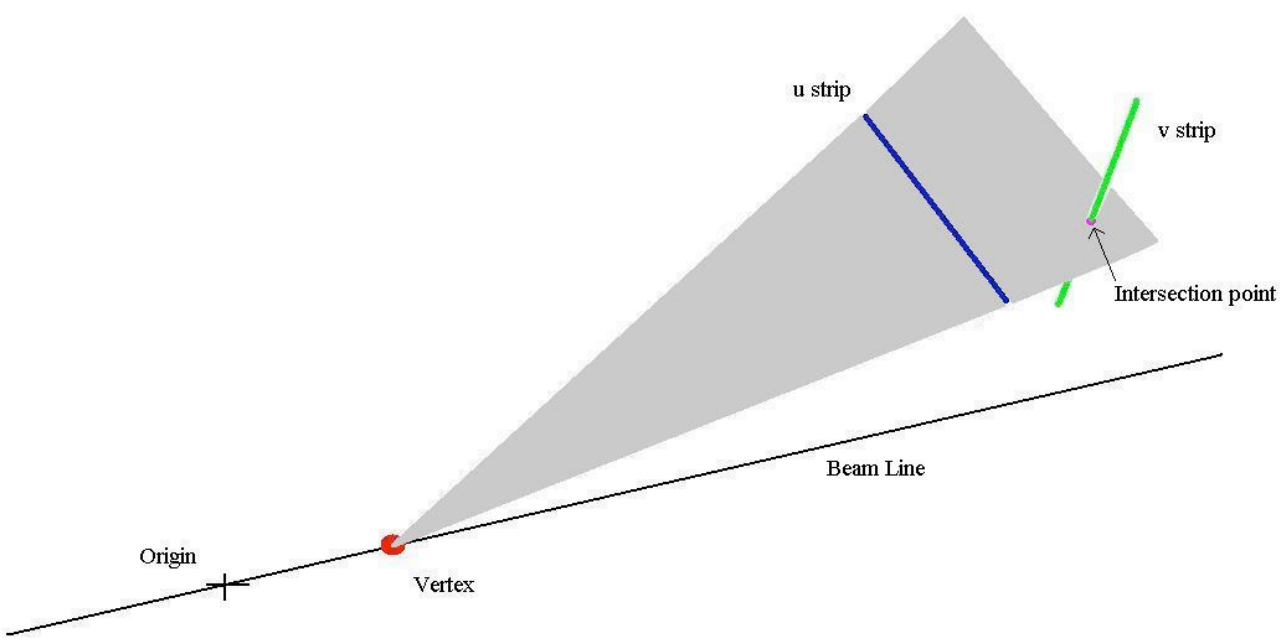
Noah's method of finding the true intersection point

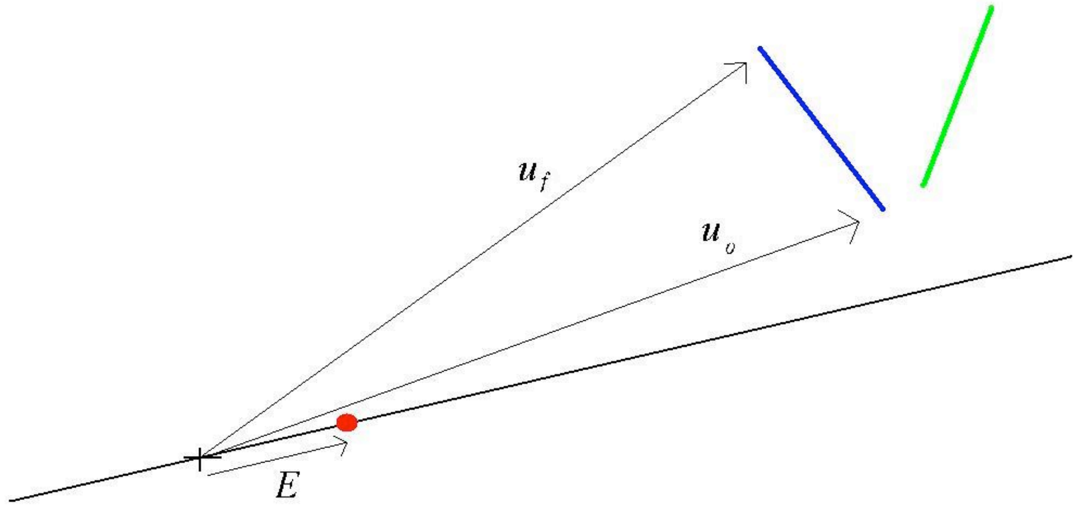
Of a particle passing through the SMD

7/6/2006

In this process I take a point in space (vertex) and two skew lines (u and v strips) and find the unique line that intersects all three. I do this by first establishing a plane by using 3 points (the vertex, the beginning of the u strip, and the end of the u strip). I form a vector that points from the vertex to the beginning of the u strip and a vector that points from the vertex to the end of the u strip. These two lines establish a unique plane. I cross these two vectors, giving me the vector normal to the plane. Using this normal vector and any point in the plane, I used the vertex, I can establish the equation of the plane in space. Next, v strip must intersect the plane at some point. I establish an equation that describes the v strip as a vector starting at a point (the beginning of the v strip). I then find the point along the v strip that satisfies the plane equation. This point is the end point of the track of the particle in question.



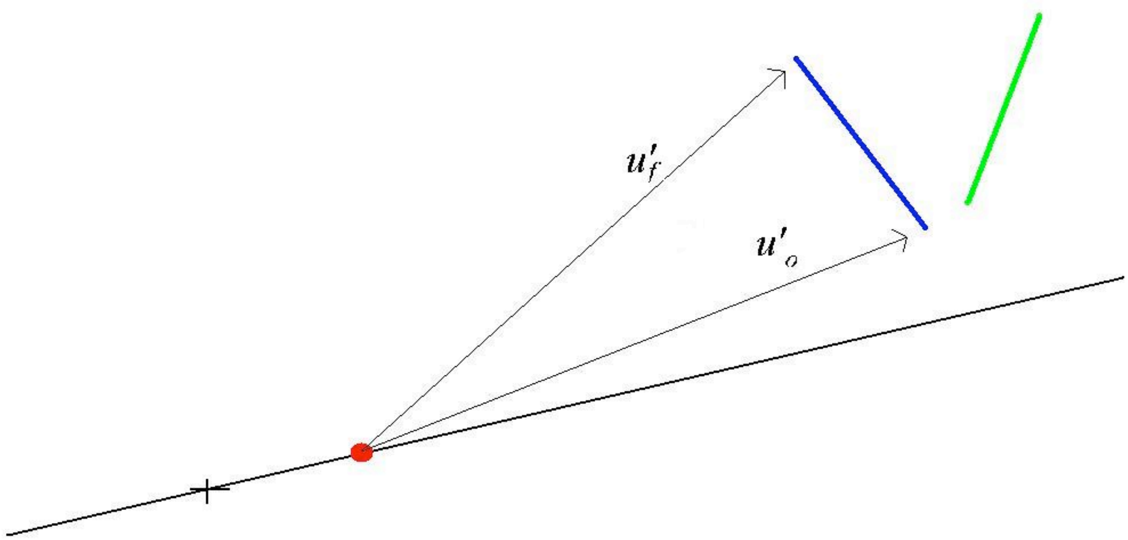
I am given vectors $\vec{u}_o$ and $\vec{u}_f$, which point from the origin to the beginning and end of a given strip, respectively, as shown. The vector $\vec{E}$ points from the origin to the vertex.



These vectors are three dimensional position vectors with components:

$$\vec{u}_o = u_{ox}\hat{i} + u_{oy}\hat{j} + u_{oz}\hat{k}$$
$$\vec{u}_f = u_{fx}\hat{i} + u_{fy}\hat{j} + u_{fz}\hat{k}$$
$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

I now need vectors from the vertex to the beginning of the u strip and to the end of the u strip.



I can calculate a vector $\vec{u}'_o$ by subtracting the vector from the origin to the vertex from the vector from the origin to the beginning of the u strip

$$\vec{u}'_o = \vec{u}_o - \vec{E}$$

This math can be done by components in each dimension

$$\vec{u}_o' = (u_{ox} - E_x)\hat{i} + (u_{oy} - E_y)\hat{j} + (u_{oz} - E_z)\hat{k}$$

This now establishes a new vector $\vec{u}_o'$

$$\vec{u}_o' = u_{ox}'\hat{i} + u_{oy}'\hat{j} + u_{oz}'\hat{k}$$

Where

$$u_{ox}' = u_{ox} - E_x$$
$$u_{oy}' = u_{oy} - E_y$$
$$u_{oz}' = u_{oz} - E_z$$

The same process can be done to find a vector $\vec{u}_f'$

$$\vec{u}_f' = \vec{u}_f - \vec{E}$$
$$\vec{u}_f' = (u_{fx} - E_x)\hat{i} + (u_{fy} - E_y)\hat{j} + (u_{fz} - E_z)\hat{k}$$
$$\vec{u}_f' = u_{fx}'\hat{i} + u_{fy}'\hat{j} + u_{fz}'\hat{k}$$

Where

$$u_{fx}' = u_{fx} - E_x$$
$$u_{fy}' = u_{fy} - E_y$$
$$u_{fz}' = u_{fz} - E_z$$

I now establish the normal vector $\vec{N}$ from the cross product of $\vec{u}_o'$ and $\vec{u}_f'$.

$$\vec{N} = \vec{u}_o' \times \vec{u}_f'$$
$$\vec{N} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_{ox}' & u_{oy}' & u_{oz}' \\ u_{fx}' & u_{fy}' & u_{fz}' \end{vmatrix} = (u_{oy}' \cdot u_{fz}' - u_{oz}' \cdot u_{fy}')\hat{i} + (u_{oz}' \cdot u_{fx}' - u_{ox}' \cdot u_{fz}')\hat{j} + (u_{ox}' \cdot u_{fy}' - u_{oy}' \cdot u_{fx}')\hat{k}$$

Which establishes $\vec{N}$

$$\vec{N} = N_x\hat{i} + N_y\hat{j} + N_z k$$

Where

$$N_x = u'_{oy} \cdot u'_{fz} - u'_{oz} \cdot u'_{fy}$$
$$N_y = u'_{oz} \cdot u'_{fx} - u'_{ox} \cdot u'_{fz}$$
$$N_z = u'_{ox} \cdot u'_{fy} - u'_{oy} \cdot u'_{fx}$$

The general equation for a plane is

(1) $$A(x - x_o) + B(y - y_o) + C(z - z_o) = 0$$

Where A, B, and C are coefficients and x_o, y_o, and z_o are the components of a point in space.

The coefficients of $\vec{N}$ become A, B, and C.

$$A = N_x$$
$$B = N_y$$
$$C = N_z$$

The point in space I used was the vertex, described by $\vec{E}$

$$x_o = E_x$$
$$y_o = E_y$$
$$z_o = E_z$$

Substitute the coefficients of $\vec{N}$ and the vertex point into eq. 1

$$N_x(x - E_x) + N_y(y - E_y) + N_z(z - E_z) = 0$$
$$N_x \cdot x - N_x \cdot E_x + N_y \cdot y - N_y \cdot E_y + N_z \cdot z - N_z \cdot E_z = 0$$
$$N_x \cdot x + N_y \cdot y + N_z \cdot z = N_x \cdot E_x + N_y \cdot E_y + N_z \cdot E_z$$

The right hand side of this equation is a constant, which I now call D

$$D = N_x \cdot E_x + N_y \cdot E_y + N_z \cdot E_z$$

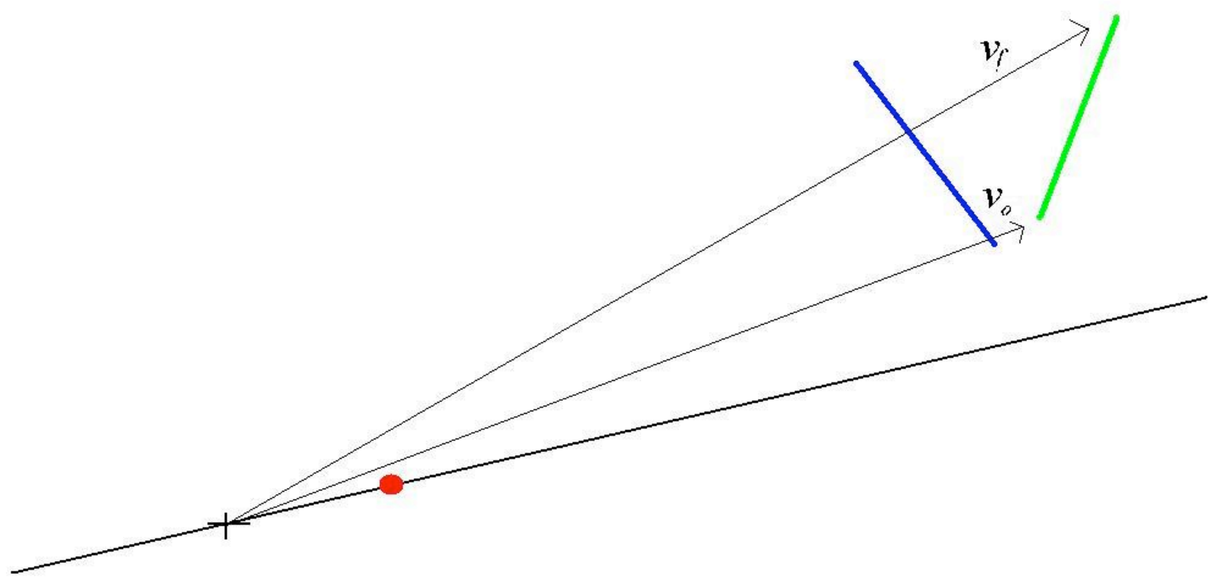
Which simplifies the plane equation to

$$N_x \cdot x + N_y \cdot y + N_z \cdot z = D$$

From now on I will refer to N_x, N_y, and N_z as A, B, and C respectively, resulting in a plane equation of

(2) $$Ax + By + Cz = D$$

Now I turn my attention to the V strip



The vectors v_o and v_f point from the origin to the beginning and from the origin to the end of the v strip. In component form these vectors are

$$\vec{v}_o = v_{ox}\hat{i} + v_{oy}\hat{j} + v_{oz}\hat{k}$$
$$\vec{v}_f = v_{fx}\hat{i} + v_{fy}\hat{j} + v_{fz}\hat{k}$$

For this process it is helpful to think of the V strip not as a vector but as a line between two points, $\vec{P}_o$ and $\vec{P}_f$.

$$\vec{P}_o = P_{ox}\hat{i} + P_{oy}\hat{j} + P_{oz}\hat{k}$$
$$\vec{P}_f = P_{fx}\hat{i} + P_{fy}\hat{j} + P_{fz}\hat{k}$$

These two points correspond directly to v_o and v_f.

$$\vec{P}_o = \vec{v}_o$$
$$\vec{P}_f = \vec{v}_f$$

Where

$$P_{ox} = v_{ox} \qquad P_{fx} = v_{fx}$$
$$P_{oy} = v_{oy} \qquad P_{fy} = v_{fy}$$
$$P_{oz} = v_{oz} \qquad P_{fz} = v_{fz}$$

Now the v strip is a line from $\vec{P}_o$ to $\vec{P}_f$, which can be described by

(3) $$\vec{P} = \vec{P}_o + s\left(\vec{P}_f - \vec{P}_o\right)$$

Where $\vec{P}$ is any point along the line and s is a scaling constant that determines how far between $\vec{P}_o$ and $\vec{P}_f$ that point is. This equation can be broken down into components along the x, y, and z directions

$$\vec{P}_x = \vec{P}_{ox} + s\left(\vec{P}_{fx} - \vec{P}_{ox}\right)$$
$$\vec{P}_y = \vec{P}_{oy} + s\left(\vec{P}_{fy} - \vec{P}_{oy}\right)$$
$$\vec{P}_z = \vec{P}_{oz} + s\left(\vec{P}_{fz} - \vec{P}_{oz}\right)$$

I'm looking for some point along the v strip that satisfies our plane equation (equation 2). I know that equation 3 describes all points along the v strip, and one must intersect the plane. I then substitute the components of equation 3 into equation 2 to find that point.

$$P_x = x$$
$$P_y = y$$
$$P_z = z$$

Making the proper substitutions I found

$$A\left(P_{ox} + s\left(P_{fx} - P_{ox}\right)\right) + B\left(P_{oy} + s\left(P_{fy} - P_{oy}\right)\right) + C\left(P_{oz} + s\left(P_{fz} - P_{oz}\right)\right) = D$$

Distribute A, B, and C

$$A \cdot P_{ox} + A \cdot s\left(P_{fx} - P_{ox}\right) + B \cdot P_{oy} + B \cdot s\left(P_{fy} - P_{oy}\right) + C \cdot P_{oz} + C \cdot s\left(P_{fz} - P_{oz}\right) = D$$

Move all s terms to the right hand side of the equation

$$A \cdot P_{ox} + B \cdot P_{oy} + C \cdot P_{oz} - D = s\left(A\left(P_{ox} - P_{fx}\right) + B\left(P_{oy} - P_{fy}\right) + C\left(P_{oz} - P_{fz}\right)\right)$$

Solve for s

(4) $$s = \frac{A \cdot P_{ox} + B \cdot P_{oy} + C \cdot P_{oz} - D}{A\left(P_{ox} - P_{fx}\right) + B\left(P_{oy} - P_{fy}\right) + C\left(P_{oz} - P_{fz}\right)}$$

Equation 4 is in terms of things we know (after calculating A, B, and C from the earlier calculation). I now establish s for the point in question, and substitute that back into equation 3. With all this in place I can determine the unique point in space along the v strip that intersects the plane, all from given information. I return a three dimensional vector in space that represents the absolute position the particle in question had along the v strip. To find this point on the u strip, I can easily reverse the process and form the plane with the v strip and the intersection with the u strip.