can neglect the longitudinal derivatives.

INDUCED MAGNETIC FIELD

The result of matching of the two solutions



Not taken into account: dependence of conductivity on B.

see e.g. K.T. "On viscous flow and azimuthal anisotropy of quark-gluon plasma in strong magnetic field", arXiv:1108.4394

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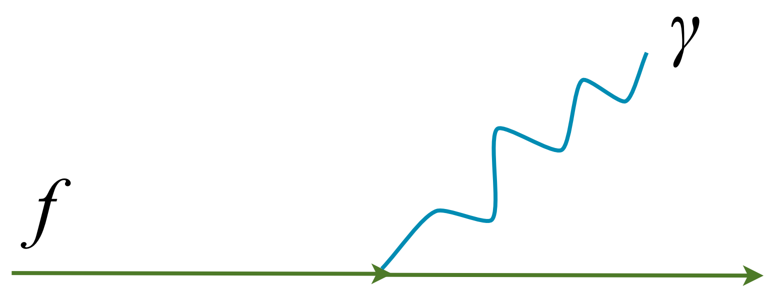
SYNCHROTRON RADIATION

KT (2010,2012)

Synchrotron radiation:



$$f(e_f, j, p) \to f(e_f, k, q) + \gamma(\boldsymbol{k})$$

QGP is transparent to the emitted electromagnetic radiation because its absorption coefficient is suppressed by α^2 (I'll show the precise calculation later).

Spacing between the Landau levels ~ ***eB/ε***, while their thermal width ~ ***T***. When ***eB/ε≳T*** it is essential to account for quantization of fermion spectra.

Fermion spectrum quantization is important not only for hard and electromagnetic probes but also for the bulk properties of QGP.

Kinematics

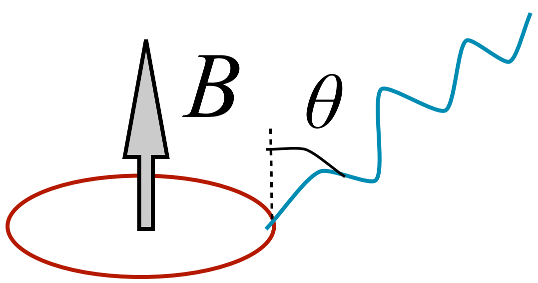
$$\varepsilon_j = \sqrt{m^2 + p^2 + 2je_fB}\,, \quad \varepsilon_k = \sqrt{m^2 + q^2 + 2ke_fB}$$

j (k) is the quantum number of Landau orbit of *initial (final)* charged fermion.

p (q) is the projection of *initial (final)* fermion momentum on the direction of *B*

Magnetic field doesn't do work ➞ energy is conserved. Magnetic Lorentz force has no component along the *B*-direction ➞ momentum along B is conserved

$$\varepsilon_j = \omega + \varepsilon_k\,, \quad p = q + \omega\cos\theta$$



Angular distribution of the power spectrum:

$$\frac{dI^j}{d\omega d\Omega} = \sum_f \frac{z_f^2\alpha}{\pi}\omega^2 \sum_{k=0}^{j} \Gamma_{jk} \left\{|\mathcal{M}_\perp|^2 + |\mathcal{M}_\parallel|^2\right\} \delta(\omega - \varepsilon_j + \varepsilon_k)$$

Matrix elements for synchrotron transitions corresponding to photon polarization perpendicular and parallel to B

Sokolov, Ternov (1968) and others

$$4\varepsilon_j\varepsilon_k|\mathcal{M}_\perp|^2 = (\varepsilon_j\varepsilon_k - pq - m^2)[I_{j,k-1}^2 + I_{j-1,k}^2] + 2\sqrt{2je_fB}\sqrt{2ke_fB}[I_{j,k-1}I_{j-1,k}]\,.$$

$$\begin{aligned}4\varepsilon_j\varepsilon_k|\mathcal{M}_\parallel|^2 = \cos^2\theta\big\{&(\varepsilon_j\varepsilon_k - pq - m^2)[I_{j,k-1}^2 + I_{j-1,k}^2] - 2\sqrt{2je_fB}\sqrt{2ke_fB}[I_{j,k-1}I_{j-1,k}]\big\}\\ &- 2\cos\theta\sin\theta\big\{p\sqrt{2ke_fB}[I_{j-1,k}I_{j-1,k-1} + I_{j,k-1}I_{j,k}]\\ &+ q\sqrt{2je_fB}[I_{j,k}I_{j-1,k} + I_{j-1,k-1}I_{j,k-1}]\big\}\\ &+ \sin^2\theta\big\{(\varepsilon_j\varepsilon_k + pq - m^2)[I_{j-1,k-1}^2 + I_{j,k}^2] + 2\sqrt{2je_fB}\sqrt{2ke_fB}(I_{j-1,k-1}I_{j,k})\big\}\end{aligned}$$

$$I_{j,k} \equiv I_{j,k}(x) = (-1)^{j-k}\sqrt{\frac{k!}{j!}}e^{-\frac{x}{2}}x^{\frac{j-k}{2}}L_k^{j-k}(x).$$

Laguerre polynomials (recall Schrödinger equation for hydrogen)

$$x = \frac{\omega^2}{2e_fB}\sin^2\theta$$

PHOTON NUMBER SPECTRUM

We are interested in the photon number spectrum radiated from QGP

$$\frac{dN^{\rm synch}}{dt d\Omega d\omega} = \sum_f \int_{-\infty}^{\infty} dp \frac{e_f B(2N_c)V}{2\pi^2} \sum_{j=0}^{\infty} \sum_{k=0}^{j} \frac{dI^j}{\omega d\omega d\Omega}(2-\delta_{j,0}) f(\varepsilon_j)[1-f(\varepsilon_k)]$$

$$f(\varepsilon) = \frac{1}{e^{\varepsilon/T}+1}$$

To take integral over p write

$$\delta(\omega - \varepsilon_j + \varepsilon_k) = \sum_{\pm} \frac{\delta(p - p_\pm^*)}{\left|\frac{p}{\varepsilon_j} - \frac{q}{\varepsilon_k}\right|}$$

$$p_\pm^* = \Big\{ \cos\theta(m_j^2 - m_k^2 + \omega^2 \sin^2\theta) \pm \sqrt{[(m_j+m_k)^2 - \omega^2 \sin^2\theta][(m_j - m_k)^2 - \omega^2\sin^2\theta]} \Big\} / (2\omega \sin^2\theta)$$

$$m_j^2 = m^2 + 2je_f B\,, \quad m_k^2 = m^2 + 2ke_f B$$

$p_\pm$ is real in two cases:

$$\text{(i)}\ m_j - m_k \geq \omega \sin\theta\,, \quad \text{or} \quad \text{(ii)}\ m_j + m_k \leq \omega \sin\theta$$

synchrotron radiation

one-photon pair annihilation

In case (i) the $j \to k$ transition must satisfy

$$\omega \leq \omega_{s,jk} \equiv \frac{m_j - m_k}{\sin\theta} = \frac{\sqrt{m^2 + 2je_fB} - \sqrt{m^2 + 2ke_fB}}{\sin\theta}$$

in particular $j{=}k$ transition is forbidden.

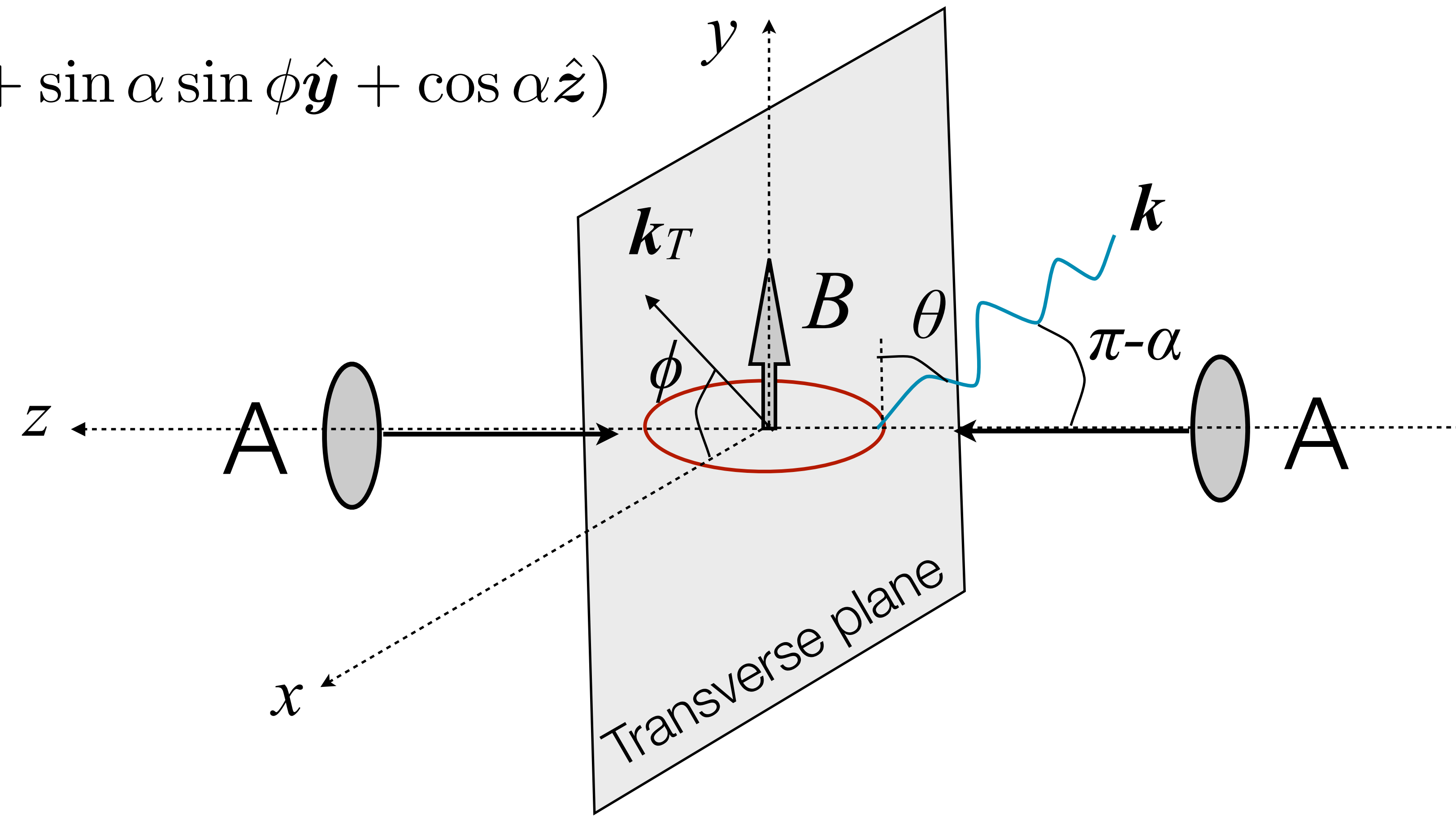
Spectral distribution of the synchrotron radiation rate per unit volume:

$$\frac{dN^{\mathrm{synch}}}{V dt d\Omega d\omega} = \sum_f \frac{2N_c z_f^2 \alpha}{\pi^3} e_f B \sum_{j=0}^{\infty} \sum_{k=0}^{j} \omega (1+\delta_{k0})\, \vartheta(\omega_{s,ij} - \omega) \int dp \sum_{\pm} \frac{\delta(p - p_\pm^*)}{\left|\frac{p}{\varepsilon_j} - \frac{q}{\varepsilon_k}\right|}$$

$$\times \left\{ |\mathcal{M}_\perp|^2 + |\mathcal{M}_\parallel|^2 \right\} f(\varepsilon_j)[1 - f(\varepsilon_k)]\,,$$

HIGH-ENERGY REFERENCE FRAME

$$\boldsymbol{k} = \omega(\sin\alpha\cos\phi\hat{\boldsymbol{x}} + \sin\alpha\sin\phi\hat{\boldsymbol{y}} + \cos\alpha\hat{\boldsymbol{z}})$$



$$\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{y}} = \cos\theta \;\Rightarrow$$

$$\cos\theta = \sin\alpha\sin\phi$$

Thus, azimuthal dependence (ϕ) of the spectrum is an artifact of the frame choice!

$$k_\perp = \sqrt{k_x^2 + k_y^2} = \frac{\omega\cos\theta}{\sin\phi}\,, \quad y = -\ln\tan\frac{\alpha}{2}$$

$$\frac{dN^{\rm synch}}{dV\,dt\,d^2k_\perp dy} = \omega\frac{dN^{\rm synch}}{dV\,dt\,d^3k} = \frac{dN^{\rm synch}}{dV\,dt\,\omega d\omega d\Omega}$$

SYNCHROTRON SPECTRUM

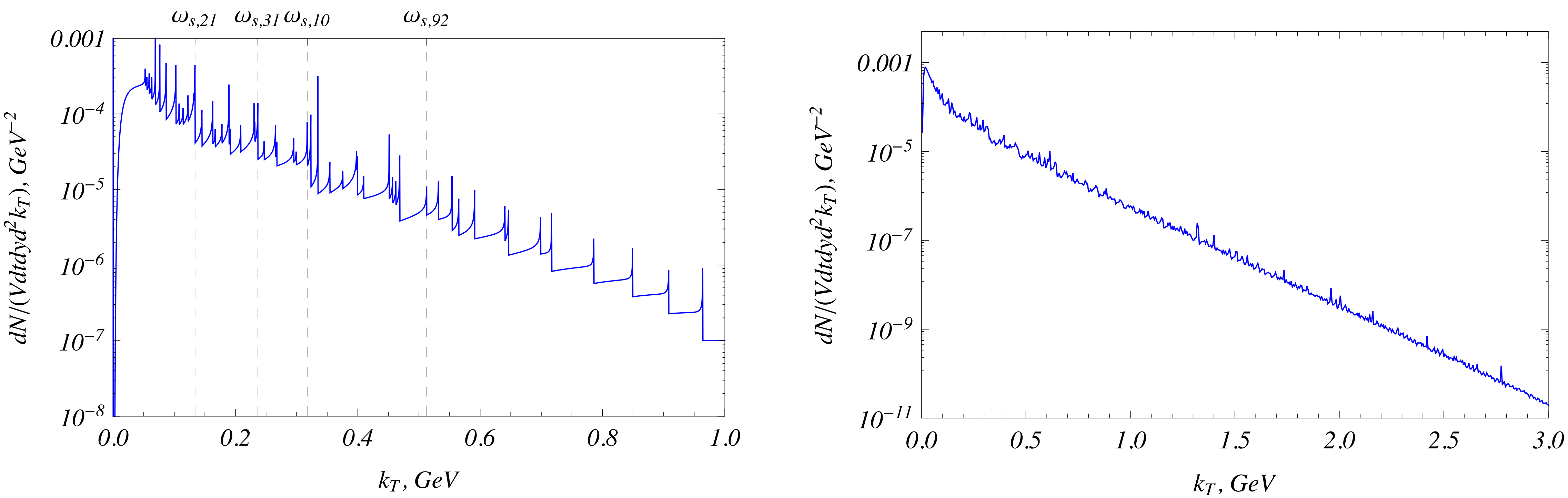


FIG. 1: Spectrum of synchrotron radiation by u quarks at $eB = m_\pi^2$, $y = 0$, $\phi = \pi/3$: (a) contribution of 10 lowest Landau levels $j \leq 10$; several cutoff frequencies are indicated; (b) summed over all Landau levels. $m_u = 3$ MeV, $T = 200$ MeV.

ANGULAR DISTRIBUTION OF SR

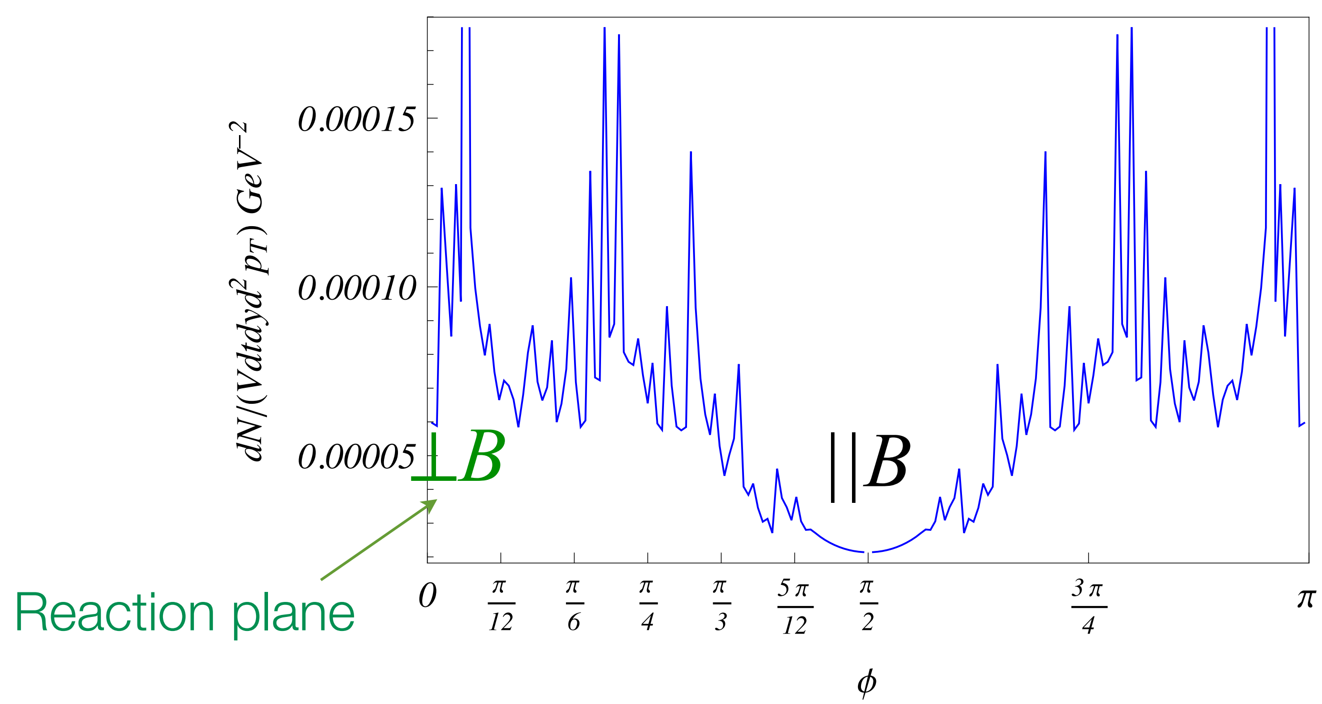
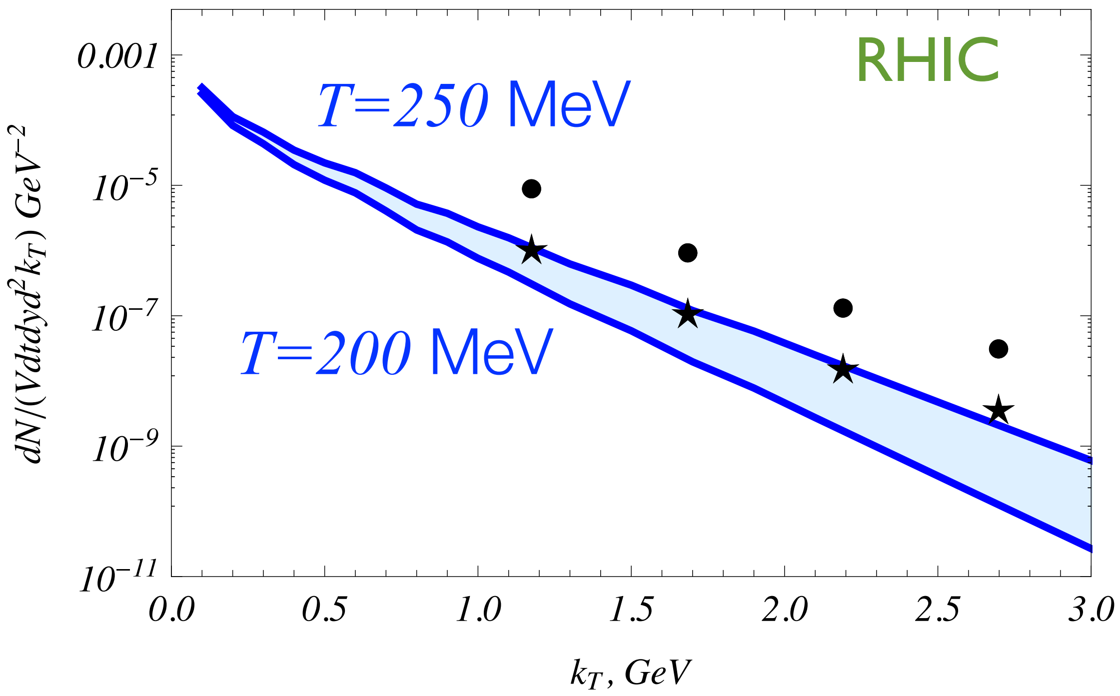


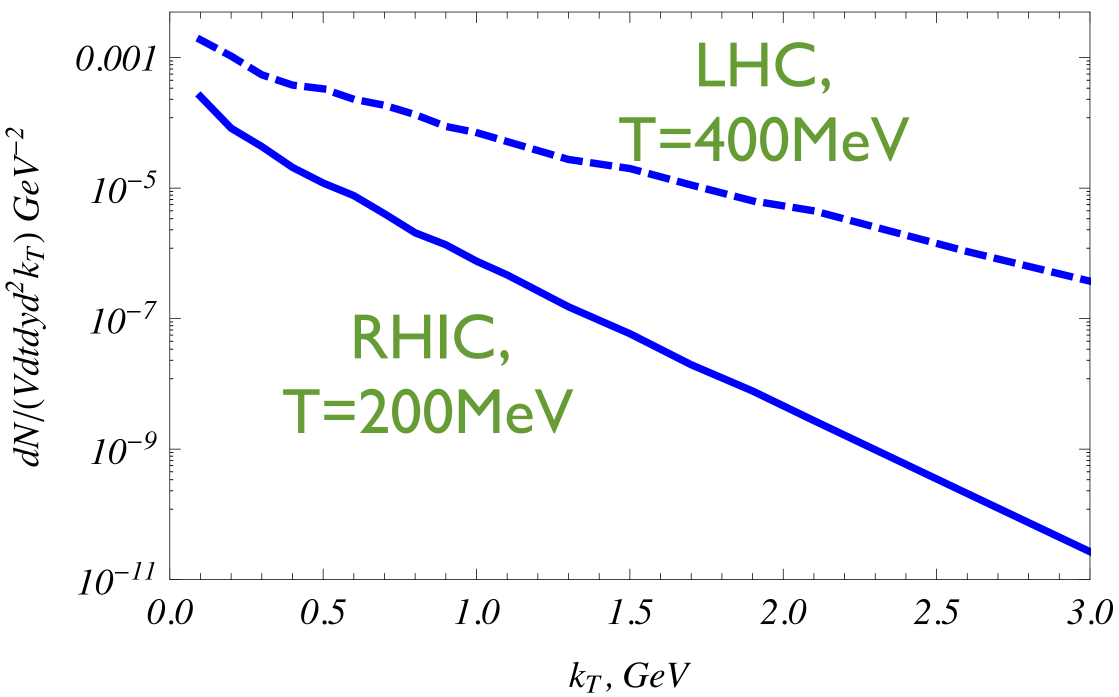
FIG. 2: Azimuthal distribution of synchrotron radiation by u-quarks at $k_\perp = 0.2$ GeV, $eB = m_\pi^2$, $y = 0$. $m_u = 3$ MeV.

This distribution implies that $v_2 > 0$ (to be calculated)

Since ϕ appears only in $\cos^2\phi$ term, there is clear symmetry $\phi \to \pi - \phi \Rightarrow$ only even harmonics survive. Odd harmonic arise from B fluctuations.

SYNCHROTRON SPECTRUM





● $Vt=25\pi\ fm^4$

★ $Vt=9\times 25\pi\ fm^4$

Photon spectrum is very sensitive to the QGP temperature.

HOW MANY LANDAU LEVELS CONTRIBUTE?

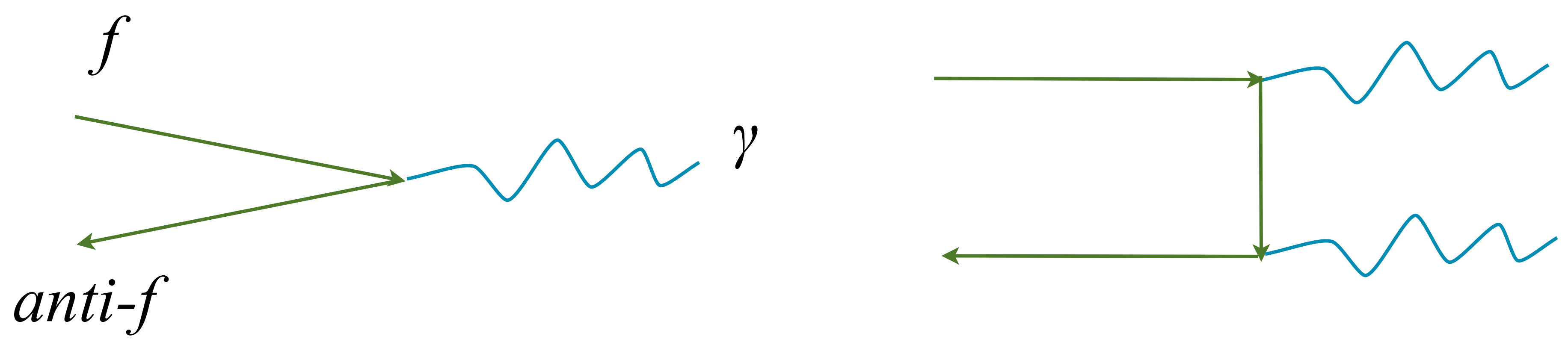
$$\frac{dN^{\rm synch}}{dt d\Omega d\omega} = \sum_f \int_{-\infty}^{\infty} dp \frac{e_f B (2N_c) V}{2\pi^2} \sum_{j=0}^{j_{\rm max}} \sum_{k=0}^{j} \frac{dI^j}{\omega d\omega d\Omega} (2-\delta_{j,0}) f(\varepsilon_j)[1-f(\varepsilon_k)]$$

f	u	u	u	u	u	u	s	u	u	s
eB/m_π^2	1	1	1	1	1	1	1	15	15	15
T, GeV	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.4	0.4	0.4
ϕ	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{6}$	$\frac{\pi}{12}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$
$k_\perp$, GeV	0.1	1	2	3	1	1	1	1	2	1
x	0.096	9.6	38	86	29	35	19	0.64	2.6	1.3
$j_{\rm max}$	30	40	90	150	120	200	90	8	12	16

TABLE I: The upper summation limit in (18) that yields the 5% accuracy. $j_{\rm max}$ is the highest Landau level of the initial quark that is taken into account at this accuracy. Throughout the table $y = 0$.

Large j,k correspond to quasi-classical limit.

PAIR ANNIHILATION



One and two-photon annihilation: At $eB \gg m^2$ one-photon annihilation dominates.

One-photon annihilation is a cross-channel of synchrotron radiation. The corresponding matrix elements are straightforward to calculate.

$$\frac{dN^{\rm annih}}{V\,dt\,d\omega\,d\Omega} = \sum_f \frac{\alpha z_f^2 \omega N_c}{4\pi e_f B} \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} \int dp\, \frac{2e_f B}{2\pi^2} f(\varepsilon_j) \int dq\, \frac{2e_f B}{2\pi^2} f(\varepsilon_k)$$

$$\times \delta(p+q-\omega\cos\theta)\,\delta(\varepsilon_j+\varepsilon_k-\omega)\{|\mathcal{T}_\perp|^2 + |\mathcal{T}_\parallel|^2\}$$

PAIR ANNIHILATION SPECTRUM

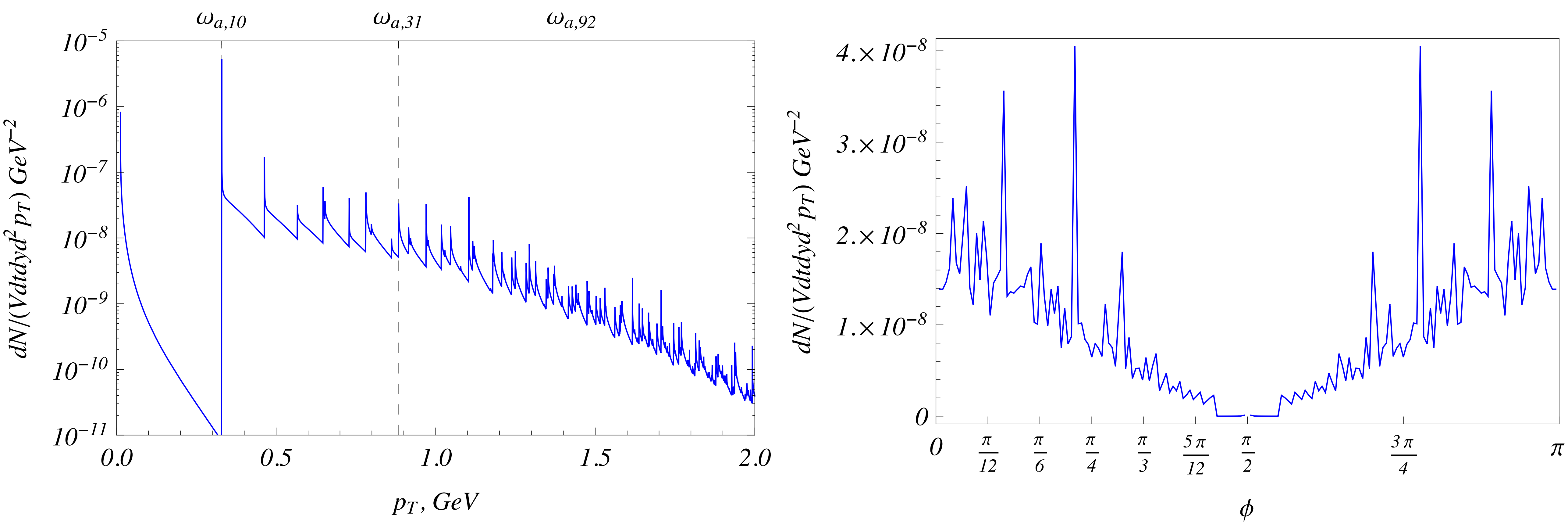


FIG. 5: Photon spectrum in one-photon annihilation of u and $\bar{u}$ quarks. $eB = m_\pi^2$, $y = 0$. (a) $k_\perp$-spectrum at $\phi = \pi/3$, (b) azimuthal angule distribution at $k_\perp = 1$ GeV.

Pair annihilation is numerically much smaller than synchrotron radiation.

CONCLUSIONS I

- Photon production by QGP due to its interaction with external magnetic field gives a considerable contribution to the total photon multiplicity in heavy-ion collisions.

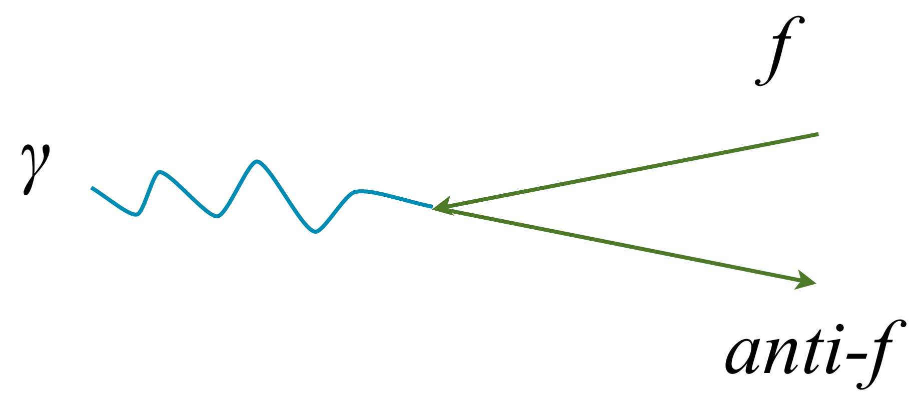
- In the kinematic region relevant for the current high energy heavy-ion experiments, contribution of the synchrotron radiation is about two orders of magnitude larger than that of pair annihilation.

- One possible way to ascertain the contribution of electromagnetic radiation in external magnetic field is to isolate the azimuthally symmetric component with respect to the direction of the magnetic field by rotating the reference frame, so that z-axis coincides with B-direction.

DILEPTON PRODUCTION VIA REAL PHOTON DECAY

KT (2010)

Photon decay is another cross-channel of the synchrotron radiation

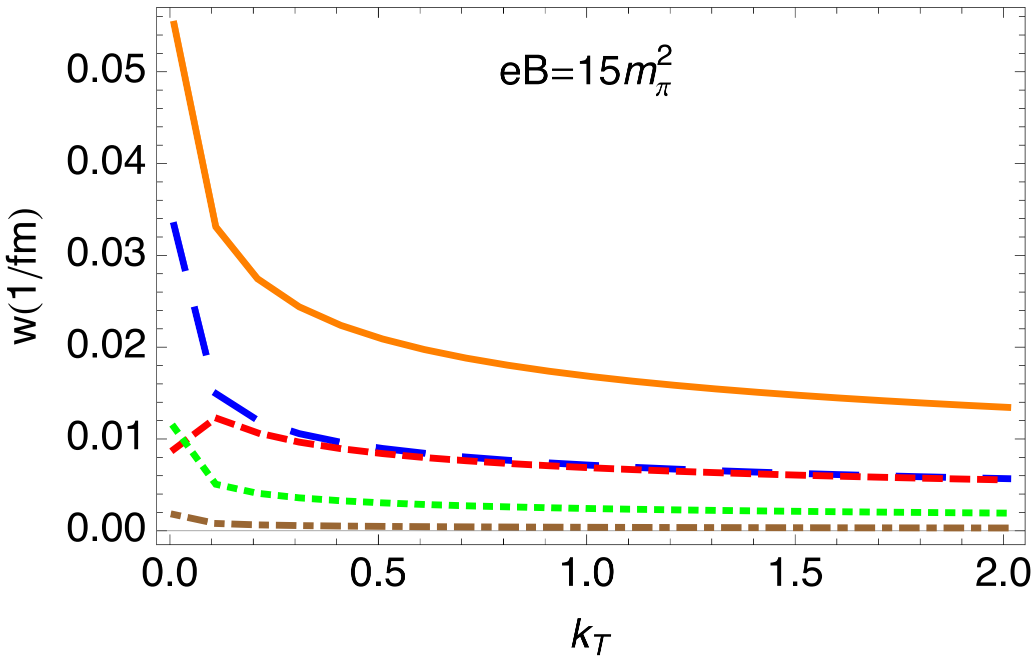


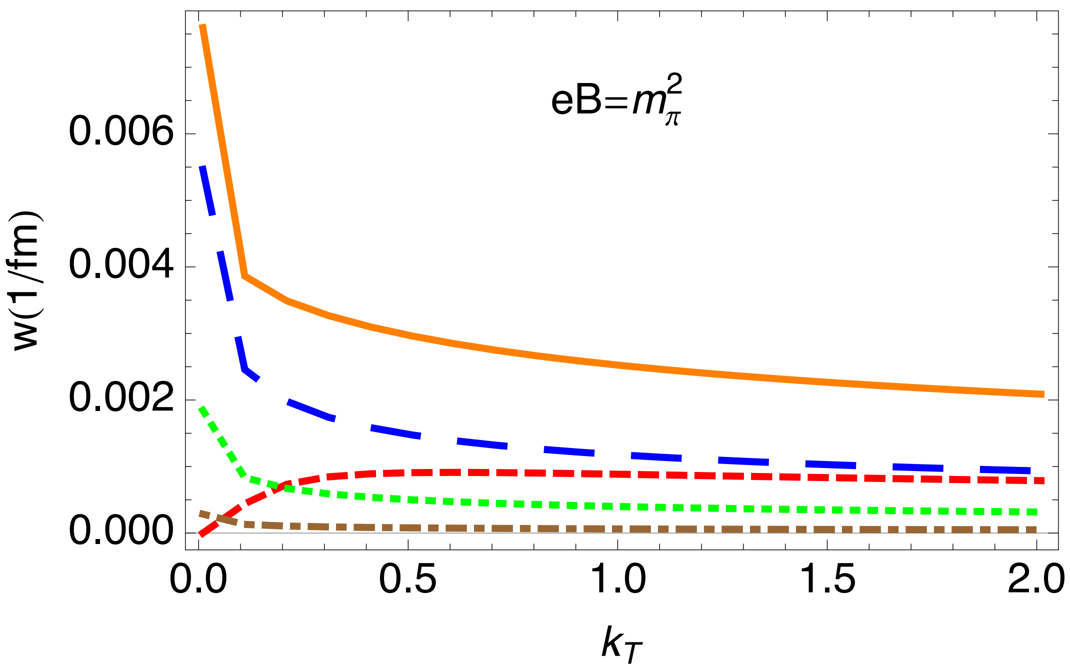
Rate in the quasi-classical ultra-relativistic approximation (discussed later):

$$w = -\sum_a \frac{\alpha_{\rm em}\, z_a^3\, eB}{m_a \varkappa_a} \int_{(4/\varkappa_a)^{2/3}}^{\infty} \frac{2(x^{3/2}+1/\varkappa_a)\,\mathrm{Ai}'(x)}{x^{11/4}(x^{3/2}-4/\varkappa_a)^{3/2}}$$

$$\varkappa_a^2 = -\frac{\alpha_{\rm em} z_a^2 \hbar^3}{m_a^6}\,(F_{\mu\nu}k^\nu)^2 = \frac{\alpha_{\rm em} z_a^2 \hbar^3}{m_a^6}(\boldsymbol{k}\times\boldsymbol{B})^2$$

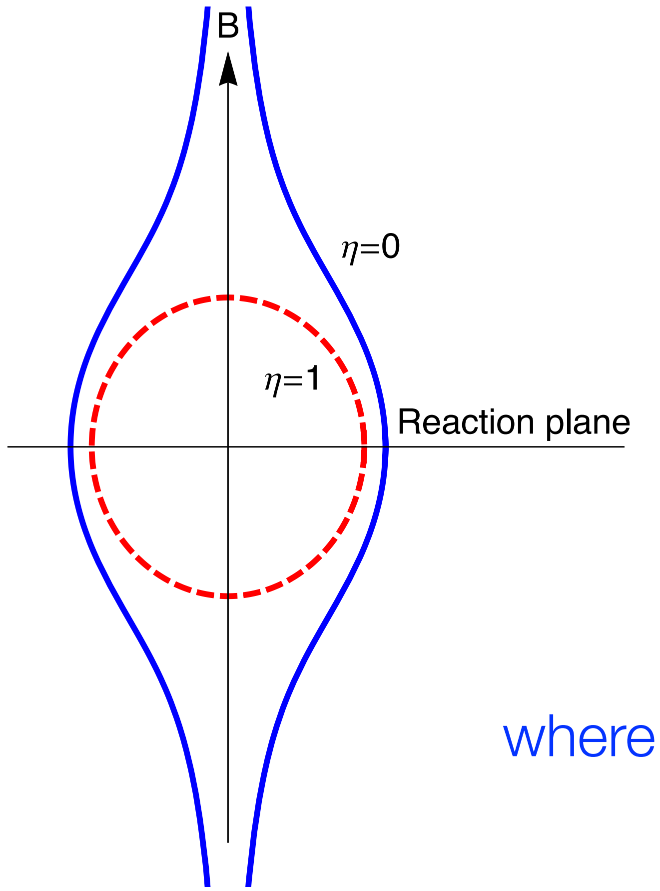
PHOTON DECAY RATE





Survival probability: $P=1\text{-}w\Delta t$, where $\Delta t \approx 5fm \Rightarrow P$(RHIC)$=98.5\%$, P(LHC)$=90\%$

AZIMUTHAL ASYMMETRY DUE TO PHOTON DECAY



Azimuthal asymmetry of the decay rate w:

$$w = \frac{1}{2} w_0 \left[1 - \sum_{k=1}^{\infty} \frac{\sqrt{\pi}\,\Gamma\left(-\frac{1}{6}\right)}{2^{2/3} B\left(\frac{5}{6}+k, \frac{5}{6}-k\right)} \cos(2k\varphi) \right]$$

Azimuthal asymmetry of the survival probability P:

$$P = \bar{P}\left(1 + \sum_{k=1}^{\infty} v_{2k}\cos(2\varphi k)\right) , \quad v_{2k} = -\frac{1-\bar{P}}{\bar{P}}\frac{2\,w_{2k}}{w_0}$$

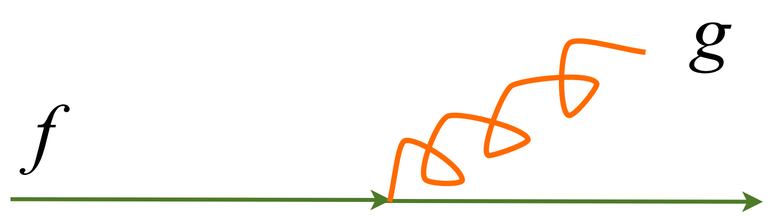
where azimuthal average is $\bar{P} = \langle 1 - w\Delta t\rangle_\varphi = 1 - w_0 \Delta t \approx 1$

$$v_2 = \Delta t \, \frac{2\,6^{2/3}\Gamma\left(\frac{2}{3}\right)}{7\pi} \sum_a \frac{\alpha_{\rm em}(eB)^{2/3} z_a^{8/3}}{(k_T)^{1/3}}$$

For k_T=1GeV and $\Delta t \approx$ 5fm ⇒ v_2(RHIC)=1%, v_2(LHC)=7%

Work in progress: $q \to q + \gamma \to q + \ell^+ + \ell^-$

SYNCHROTRON RADIATION OF GLUON BY FAST QUARKS



- General formulas for synchrotron radiation simplify if quark is **ultra-relativistic** $\varepsilon \gg m$ *before* and *after* gluon radiation.

  This is always true in week fields $eB \ll m^2$

  In strong fields $eB \gg m^2$ this approximation breaks down at the threshold $\omega \sim \varepsilon$, i.e. gluon carries away almost all quark energy ⇒ energy loss in this approximation must satisfy $\Delta\varepsilon \ll \varepsilon$

- Synchrotron radiation is **quasi-classical** if

  1.Spacing between Landau levels eB/ε is much smaller than ε =>$\varepsilon^2 \gg eB$

  2. Recoil due to gluon emission is small: $\omega \ll \varepsilon$ (i.e. far from the threshold)

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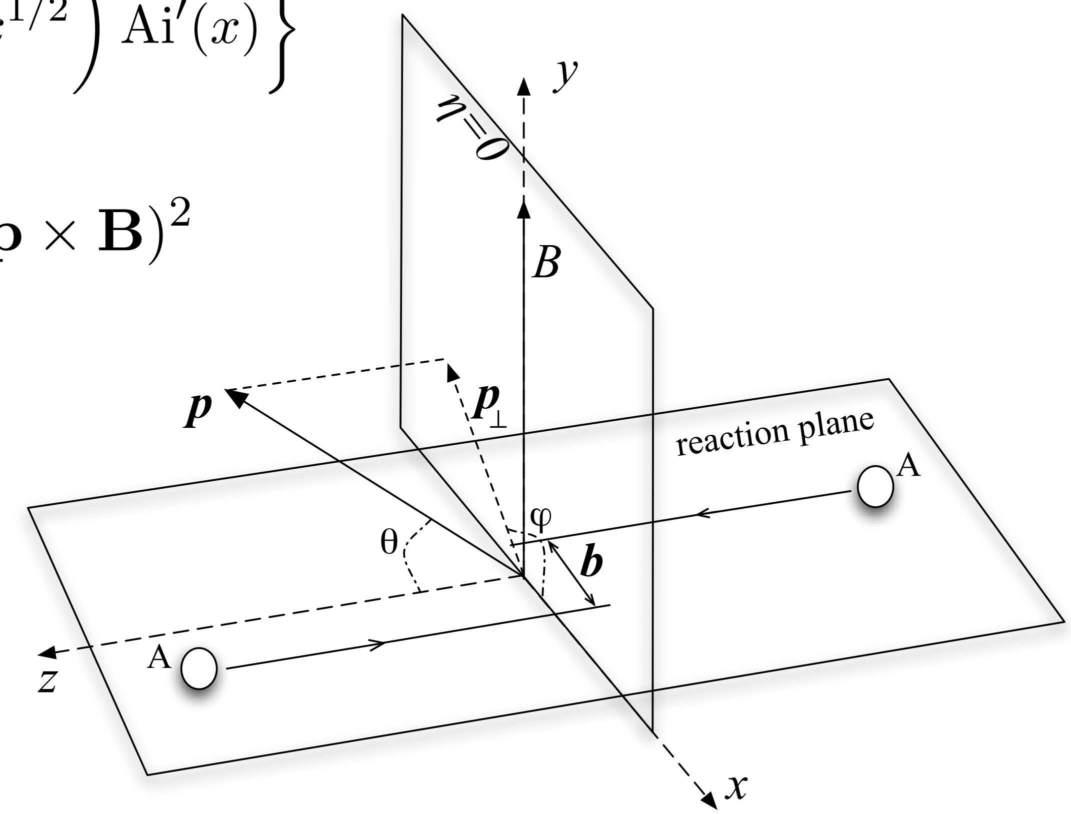
ULTRA-RELATIVISTIC + QUASI-CLASSICAL LIMIT

KT (2010,2012)

In the quasi-classical approximation $j \gg 1$, $k \gg 1$. Taking also the UR limit Laugerre polynomials reduce to Airy functions:

$$\frac{dI}{d\omega} = -\alpha_s C_F \frac{m^2\,\omega}{\varepsilon^2}\left\{\int_x^\infty \mathrm{Ai}(\xi)\,d\xi + \left(\frac{2}{x}+\frac{\omega}{\varepsilon}\,\chi\,x^{1/2}\right)\mathrm{Ai}'(x)\right\}$$



Invariant parameter $\chi^2 = -\frac{\alpha_{\mathrm{em}} Z_q^2 \hbar^3}{m^6}(F_{\mu\nu}p^\nu)^2 = \frac{\alpha_{\mathrm{em}} Z_q^2 \hbar^3}{m^6}(\mathbf{p}\times\mathbf{B})^2$

$$\chi^2 = \frac{\hbar^2(eB)^2}{m^6}\,p_\perp^2\,(\sinh^2\eta + \cos^2\varphi)$$

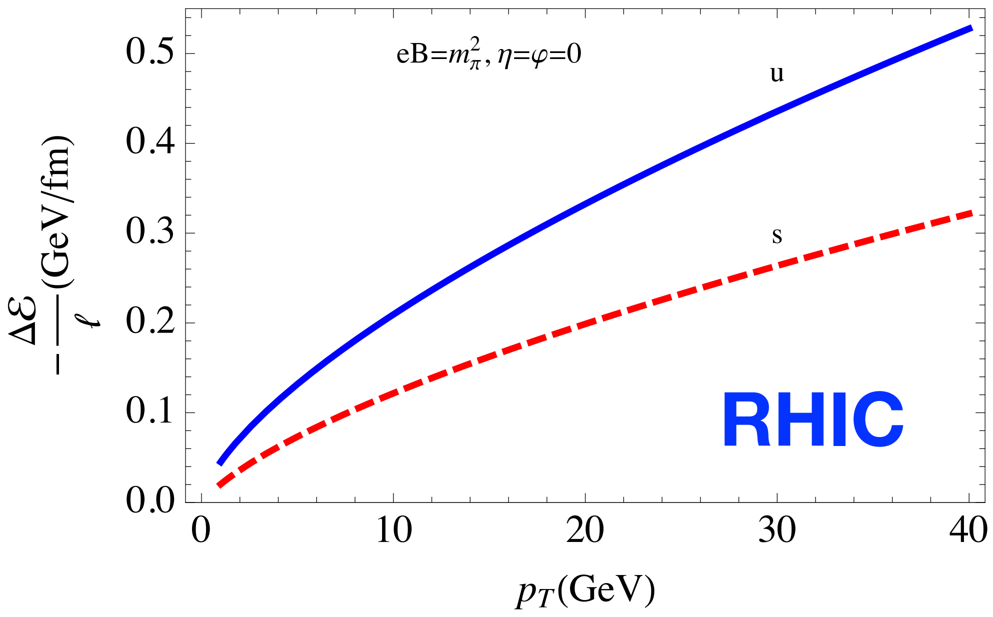
Energy loss due to synchrotron radiation

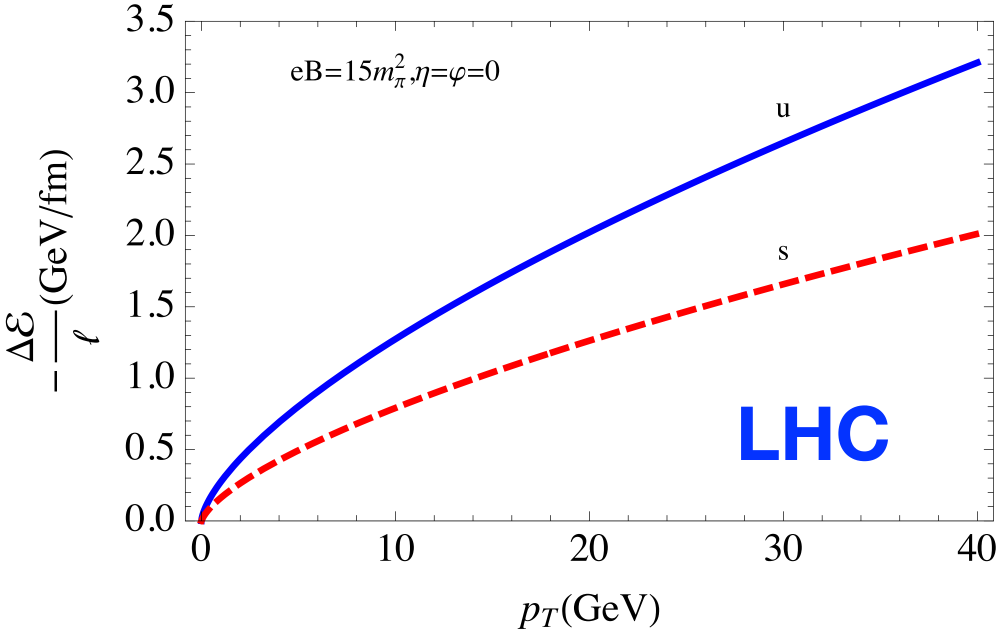
$$\frac{d\varepsilon}{dl} = -\int_0^\infty d\omega \frac{dI}{d\omega} = \alpha_s C_F \frac{m^2\chi^2}{2}\int_0^\infty \frac{4+5\chi\,x^{3/2}+4\chi^2\,x^3}{(1+\chi\,x^{3/2})^4}\,\mathrm{Ai}'(x)\,x\,dx$$

$$\frac{d\varepsilon}{dl} = -\frac{2\,\alpha_s\,\hbar\,C_F\,(Z_q eB)^2\varepsilon^2}{3m^4}\,, \quad \chi \ll 1\,, \qquad \text{Weak fields}$$

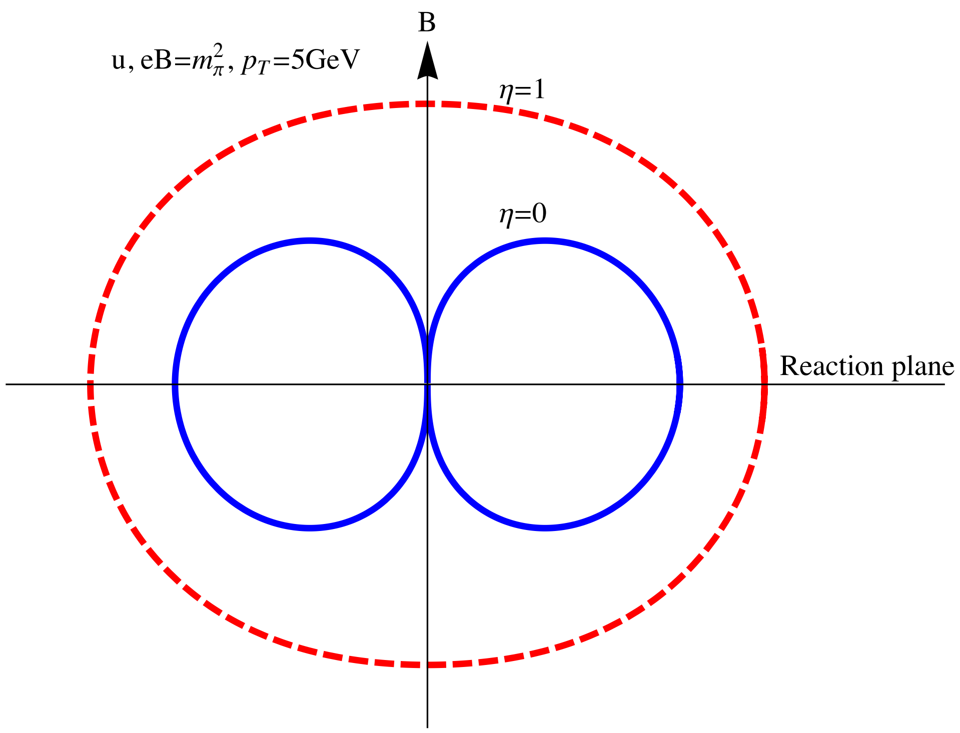
$$\frac{d\varepsilon}{dl} = -0.37\,\alpha_s\,\hbar^{-1/3}C_F\,(Z_q eB\,\varepsilon)^{2/3}\,, \quad \chi \gg 1 \qquad \text{Strong fields}$$

Energy loss in magnetic field





Azimuthal asymmetry:

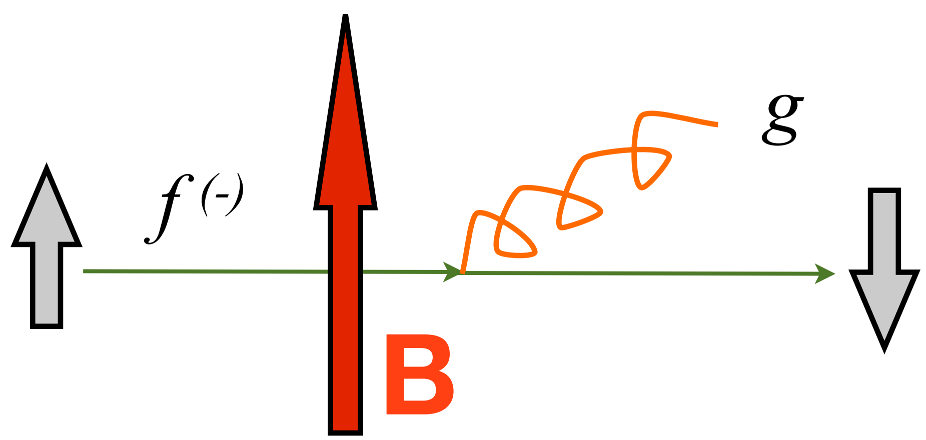


POLARIZATION OF LEPTONS AND LIGHT QUARKS

Spin-flip probability per unit time

$$w = \frac{5\sqrt{3}\alpha_s C_F}{16}\frac{\hbar^2}{m^2}\left(\frac{\varepsilon}{m}\right)^5\left(\frac{Z_q e\,|\boldsymbol{v}\times\boldsymbol{B}|}{\varepsilon}\right)^3\left(1-\frac{2}{9}(\boldsymbol{\zeta}\cdot\boldsymbol{v})^2-\frac{8\sqrt{3}}{15}\mathrm{sign}\,(e_q)\,(\boldsymbol{\zeta}\cdot\boldsymbol{b})\right)$$

Sokolov, Ternov (1964)



NR case: $H = -\boldsymbol{\mu}\cdot\boldsymbol{B} = -\left(\frac{geZ_q\hbar}{2m}\right)\boldsymbol{s}\cdot\boldsymbol{B}$

Spin-asymmetry: $A = \frac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow}$ n(↑)/n(↓) be the number of fermions with given momentum and spin direction parallel /anti-parallel to the field in a given event.

$$A = \frac{8}{5\sqrt{3}} = 92\%$$ **A very strong polarization of quarks and leptons!**

BEYOND THE QUASI-CLASSICAL APPROXIMATION

- **In strong fields** $B \gg e/m^2$ **near the threshold** $\omega = \varepsilon$:

  Quark looses almost all its energy due to synchrotron radiation and falls on one of the lowest Landau levels.

  This brakes both the quasi-classical and ultra-relativistic approximation.

- Transition to the ground state occurs with probability

$$w_{n0} = \frac{\alpha_s}{2}\frac{m^2}{\varepsilon}\frac{B}{B_c}e^{-B_c/B}$$

Sokolov, Borisov, Zhukovskii (1975)

where $B_c = e/m^2$

- In heavy-ion collisions B is stronger than B_c, so such transitions must be taken into account.

☞ Future project.

CONCLUSIONS II

- Synchrotron radiation of gluons contributes to the quark energy loss and is azimuthally asymmetric.

- Polarization of leptons escaping the QGP is a sensitive probe of magnetic field.

SUMMARY

- Magnetic field in relativistic heavy-ion collisions is super-critical and slowly varying in time.

- Synchrotron photons maybe a significant part of the total photon spectrum at low p_T.

- Fast quarks and leptons loose a lot of energy and get polarized in magnetic field. **`Can the polarization be measured?`**