Autocorrelation Multiplicity Trigger One-Dimensional Simulations

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Abstract

I review here a possible fast trigger scheme, based on a midrapidity multiplicity distribution, which is insensitive to the Poisson component in the distribution and can select for various non-Poisson distribution features. I consider a distribution of particle hits over some 2-D space such as (η, ϕ). This space is binned to obtain a discrete distribution over a finite 2-D domain. I formulate a differential autocorrelation function on the finite domain and obtain the corresponding power spectrum. The trigger is based on the total power and features of the differential power spectrum as signatures of departure from Poisson. The present work is carried out for the 1-D case with various simulations. An extension of this work will consider the full two-dimensional problem. One can also consider optimization of domain sizes and bin numbers in the context of specific physics models.

1 Introduction

Large detector systems for high energy nuclear collisions require complex trigger systems spanning time scales from nanoseconds to seconds. The charged particle multiplicity distribution provides one basis for a fast (10 μsec) trigger. For the baseline **STAR** detector system this is the only substantive trigger basis until track information is available ($\approx$ 10 ms). Therefore, it is important to try to squeeze as much information as possible from this distribution. The distribution integral, or total multiplicity, provides a basis for a centrality trigger. The logical "OR" of all multiplicity

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elements provides a minimum bias trigger. This leaves the detailed spatial dependence of the multiplicity to provide some indication of event topology.

The event topology should provide an important trigger criterion, although the relationship between event topology and multiplicity distribution is not clear for RHIC experiments as it is for $p - \bar{p}$ or $e^+ - e^-$ for instance. For LEP events as an example one can trigger preferentially on single QCD vertices formed via the Z_0 resonance. One can then examine the topology of products accompanying the basic two-jet vertex to elucidate the detailed structure of QCD. In these simpler cases the trigger can be based on isolated jets and rely on fairly sparse multiplicity distributions—a situation very different from RHIC events dominated by a large Poisson component in the distribution.

At the high RHIC multiplicities one needs to have a basis for "removing" the Poisson contribution from the multiplicity distribution to provide a differential trigger signal. I consider here a strategy to accomplish this based on a "differential autocorrelation function."

2 Binning

Consider a particle hit distribution on some domain in (η, ϕ). The size of this domain is determined by the chosen detector configuration. This configuration also determines the nature of the binning. Ideally, this binning is uniform for mathematical convenience, and optimized in bin size to obtain maximum trigger information. Practical considerations usually result in significant departures from the ideal case.

For the present study I consider a one-dimensional simulation with 100 bins. The total multiplicity is taken to be 1000, and the distribution is assumed to be uniform over the domain. The multiplicity in bin i is f_i.

3 Statistics

I review the usual statistical quantities for the population $\{f_i\}$. The number of bins defined on the 1-D domain is taken to be N.

$$< f >= \frac{1}{N} \sum_{i=1}^{N} f_i \tag{1}$$

$$< f^2 >= \frac{1}{N} \sum_{i=1}^{N} f_i^2 \tag{2}$$

$$\sigma^2 = \frac{1}{N-1} \sum_{i=1}^{N} (f_i - < f >)^2 \tag{3}$$

$$< f^2 >=< f >^2 + \frac{N-1}{N}\sigma^2 \tag{4}$$

4 Autocorrelation Function

I now develop two autocorrelation functions for the population $\{f_i\}$. In what follows I begin with a generalized set $\{f_i\}$ with complex elements. Later I consider the special case that the elements are purely real, corresponding to the present application. The autocorrelation function can be defined in two ways in terms of the set of product pairs from $\{f_i\}$ which I denote by $\{f_i \cdot f_j^*\}$. In the first (usual) scheme the $\{f_i\}$, with $2N$ independent elements, are considered to be part of an ordered, infinite, periodic sequence of discrete complex elements with period N. The autocorrelation function is then defined on this infinite sequence as

$$A_m = \frac{1}{N}\sum_{i=1}^{N} f_i \cdot f_{i+m}^*. \tag{5}$$

We have the trivial result

$$A_0 =< f^2 >, \tag{6}$$

and there are only N independent elements A_m, since in this case

$$A_m = A_{N-m}^* = A_{-m}^*. \tag{7}$$

The second definition of autocorrelation accommodates explicitly the finite domain of the $\{f_i\}$,

$$B_m = \frac{1}{N-m}\sum_{i=1}^{N-m} f_i \cdot f_{i+m}^*. \tag{8}$$

These quantities, unlike the A_m, are independent over the entire domain $\{0, N-1\}$. Therefore, they represent $2N$ independent elements.

5 Fourier Transforms

The trigger criteria to be developed for the multiplicity distribution are based on the power spectrum of the differential autocorrelation function. I begin with a brief review of discrete Fourier transforms between dual vector spaces. Then I discuss the general relationships among the autocorrelations and power spectra defined for these spaces. In the following section I define **differential** autocorrelation functions which will provide the basis for a multiplicity trigger.

Each population $\{f_i\}$ can be considered to be a complex vector $\mathbf{f}$ in an N-dimensional space. One can define a dual space with elements $\hat{\mathbf{f}}$ related to the first by the discrete Fourier transform (FT). The FT is given explicitly as

$$\hat{f}_k = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} f_j e^{-\mathbf{i}\frac{2\pi}{N}k\cdot j} \tag{9}$$

with the inverse

$$f_j = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} \hat{f}_k e^{\mathbf{i}\frac{2\pi}{N}j\cdot k}. \tag{10}$$

In either space one can consider the autocorrelation function to be a kind of product vector ($\mathbf{A} = \mathbf{f} \circ \mathbf{f}^*$ or $\hat{\mathbf{A}} = \hat{\mathbf{f}} \circ \hat{\mathbf{f}}^*$) with components A_m or $\hat{A}_m$. The dual quantity for the autocorrelation in either space is the power spectrum in the dual space. This product may be represented by $\mathbf{P} = \mathbf{f} \cdot \mathbf{f}^*$ or $\hat{\mathbf{P}} = \hat{\mathbf{f}} \cdot \hat{\mathbf{f}}^*$ with components P_k or $\hat{P}_k$. This product is then defined by

$$P_k = f_k \cdot f_k^* = |f_k|^2. \tag{11}$$

With these definitions it can be shown that the autocorrelation and power spectra are Fourier transform pairs. This relationship, especially when the dual spaces are indexed by the conjugate variables time/frequency, is expressed as the Wiener-Khintchin theorem, important for signal analysis.

$$\hat{P}_k = \sum_{m=1}^{N} A_m e^{\mathbf{i}\frac{2\pi}{N}k\cdot m} \tag{12}$$

$$A_m = \frac{1}{N} \sum_{k=1}^{N} \hat{P}_k e^{-\mathbf{i}\frac{2\pi}{N}m\cdot k} \tag{13}$$

For the special case $m = 0$ the latter equation represents the Parseval-Plancherel theorem, which states that the Hermitian magnitudes of dual vectors are equal.

$$\sum_{j=1}^{N} f_j \cdot f_j^* = \sum_{k=1}^{N} \hat{f}_k \cdot \hat{f}_k^* \tag{14}$$

If we use the notation $\mathbf{f} \leftarrow FT \rightarrow \hat{\mathbf{f}}$ to denote the relationship between the dual vectors $\mathbf{f}$ and $\hat{\mathbf{f}}$ then we can represent the relationships among the products in the two spaces by the diagram

$$\begin{array}{ccccc} \mathbf{A} = \mathbf{f} \circ \mathbf{f}^* & & & & \hat{\mathbf{A}} = \hat{\mathbf{f}} \circ \hat{\mathbf{f}}^* \\ & \nwarrow & & \nearrow & \\ & & FT & & \\ & \swarrow & & \searrow & \\ \mathbf{P} = \mathbf{f} \cdot \mathbf{f}^* & & & & \hat{\mathbf{P}} = \hat{\mathbf{f}} \cdot \hat{\mathbf{f}}^*. \end{array}$$

In order to symmetrize this treatment I have defined the autocorrelations A_m and B_m in terms of complex quantities, thus neither of the dual spaces is assumed to contain only purely real vectors. I now comment briefly on the general structures of the autocorrelations and power spectra arising from the nature of the general, complex $\{f_i\}$ and the special cases for real elements $\{f_i\}$.

The power spectra P_k and $\hat{P}_k$ are purely real by definition. Under the assumption that the $\{f_i\}$ are part of an infinite periodic sequence the A_m are "conjugation symmetric" about $N/2$ or 0.

$$A_m = A^*_{N-m} = A^*_{-m}. \tag{15}$$

This is also consistent with the reality of the dual quantities $\hat{P}_k$. If the $\{f_i\}$ are not purely real then the $\hat{P}_k$ are not symmetric about $N/2$. Instead

$$\hat{P}_{N-k} - \hat{P}_k = \frac{1}{N} \sum_{l,j} 2\mathbf{i} Im(f_l \cdot f^*_j) e^{-\mathbf{i}\frac{2\pi}{N}(l-j)\cdot k} \tag{16}$$

$$= \sum_{j<l} Im(f_l \cdot f^*_j) Sin(\frac{2\pi}{N}(l-j) \cdot k). \tag{17}$$

If the elements of $\{f_i\}$ are purely real on the other hand then both the A_m and the $\hat{P}_k$ are simply symmetric about $N/2$. This situation is reflected in the Nyquist principle that for a real data set $\{f_i\}$

$$\hat{P}_{N-k} = \hat{P}_k, \tag{18}$$

and that independent information in the power spectrum of a time sample extends only to 1/2 of the sample frequency.

While the above notation completely symmetrizes the two dual spaces, the usual presentation and terminology are asymmetric, especially when associated with time/frequency indices and real data elements f_i, since the autocorrelation is intended to reveal the predictability of a time sequence, such as a measurement process in time, and the power spectrum is connected with the frequency variable. In the present context we are describing a measurement process in space. The data and autocorrelation are indexed by position variables. And the dual space and the power spectrum can be considered to be indexed by the wave number.

6 Differential Autocorrelations

In this section the elements of $\{f_i\}$ are taken to be purely real to correspond to a multiplicity distribution. If the elements of $\{f_i\}$ are taken in product pairs $f_i \cdot f_j$ then there are N^2 of these pairs in the population $\{f_i \cdot f_j\}$. N of these pairs are the special elements f_i^2. There remain $N(N-1)/2$ independent pairs, since $f_i \cdot f_j = f_j \cdot f_i$. If the f_i are considered to be a Poisson sample with mean value $< f >$ then the product pairs can also be considered to be a sample from a parent population of product pairs. I now want to relate the statistics for this product sample $\{f_i \cdot f_j\}$ to the statistics for the population $\{f_i\}$. I start with the square of the sample mean

$$< f >^2 = \frac{1}{N^2} \sum_{i \neq j} f_i \cdot f_j + \frac{1}{N^2} \sum_{i=1}^{N} f_i^2, \tag{19}$$

and I define the mean value of the product sample as

$$< f_i \cdot f_j > = \frac{1}{N(N-1)} \sum_{i \neq j} f_i \cdot f_j. \tag{20}$$

This definition is chosen in part because in a truly random sample of pairs from the parent population for $\{f_i\}$ it is much less likely that nearly identical values of f_i would be selected for pairs than is represented by the N elements f_i^2 in $\{f_i \cdot f_j\}$. The more important motivation is that this definition is most closely related to the mean values of the A_m for a Poisson sample. Using this definition we have

$$< f >^2 = \frac{1}{N}((N-1) < f_i \cdot f_j > + < f^2 >) \tag{21}$$

and

$$< f_i \cdot f_j > = < f >^2 - \frac{\sigma^2}{N}. \tag{22}$$

If the $\{f_i\}$ constitute a Poisson sample then all A_m except A_0 represent estimates of the mean value of the product pair population $< f_i \cdot f_j >$ as defined above. This presents the opportunity to define a **differential** autocorrelation function with mean value 0 if the $\{f_i\}$ are Poisson distributed. Statistically significant nonzero values of these differential quantities should represent a departure of the $\{f_i\}$ population from Poisson. Substituting the Poisson value $< f >$ for σ^2 I define the differential autocorrelation as

$$a_m = A_m - < f >^2 + \frac{< f >}{N} - < f > \delta_{0,m} \qquad m \in \{0, N-1\}. \tag{23}$$

Corresponding differential quantities can be defined in connection with the alternative B_m. I start by noting that with the definition of the B_m given above we have

$$A_m = \frac{N-m}{N} B_m + \frac{m}{N} B_{N-m}. \tag{24}$$

I can therefore write the a_m in the form

$$a_m = \frac{N-m}{N}[B_m - <f>^2 + \frac{\sigma^2}{N}] + \frac{m}{N}[B_{N-m} - <f>^2 + \frac{\sigma^2}{N}]. \tag{25}$$

This suggests that I define the differential quantities associated with the B_m as

$$b_m = \frac{N-m}{N}(B_m - <f>^2 + \frac{<f>}{N} - <f> \delta_{0,m}) \qquad m \in \{0, N-1\}. \tag{26}$$

This is consistent with

$$a_m = b_m + b_{N-m} \tag{27}$$

and

$$a_m = a_{N-m} = a_{-m}. \tag{28}$$

So we see that the a_m, with $N/2$ independent elements, are just the symmetric parts of the b_m with N independent elements. We can, for completeness, define the antisymmetric parts of the b_m as

$$\bar{a}_m = b_m - b_{N-m}. \tag{29}$$

One should note that the dual of a purely real power spectrum must be conjugation symmetric about $N/2$ as is A_m defined above. The b_m do not have this symmetry and therefore are not strictly speaking an autocorrelation function. The FT for these quantities is not purely real, and so cannot properly be called a power spectrum. However, these quantities show a superior sensitivity to nonPoisson contributions to the $\{f_i\}$ and so are denoted by the terminology applied to the more standard quantities. The "power spectrum" of the b_m shown below is actually the **modulus** of the FT of the b_m.

In the simulations to follow I compare the general differential autocorrelation b_m and its symmetric part a_m. I calculate the power spectrum $\hat{\mathbf{P}}_b$ of $\mathbf{b}$ for various multiplicity distributions. And I examine the effectiveness of the total power as a trigger parameter for various nonPoisson features in a multiplicity distribution.

7 Simulations

The basis for the simulations is a population $\{f_i\}$ from a random number generator with $<f>=10$ over $N = 100$ bins. This forms the Poisson multiplicity distribution. To this are added various

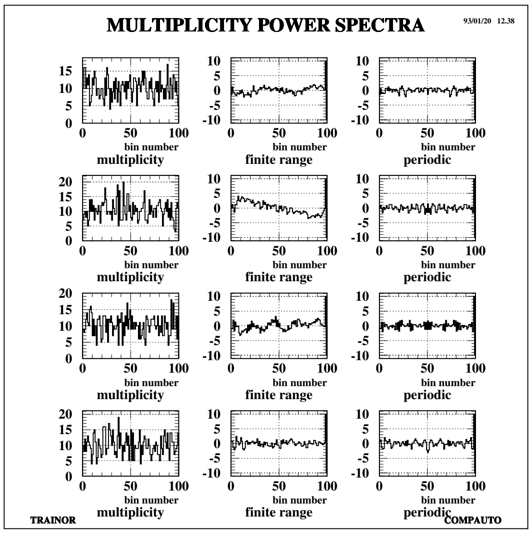

Figure 1: Poisson multiplicity

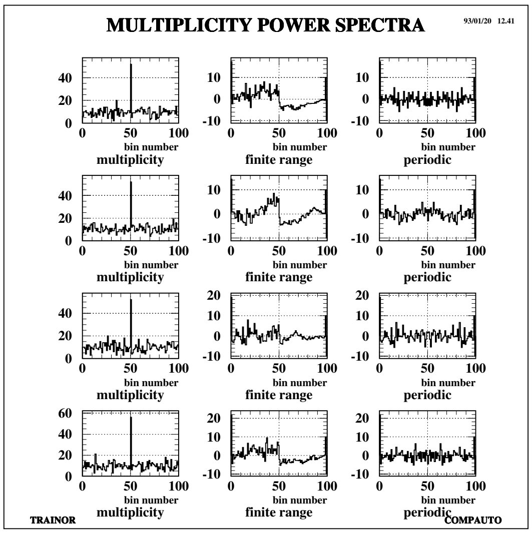

Figure 2: Poisson plus 45 ct delta

"features" consisting of 45 additional counts in one or more bins. The case of 45 counts placed in a single bin is labeled "delta." A "standard" feature is about 4 bins FWHM. I also used double, quadruple and half width features.

A single feature was always placed at bin 50. Two features were placed at bins 20 and 50. These details will be apparent in the following figures.

I begin with a comparison of a_m and b_m in the first four figures (1-4). Each figure is composed of four generator runs of 1000 hits into 100 bins. The multiplicity is displayed in the left graphs. The central graphs display the b_m values, labeled "finite range," for each of the runs. The right graphs display the a_m values, labeled "periodic." I note that the a_m are rigorously symmetric about $N/2$, as expected, whereas the b_m are, if anything, approximately antisymmetric about the central bin.

This group of four figures corresponds to four different multiplicity distributions. Fig. 1 corresponds to a purely Poisson distribution, while Fig. 2 includes also a 45 count delta at bin 50. Fig. 3 includes a 45 count "peak" or feature centered at bin 50, with a 4 bin width. And Fig. 4 includes a second identical feature at bin 20. The additional counts for the features are the same for each of the four runs.

Note that the finite domain correlation approximately integrates the feature, but there is a tendency for this autocorrelation also to differentiate broad trends in the distribution. The periodic correlation exhibits a significantly smaller response to simple features but becomes comparable to the finite domain case as the distribution becomes more complex. The poorer sensitivity, plus the reduced information available from the a_m recommends the b_m for use in a trigger application. For

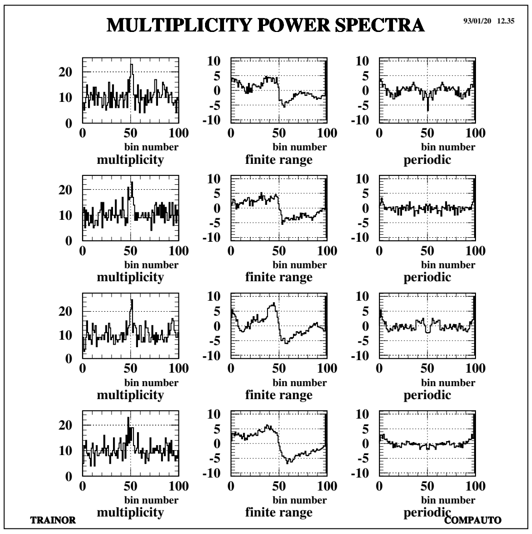

Figure 3: Poisson plus one 45 ct peak

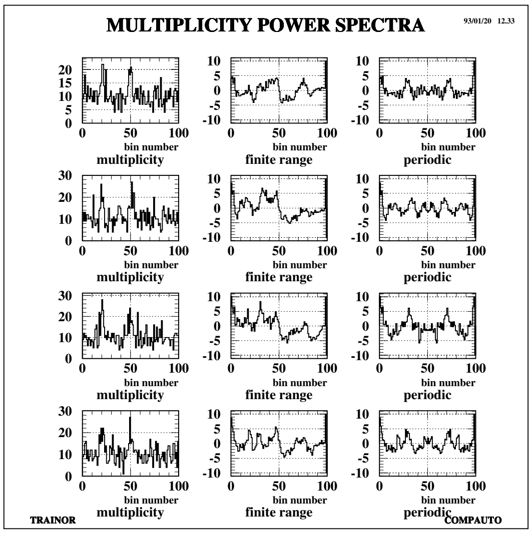

Figure 4: Poisson plus two 45 ct peaks

what follows then I use the latter correlation.

For the next set of four figures (5-8) I consider the same selection of distribution features, with the finite domain autocorrelation and the corresponding Fourier transform power spectrum. This last replaces the periodic correlation at the right in the figures. The total power is also given on the power spectrum graph.

The "total power" is the sum of spectrum bin contents from some threshold up to the last bin. This threshold is meant to reduce sensitivity to large wavelength disturbances visible in the autocorrelations for the purely Poisson runs. This threshold is set at bin 3 in the current simulations. So the sum includes bin 4 and above.

Looking at Fig. 8, corresponding to two peak features, one sees that the power density extends to higher wave number, as one might expect for the more complex distribution. This suggest that higher moments of the power spectrum than the total integral might also serve as trigger criteria. The value of these criteria of course depend on the statistical significance of the features one actually encounters in an experiment. HIJING may provide some information along these lines.

Each of the next set of four figures (9-12) shows the histogramed total powers for 100 runs. These figures correspond to the same multiplicity distribution features as for the previous figures (5-8). They illustrate the capability of the autocorrelation as a trigger criterion.

Fig. 9 shows the distribution of total powers for 100 Poisson-only runs. About 90% of the runs had total powers below 10 units. In Fig. 10 100% of the runs for a 45 count delta are above 15 units. Examining Figs. 11 and 12 one can see that the total power is a good criterion for selecting

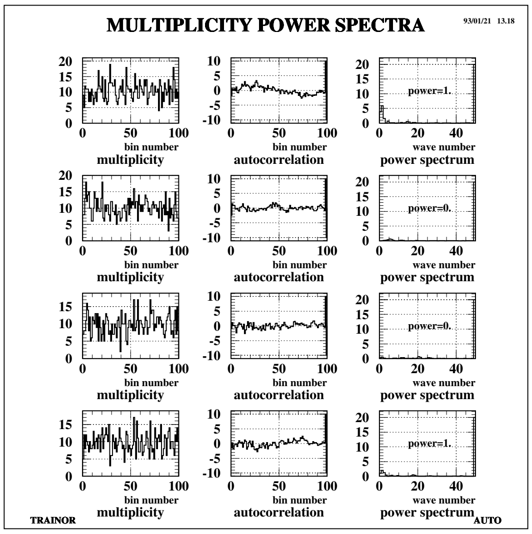

Figure 5: Poisson only

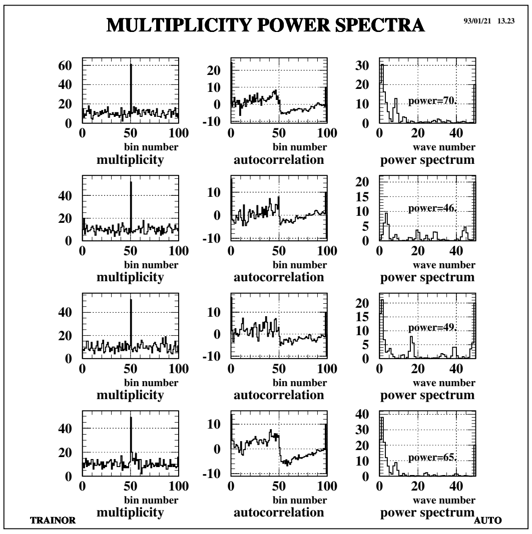

Figure 6: Poisson plus 45 ct delta

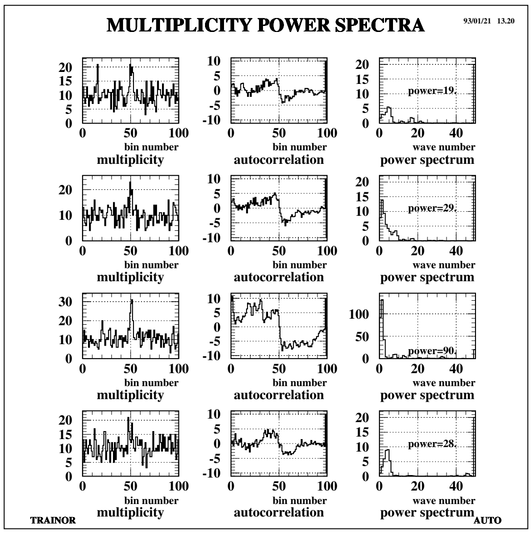

Figure 7: Poisson plus one 45 ct peak

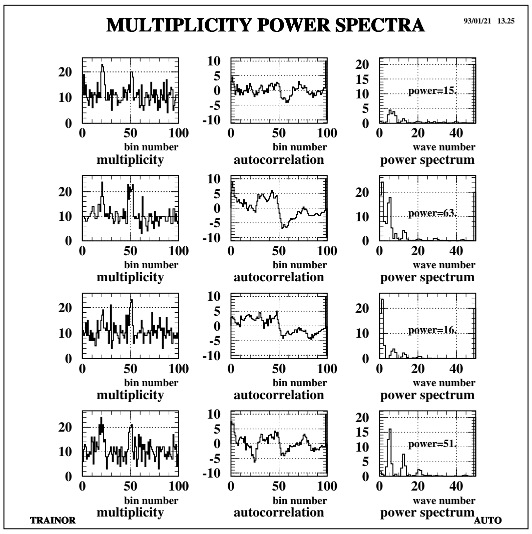

Figure 8: Poisson plus two 45 ct peaks

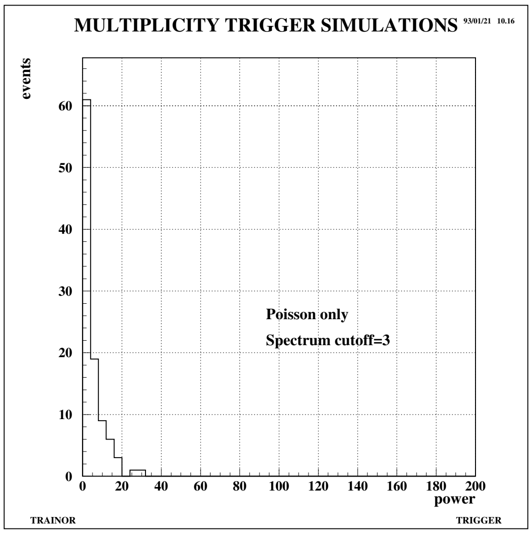

Figure 9:

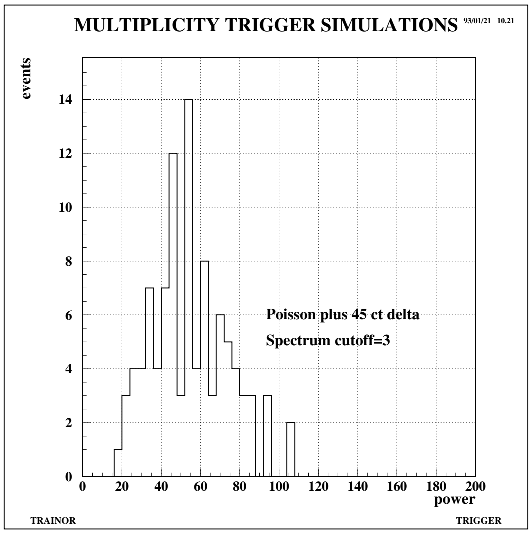

Figure 10:

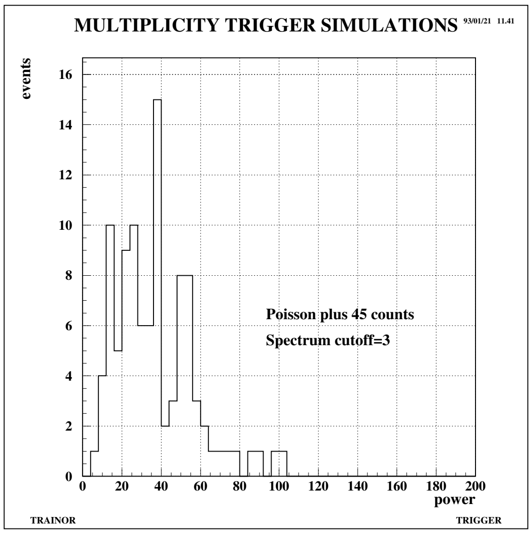

Figure 11:

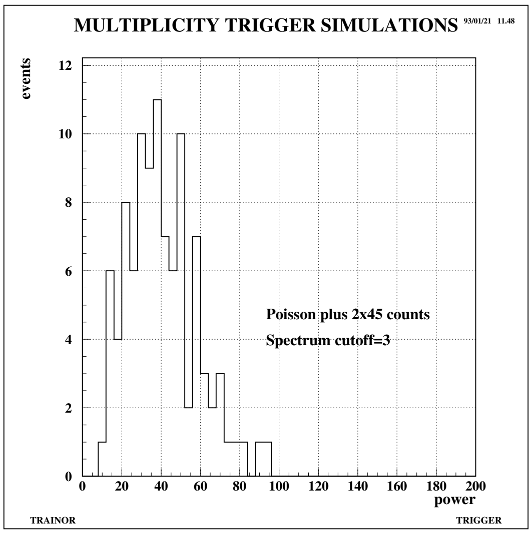

Figure 12:

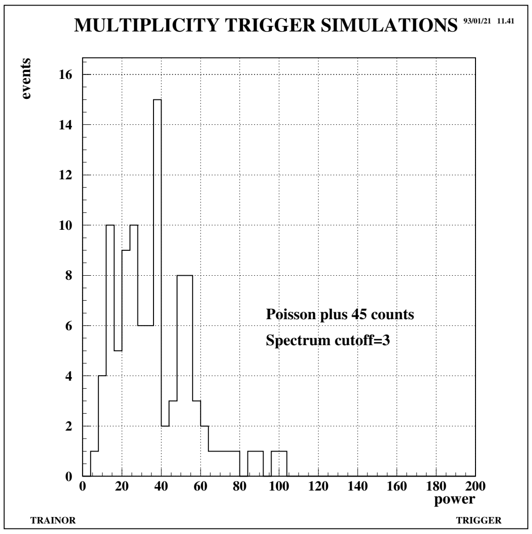

Figure 13:

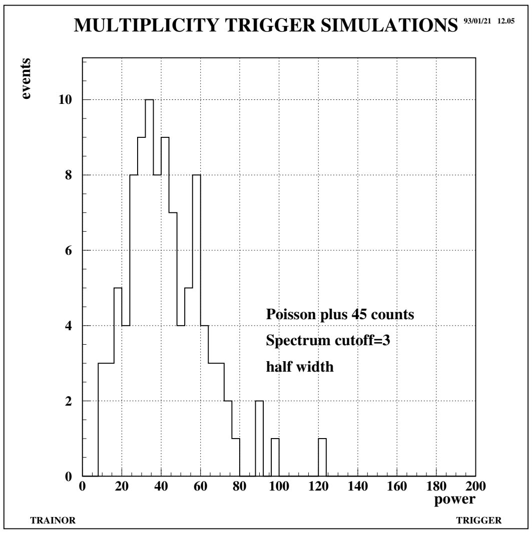

Figure 14:

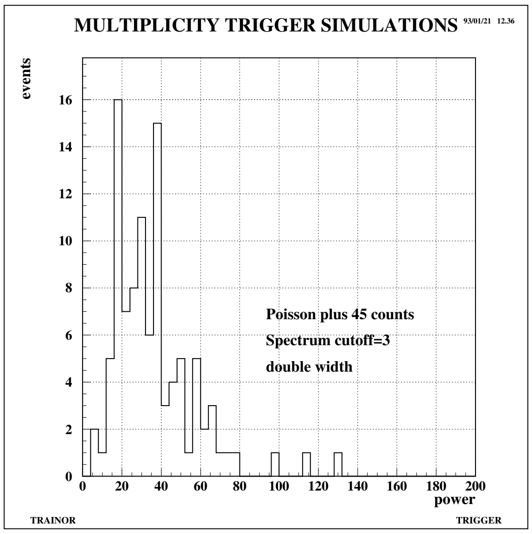

Figure 15:

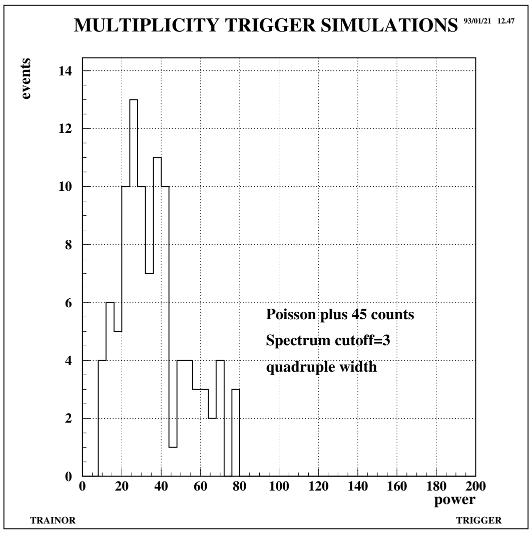

Figure 16:

peak features with even as few as 45 counts (4.5% of the total multiplicity).

Figs. 13-16 represent an examination of the feature width dependence of the total power. The conclusion is that even with a width as large as 10 bins the power distribution is still well above that for Poisson only. This feature distribution represents 10 bins, each 1.5 sigma (4.5 counts) above the background. It seems that as long as the excess counts are deposited in a region small compared to the domain size the autocorrelation provides a good trigger criterion.

8 Conclusions

Trigger design is a problem in pattern recognition. I address here the problem of distinguishing nonrandom features superposed on a random background. I have approached this problem from the standard treatments of electrical signals and noise, but have developed by extension differential quantities which offer the possibility of enhanced sensitivity to interesting events. This treatment requires no initial hypothesis, other than the nature of the random background which is to be "subtracted" from the measurement.

I find that this approach does represent a sensitive test for nonrandomness, as illustrated by the simulation results. Features just at the level of confident identification by eye are reliably identified by a simple algebraic criterion.

More detailed tests based on generators such as HIJING which provide structures related to some nuclear model are now required. This treatment should also be extended to the 2-D case, and to nonuniform binning, which results from the irregularities of real detector systems. Criteria for optimum binning should also be addressed from the standpoint of typical "sizes" of features expected on the measurement domain.

In summary, I have defined new quantities, by analogy with the standard autocorrelation and power spectrum, which should provide a more sensitive means of detecting departures from a Poisson distribution.