Suppression of the Collins mechanism for SSA in $p^{\uparrow}p \to \pi X$

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(hep-ph/0408356 + work in progress)

Outline

- Generalized pQCD approach with spin and $\boldsymbol{k}_\perp$-effects in distribution and fragmentation functions and elementary dynamics;
- SSA in $pp \Rightarrow \pi X$: theory - helicity formalism;
- SSA in $pp \Rightarrow \pi X$: phenomenology of the Collins effect;
- Conclusions and outlook

Unpolarized cross sections: $k_\perp$-factorization *ansatz*

- The collinear pQCD factorization formula for $AB \to CX$ reads

$$d\sigma^{AB\to CX} = \sum_{a,b,c,d} f_{a/A}(x_a) \otimes f_{b/B}(x_b) \otimes d\hat{\sigma}^{ab\to cd}(\hat{s},\hat{t}) \otimes D_{C/c}(z)$$

$$\hat{s} = s x_a x_b, \ \hat{t} = t x_a/z, \ \hat{u} = u x_b/z$$

- Including transverse momenta we have:

$$\Rightarrow \sum_{a,b,c,d} \hat{f}_{a/A}(x_a, \boldsymbol{k}_{\perp a}) \otimes \hat{f}_{b/B}(x_b, \boldsymbol{k}_{\perp b}) \otimes d\hat{\sigma}^{ab\to cd}(\hat{s},\hat{t}) \otimes \hat{D}_{C/c}(z, \boldsymbol{k}_{\perp C})$$

$$\hat{s} \to \hat{s}\,\phi_s(\boldsymbol{k}_{\perp a}, \boldsymbol{k}_{\perp b}), \ \hat{t} \to \hat{t}\,\phi_t(\boldsymbol{k}_{\perp a}, \boldsymbol{k}_{\perp C}), \ \hat{u} \to \hat{u}\,\phi_u(\boldsymbol{k}_{\perp b}, \boldsymbol{k}_{\perp C})$$

+ Jacobian in FF + Cahn effect + Gaussian $k_\perp$-depend.; + factoriz. ...

see *U. D., F. Murgia*: hep-ph/0408092, PRD in press

$\boldsymbol{k}_\perp$ - effects

1. $\boldsymbol{k}_\perp$ alters $\langle x \rangle$: small shift but important at large x where PDF are varying rapidly.
   $\rightarrow$ *enhanced higher-twist* effect (up to a factor of 10 in a cross section);

2. the partonic scattering angle in the pp c.m. frame might be much smaller than the hadronic production angle, thus enhancing the moderately large p_T inclusive production of particles;

3. spin and $\boldsymbol{k}_\perp$-effects generated by soft mechanisms: SSA ($pp \rightarrow hX$)
   (a) Sivers PDF: $\boldsymbol{S} \cdot (\boldsymbol{P} \times \boldsymbol{k}_{\perp q})$
   (b) Collins FF: $\boldsymbol{s}_q \cdot (\boldsymbol{p}_q \times \boldsymbol{k}_{\perp h})$
   (c) Boer-Mulders PDF: $\boldsymbol{s}_q \cdot (\boldsymbol{P} \times \boldsymbol{k}_{\perp q})$
   (d) polarizing FF: $\boldsymbol{S}_h \cdot (\boldsymbol{p}_q \times \boldsymbol{k}_{\perp h})$

Polarized cross sections: Helicity formalism

◇ **Helicity density matrices**, in the $\boldsymbol{k}_\perp$-factorization scheme, to describe parton spin states for a polarized cross section:

$$d\sigma^{A,S_A+B,S_B\to C+X} = \sum_{a,b,c,d,\{\lambda\}} \rho^{a/A,S_A}_{\lambda_a,\lambda'_a}\ \hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a}) \otimes \rho^{b/B,S_B}_{\lambda_b,\lambda'_b}\ \hat{f}_{b/B,S_B}(x_b,\boldsymbol{k}_{\perp b})$$

$$\otimes\ \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\ \hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b} \otimes \hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C})$$

- $\rho^{a/A,S_A}_{\lambda_a,\lambda'_a}$: helicity density matrix of parton a inside hadron A with polarization S_A
- $\frac{d\hat{\sigma}^{ab\to cd}}{d\hat{t}} \simeq \sum_{\lambda_a,\lambda_b,\lambda_c,\lambda_d} |\hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}|^2$ (scattering amplitudes)
- $\hat{D}^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c} \equiv \sum\!\!\!\!\!\!\int_{X,\lambda_X} \hat{\mathcal{D}}_{\lambda_C,\lambda_X;\lambda_c}\ \hat{\mathcal{D}}^*_{\lambda'_C,\lambda_X;\lambda'_c}$

  i.e. the product of *fragmentation amplitudes* for the $c \to C + X$ process

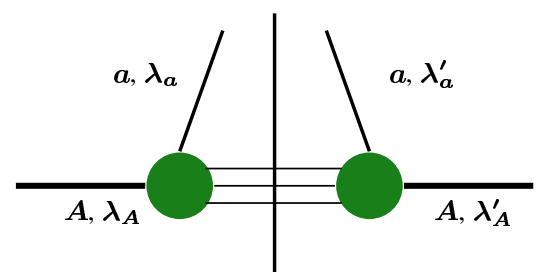
Helicity density matrices: PDF sector

◇ Partonic distribution function at L.O. (and twist two) can be interpreted as the inclusive cross section for the process $A \to a + X$.

Helicity density matrix of parton $a \leftrightarrow$ to the helicity density matrix of hadron A.

$$\rho^{a/A,S_A}_{\lambda_a,\lambda'_a} \qquad \hat{f}_{a/A,S_A}(x_a, \boldsymbol{k}_{\perp a}) = \sum_{\lambda_A,\lambda'_A} \rho^{A,S_A}_{\lambda_A,\lambda'_A} \hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}$$

$$\hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A} \equiv \sum\!\!\!\!\!\!\int_{X_A,\lambda_X} \hat{\mathcal{F}}_{\lambda_a,\lambda_X;\lambda_A} \hat{\mathcal{F}}^*_{\lambda'_a,\lambda_X;\lambda'_A}$$



Each helicity density matrix describes the spin orientation of a particle in its own helicity frame.

For a spin-1/2 particle, $\mathrm{Tr}\,(\sigma_i \rho) = P_i$ is the i-component of the spin-polarization vector $\boldsymbol{P}$ in the helicity rest frame of the particle.

The distribution function of parton a inside the polarized (S_A) hadron A is given by

$$\hat{f}_{a/A,S_A}(x_a, \boldsymbol{k}_{\perp a}) = \sum_{\lambda_a, \lambda_A, \lambda'_A} \rho^{A,S_A}_{\lambda_A,\lambda'_A} \hat{F}^{\lambda_a,\lambda_a}_{\lambda_A,\lambda'_A}$$

- We have 16 $\hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}$ which by Parity constraints reduce to 6 independent products.

4 of them are *Real functions*

$\Rightarrow$ 8 independent spin-$\boldsymbol{k}_\perp$ PDF.

For instance:

$$\rho^{a/A,S_A}_{\pm,\pm} \hat{f}_{a/A,S_A}(x_a, \boldsymbol{k}_{\perp a}) \equiv \hat{f}_{a_\pm/A,S_A}(x_a, \boldsymbol{k}_{\perp a})$$

Partonic interpretation: a few examples

Spin and $k_\perp$-dependent distribution and fragmentation functions of leading twist:
[vs. notation of P.J. Mulders, R.D. Tangerman *Nucl. Phys.* **B461** (1996)]

- $\hat{f}_{q/p} = \hat{F}^{+,+}_{+,+} + \hat{F}^{-,-}_{-,-}$: unpolarized PDF
- $\Delta\hat{f}_{q/p} = \hat{F}^{+,+}_{+,+} - \hat{F}^{-,-}_{-,-}$: long. polariz. PDF
- $\hat{D}_{\pi/q} = \hat{D}^{\pi}_{++}$: unpolarized FF
- $\hat{f}_{q/p^\uparrow} - \hat{f}_{q/p^\downarrow} \equiv \Delta^N f_{q/p^\uparrow} = 4\mathrm{Im}\hat{F}^{+,+}_{+,-} \propto (k_\perp/M) f_{1T}^\perp$: Sivers funct.
- $\hat{f}_{q^\uparrow/p} - \hat{f}_{q^\downarrow/p} \equiv \Delta^N f_{q^\uparrow/p} = 2\mathrm{Im}\hat{F}^{+,-}_{+,+} \propto (k_\perp/M) h_1^\perp$: Boer-Mulders funct.
- $\hat{D}_{h/q^\uparrow} - \hat{D}_{h/q^\downarrow} \equiv \Delta^N D_{h/q^\uparrow} = 2\mathrm{Im}D^h_{+-} \propto (k_\perp/M) H_1^\perp$: Collins funct.
- $\hat{f}_{q_+/p^\uparrow} - \hat{f}_{q_-/p^\uparrow} = 2\mathrm{Re}\hat{F}^{+,+}_{+,-} \propto (k_\perp/M) g_{1T}$
- $\hat{f}_{q^\downarrow/p_+} - \hat{f}_{q^\uparrow/p_+} = 2\mathrm{Re}\hat{F}^{+,+}_{+,-} \propto (k_\perp/M) h_{1L}^\perp$
- $\hat{f}_{q^\uparrow/p^\uparrow} - \hat{f}_{q^\downarrow/p^\uparrow} \Rightarrow \hat{F}^{+,-}_{+,-}; \hat{F}^{-,+}_{+,-} \propto h_1; (k_\perp/M)^2 h_{1T}^\perp$
- and gluons...

see also hep-ph/0410050, ("the Trento conventions")
A. Bacchetta, U.D., M. Diehl, A. Miller

The master formula

$$d\sigma^{A,S_A+B,S_B\to C+X} = \sum_{a,b,c,d,\{\lambda\}} \rho^{A,S_A}_{\lambda_A,\lambda'_A}\, \hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}(\boldsymbol{k}_{\perp a}) \otimes \rho^{B,S_B}_{\lambda_B,\lambda'_B}\, \hat{F}^{\lambda_b,\lambda'_b}_{\lambda_B,\lambda'_B}(\boldsymbol{k}_{\perp b}) \otimes \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\, \hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b} \otimes \hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}$$

On summing over $\{\lambda' s\}$ we get all combinations of spin and $\boldsymbol{k}_\perp$- mechanisms in Single and Double Spin Asymmetries for unpolarized particle production.

Notice: even for $\boldsymbol{S}_A = \boldsymbol{S}_B = 0 \to \rho^{A,S_A}_{\lambda_A,\lambda'_A} = (1/2)\delta_{\lambda_A,\lambda'_A}$ (same for B) the unpolarized cross section would develop Boer-Mulders $\otimes$ Collins effects ($h_1^\perp \otimes H_1^\perp$): negligible (checked numerically!).

[M. Anselmino, M. Boglione, U.D., E. Leader, S. Melis, F. Murgia, in preparation]

Single Spin Asymmetries: $A^\uparrow B \to \pi X$, $A_N = (d\sigma^\uparrow - d\sigma^\downarrow)/(d\sigma^\uparrow + d\sigma^\downarrow)$

- Configuration ($AB \equiv pp$):
  AB center of mass frame - A along $+Z$-axis, π in the XZ plane with $\boldsymbol{p}_T$ parallel to $+X$-axis. $S_A = \uparrow$ or $\downarrow$ along Y-axis, $S_B = 0$.

Quantities to be considered (convoluted):

- Numerator of A_N

$$\Sigma(\uparrow,0) - \Sigma(\downarrow,0) = \sum_{\{\lambda\}} \frac{(-i)}{2} \left[\hat{F}^{\lambda_a,\lambda'_a}_{+,-}(k_{\perp a}) - \hat{F}^{\lambda_a,\lambda'_a}_{-,+}(k_{\perp a}) \right] \times \hat{F}^{\lambda_b,\lambda'_b}_{\lambda_B,\lambda_B} \; \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b} \, \hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b} \; \hat{D}^\pi_{\lambda_c,\lambda'_c}$$

- Denominator of A_N (twice the unpolarized cross section)

$$\Sigma(\uparrow,0) + \Sigma(\downarrow,0) = \sum_{\{\lambda\}} \frac{1}{2} \left[\hat{F}^{\lambda_a,\lambda'_a}_{+,+}(k_{\perp a}) + \hat{F}^{\lambda_a,\lambda'_a}_{-,-}(k_{\perp a}) \right] \times \hat{F}^{\lambda_b,\lambda'_b}_{\lambda_B,\lambda_B} \; \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b} \, \hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b} \; \hat{D}^\pi_{\lambda_c,\lambda'_c}$$

Helicity scattering amplitudes with full $\boldsymbol{k}_\perp$

• $\boldsymbol{k}_\perp$'s imply: non-planar $ab \to cd$ scattering; different $\perp$ directions.

Parton helicity amplitudes in pp c.o.m frame in terms of hel. amplitudes in partonic c.o.m. frame.

1) Boost along $\boldsymbol{p}_a + \boldsymbol{p}_b$: $S \to S'$

2) Rotation to put $\boldsymbol{p}'_a$ along Z axis: $S' \to S''$

• $\hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b} = \hat{M}^0_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}(\hat{s},\hat{t})\, \mathrm{e}^{i\sum_m \lambda_m \xi_m}\, \mathrm{e}^{i(\lambda_a-\lambda_b)\phi''_c}$

Notice: various combinations of "T-odd" effects in (un)polarized cross sections but by including proper phases + numerical (VEGAS) integration:

$$d\sigma^{\mathrm{unp}} \simeq f_{a/p} \otimes f_{b/p} \otimes d\hat{\sigma} \otimes D_{\pi/c}$$

$$d\sigma^{\uparrow} - d\sigma^{\downarrow} \simeq \Delta^N f_{a/p^\uparrow} \otimes f_{b/p} \otimes d\hat{\sigma} \otimes D_{\pi/c} \qquad \text{"Sivers effect"}$$

$$+\ h_{1a} \otimes f_{b/p} \otimes d\Delta\hat{\sigma} \otimes \Delta^N D_{\pi/c^\uparrow} \qquad \text{"Collins effect"}$$

$\Rightarrow$ **Let's focus on Collins effect**

On summing over $\{\lambda\}$ [$\lambda_a \neq \lambda'_a$ and $\lambda_b = \lambda'_b$] we get ($b = q, \bar{q}, g$)

$$[\Sigma(\uparrow,0) - \Sigma(\downarrow,0)]_{qb} = \Big\{ F^{+-}_{+-}(x_a, k_{\perp a}) \cos[\phi_a + \phi''_c - \xi_a - \tilde{\xi}_a + \xi_c + \tilde{\xi}_c + \phi^H_\pi]$$

$$- F^{+-}_{-+}(x_a, k_{\perp a}) \cos[\phi_a - \phi''_c + \xi_a + \tilde{\xi}_a - \xi_c - \tilde{\xi}_c - \phi^H_\pi] \Big\}$$

$$\times \ \hat{f}_{b/B}(x_b, k_{\perp b}) \left[\hat{M}^0_1 \hat{M}^0_2(x_a, x_b, z; \boldsymbol{k}_{\perp a}, \boldsymbol{k}_{\perp b}, \boldsymbol{k}_{\perp \pi})\right]_{qb}$$

$$\times \ [2\,\mathrm{Im} D^\pi_{+-}(z, k_{\perp \pi})]$$

$$(\hat{M}^0_1)_{qb} \equiv \hat{M}^0_{+,+;+,+} = \hat{M}^0_{-,-;-,-} \qquad (\hat{M}^0_2)_{qb} \equiv \hat{M}^0_{-,+;-,+} = \hat{M}^0_{+,-;+,-}$$

The product $\hat{M}^0_1 \hat{M}^0_2$, is, for a collinear collision with scattering plane $\equiv XZ$, related to the spin transfer cross section:

$$\frac{1}{16\pi \hat{s}^2} \left[\hat{M}^0_1 \hat{M}^0_2\right]_{qb} = \frac{d\hat{\sigma}^{q^\uparrow\, b \to q^\uparrow\, b}}{d\hat{t}} - \frac{d\hat{\sigma}^{q^\uparrow\, b \to q^\downarrow\, b}}{d\hat{t}}$$

Maximization of the Collins mechanism in $p^\uparrow p \to \pi X$

In the notations of P.J. Mulders, R.D. Tangerman *Nucl. Phys.* **B461** (1996):

$$
\begin{aligned}
F_{+-}^{+-}(x,k_\perp) &= h_1(x,k_\perp) = h_{1T}(x,k_\perp) + \frac{k_\perp^2}{2M_p^2}\, h_{1T}^\perp(x,k_\perp) \\
F_{-+}^{+-}(x,k_\perp) &= \frac{k_\perp^2}{2M_p^2}\, h_{1T}^\perp(x,k_\perp) \\
2\,\mathrm{Im} D_{+-}^{\pi}(z,k_\perp) &= \Delta^N D_{\pi/q^\uparrow}(z,k_\perp) = \frac{2k_\perp}{zM_\pi}\, H_1^{\perp q}(z,k_\perp)
\end{aligned}
$$

The following positivity bounds hold [A. Bacchetta, *et al.* *Phys. Rev. Lett.* **85** (2000)]:

$$
\begin{aligned}
|h_1(x,k_\perp)| &\leq \frac{1}{2}\left[q(x,k_\perp) + \Delta q(x,k_\perp)\right] = q_+(x,k_\perp) \\
\frac{k_\perp^2}{2M_p^2}\, |h_{1T}^\perp(x,k_\perp)| &\leq \frac{1}{2}\left[q(x,k_\perp) - \Delta q(x,k_\perp)\right] = q_-(x,k_\perp) \\
|\Delta^N D_{\pi/q^\uparrow}(z,k_\perp)| &\leq 2D_{\pi/q}(z,k_\perp)
\end{aligned}
$$

Let's consider $A_N(\pi^+)$: u, d active flavor for h_1 and $h_{1T}^{\perp}$:

$$h_1^u(x,k_\perp) = u_+(x,k_\perp) \qquad h_1^d(x,k_\perp) = -d_+(x,k_\perp)$$

$$\frac{k_\perp^2}{2M_p^2}\, h_{1T}^{\perp u}(x,k_\perp) = -u_-(x,k_\perp) \qquad \frac{k_\perp^2}{2M_p^2}\, h_{1T}^{\perp d}(x,k_\perp) = +d_+(x,k_\perp)$$

being the product of elementary amplitudes $\hat{M}_1^0 \hat{M}_2^0 < 0$, to get $A_N(\pi^+) > 0$

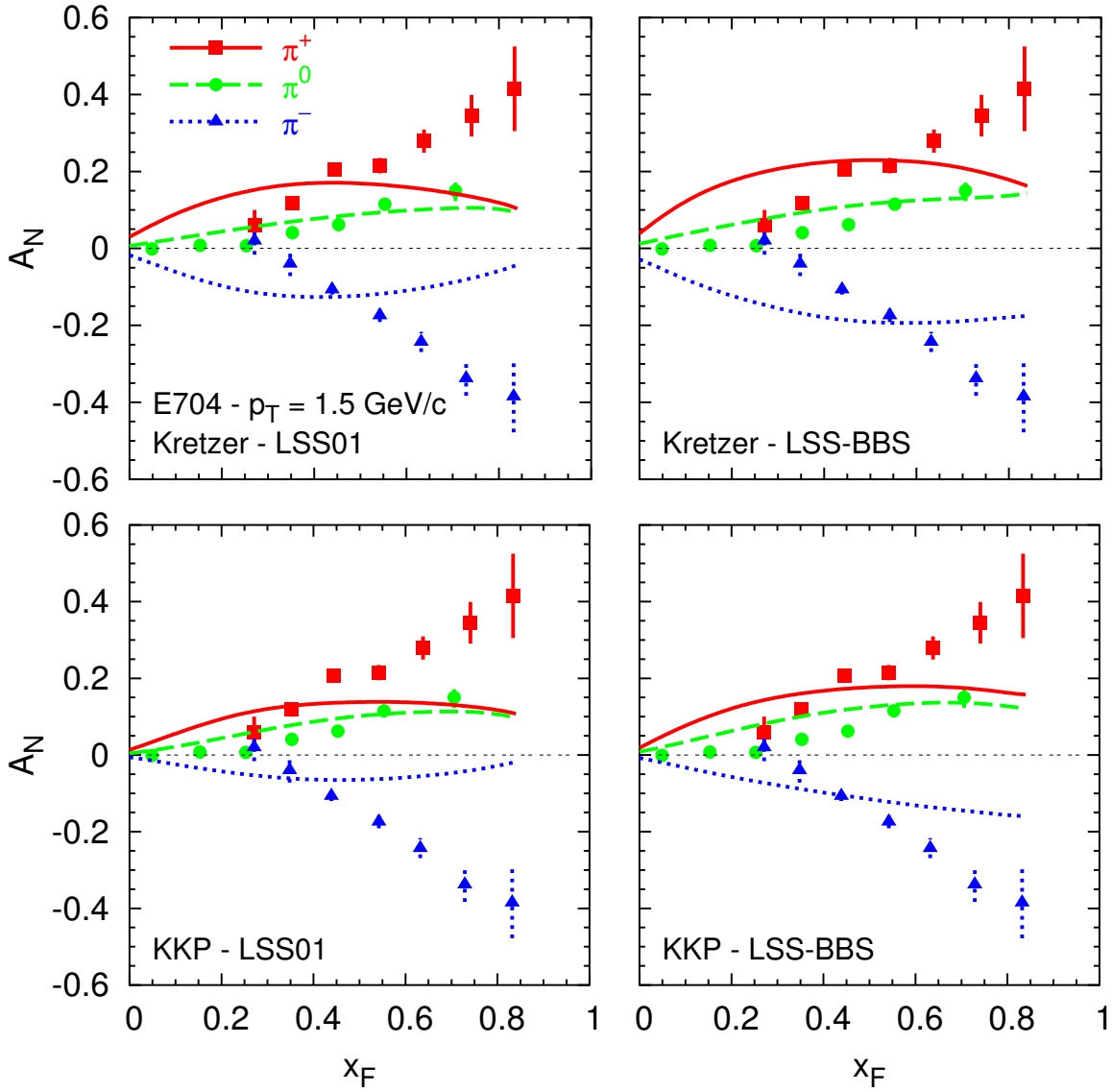
$$\Rightarrow \Delta^N D_{\pi^+/u^\uparrow}(z,k_\perp) = -2D_{\pi^+/u}(z,k_\perp)$$

Contribution (maximized) of the sub-leading channel $down \to down$ (neglected in former studies); since $h_1^d < 0$, to add in sign, we use

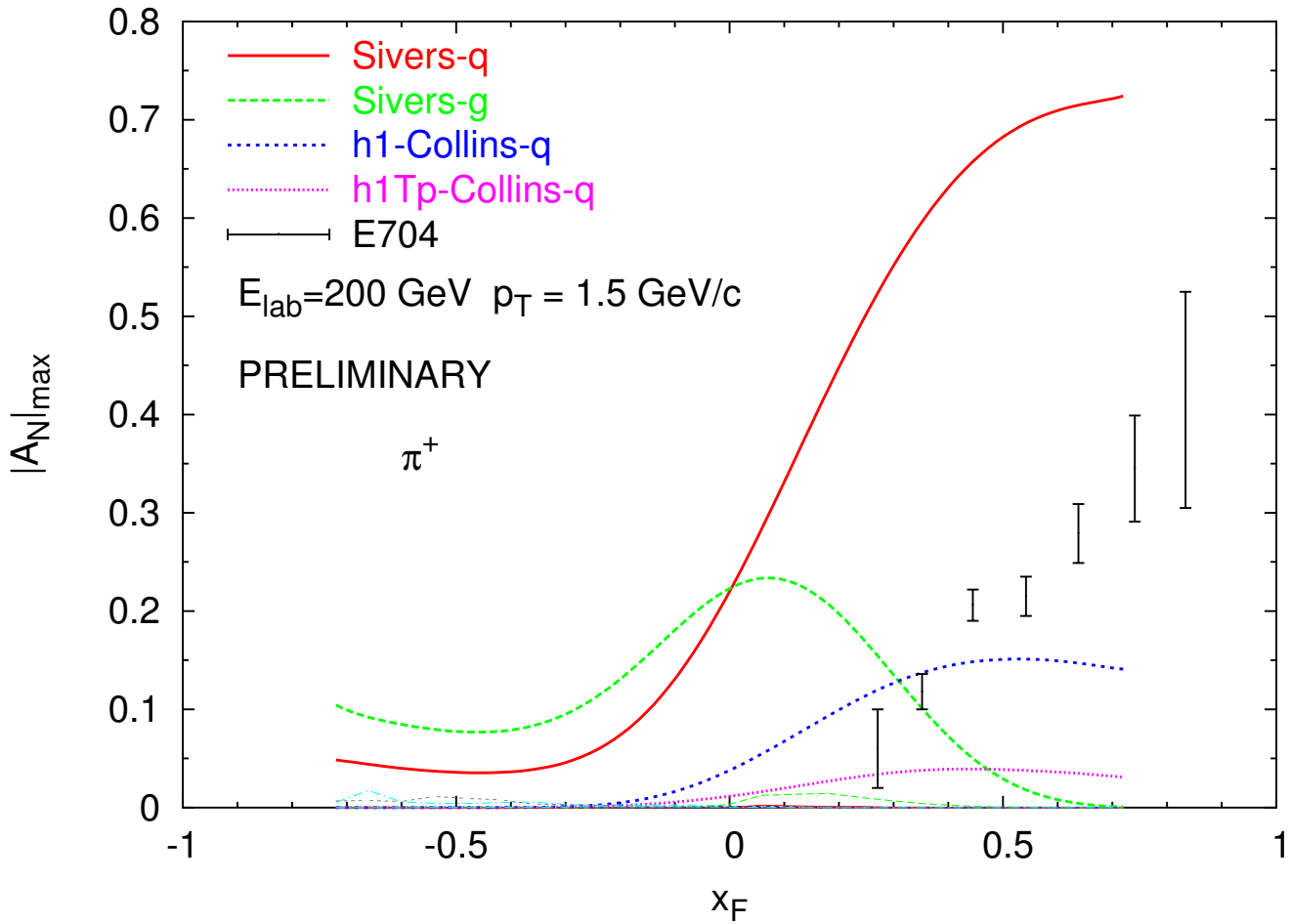
$$\Delta^N D_{\pi^+/d^\uparrow}(z,k_\perp) = +2D_{\pi^+/d}(z,k_\perp)$$

A_N for π^-'s is maximized in size (by isospin invariance).

- Pol PDF: LSS01 and LSS-BBS ($\Delta d > 0$ at large $x \to$ more relaxed bound on h_1^d)
- FF: Kretzer (sea $\simeq (1-z)$ valence) and KKP (sea suppressed)



Maximized estimates of $A_N(\pi)$ for $p^\uparrow p \to \pi X$ vs. x_F, at $\langle p_T \rangle = 1.5$ GeV/c and $\sqrt{s} = 20$ GeV. Collins mechanism full saturated. Data are from [E704] D.L. Adams *et al.*, *Phys. Lett.* **B261** (1991) and A. Bravar *et al.*, *Phys. Rev. Lett.* **77** (1996).



(Over)Maximized estimates of A_N: all possible contributions.

Conclusions and outlook

- A generalized pQCD approach to SSA and unpol. cross sections:
  LO–pQCD + spin and new $\boldsymbol{k}_\perp$-dependent PDF and FF;
- $A_N(p^\uparrow p \to \pi X)$: Complete $\boldsymbol{k}_\perp$-helicity formalism and phenomenology.
  True detailed microscopic dynamics (proper phases) $\Rightarrow$
  Sivers effect results confirmed;
  Collins effect suppressed (against former claims!);
- $p^\uparrow p \to \pi$ jet X: to extract Collins effect (less severe phases cancellations);
- $AB \to CX$ $(C = \pi, \Lambda, \rho)$: Complete $\boldsymbol{k}_\perp$-helicity formalism and phenomenology for A_{LL}, A_{TT}, A_{LT}, P (in progress). Role of "T-odd" effects in $A_{LL}(\pi)$;
- Azimuthal spin asymmetries in SIDIS: role of Collins & Sivers effect
  (+ Kotzinian & Prokudin: detailed $\boldsymbol{k}_\perp$ approach in progress);
- $p^\uparrow p-$data at large energies and at larger p_T, and $ep^\uparrow-$data with transversely polarized target are very welcome.