University of Washington Nuclear Physics Laboratory

Power Spectrum Analysis with Nonuniform Bin Size and Variable Bin Number

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Ultrarelativistic Heavy Ions Group

1 Introduction

In a previous paper ?? I described the power spectrum analysis of an ensemble of ten 'normal' and ten 'anomalous' events generated using HIJET with some rearrangement and enhancement of stopped energy from that provided by the standard generator. During that analysis I assumed a trigger detector configuration providing 100 uniform bins over the η interval [-2,2]. The result of this analysis was the construction of a pattern space in which the ten normal events were clearly localized and separated from the ten anomalous events, and the latter were dispersed in a manner which differentiated among the various features of the anomalous distributions.

This choice of bin model was convenient for a proof of principle for the autocorrelation and power spectrum trigger algorithm. The use of 100 bins assured that bin size would in no way limit the analysis by effective bandwidth reduction. However, that model is inconsistent with the presently visualized baseline trigger detector system which consists of 20 nonuniformly sized bins. The present paper describes a similar analysis performed using the correct binning model for the present baseline system of CTB and MWC and additional 12-bin and 500-bin models for overall comparison. By examination of these results we can determine whether the correct binning model in any way limits the trigger algorithm sensitivity to anomalous features contained in the present event ensemble.

2 The Trigger Analysis Chain

2.1 Binning Model

For this investigation I have used the same set of twenty events generated by Ron Longacre at BNL. These events have been passed through GEANT by Zoran Milosevich at CMU to produce hits in (η, ϕ). These hits are then binned according to the model of choice. For the power spectrum analysis I have ignored the ϕ dependence, and have considered only the one-dimensional η problem.

For this analysis the 20-bin model consists of eight uniform bins on each of the η intervals [-2,-1] and [1,2] representing the two MWCs. The interval [-1,1] is divided into four uniform bins representing the CTB. This makes a total of twenty bins. The 12-bin and 500-bin models equally divide the eta range into the stated number of bins.

This bin system is then filled with hits in η to provide a set of bin counts $\{n_i\}$ for each event. For the purpose of this analysis in the 20-bin model I have divided the resulting contents of the CTB bins by four, reflecting the four-times larger η width of these bins. This operation establishes the multiplicity distribution for each event in the selected binning model.

2.2 Null Hypothesis

The next step is to determine the null hypothesis for the differential autocorrelation. This is done for each binning model by obtaining power spectrum sine and cosine amplitudes for each member of the set of 'normal' events in the ensemble. The mean values of each of the first six amplitudes of the sines and cosines, taken over the ensemble of normal events, represent the null hypothesis.

2.3 Power/Phase Analysis

These null hypothesis amplitudes are then inserted into the power spectrum analysis algorithm, and power and phase parameters are obtained for each of the twenty events. If the null hypothesis is properly chosen the normal events have very low power magnitudes. Obviously, if any one event were chosen as the null hypothesis that event would be found to have the lowest power in this analysis. It would not have zero power unless a number of sine and cosine amplitudes was used to represent it greater than the bin number.

2.4 Pattern Space

The result of the power spectrum analysis is power and phase values for each event. The ensemble of events is then represented in a pattern space using these two parameters as coordinates. It is preferable to plot the logarithm of the power both because of the dynamic range of this parameter in this particular event ensemble and because what we designate as the 'power' in this analysis is closely related to the information entropy of the multiplicity distribution.

3 Results of This Analysis

We compare selected analysis results for the 12- 20- and 500-bin models with the result of the 100-bin analysis presented in the previous power spectrum paper.

3.1 Generator Multiplicities

These are the same events used in the original 100-bin analysis. They suffer from an artifact in the GEANT simulation in that the yield to the MWCs is high in comparison to the yield to the CTB, resulting in a normal event multiplicity which is discontinuous across the transition from one detector type to the other. This does not pose any problem for the power spectrum analysis because the effect of the discontinuity is just subsumed into the null hypothesis. Refer to the appropriate 'generator multiplicity' figures in the 100-bin paper.

3.2 Event Autocorrelations with Null Hypothesis

This shows the agreement between the events of the normal set and the null hypothesis for 12-, 20- and 500-bin binning models, and the nature of the deviations for the anomalous events in terms of the autocorrelation. These autocorrelations are all of the finite-domain type.

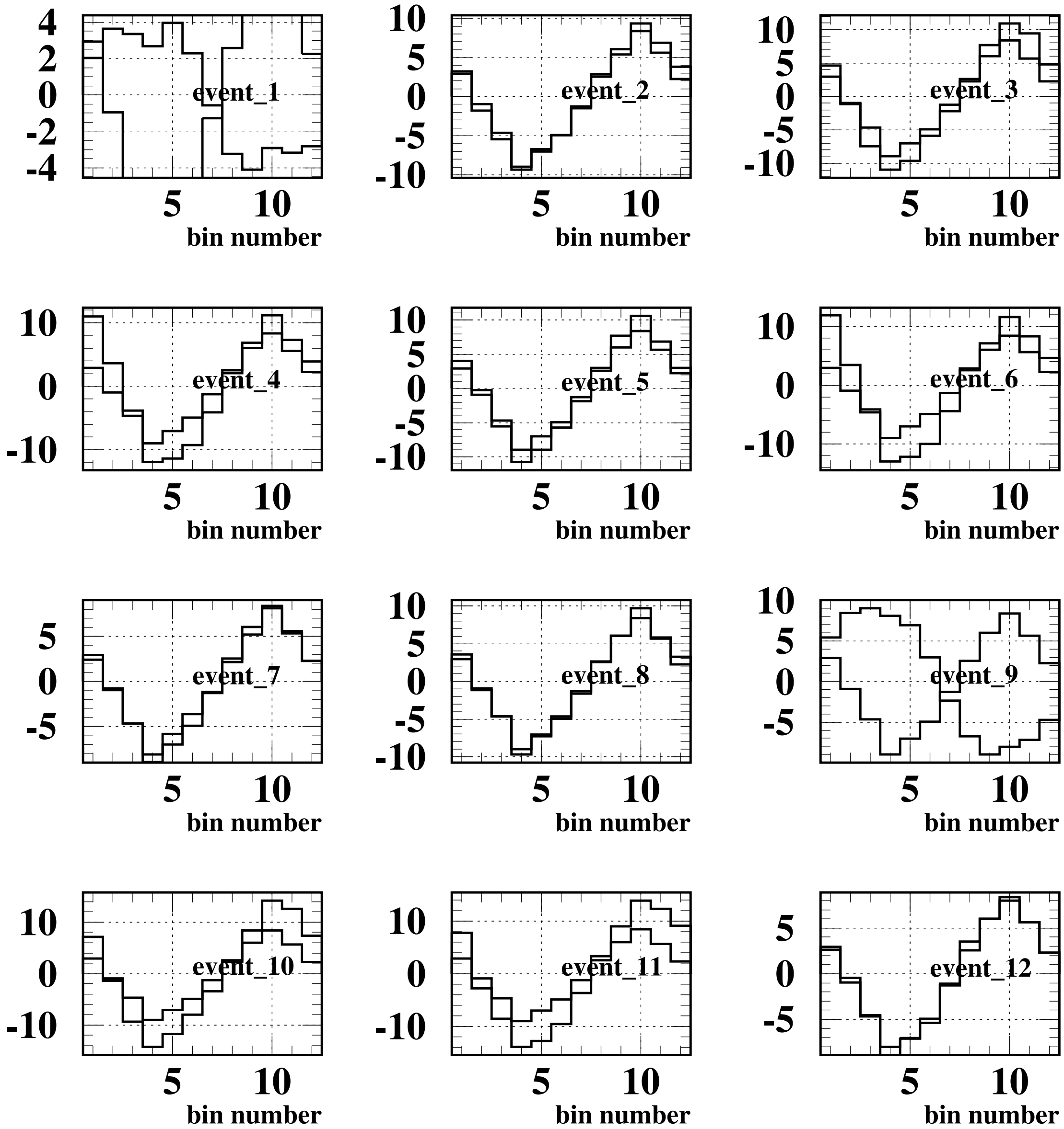

Figure 1: Comparison of autocorrelation and null hypothesis, 12 bins.

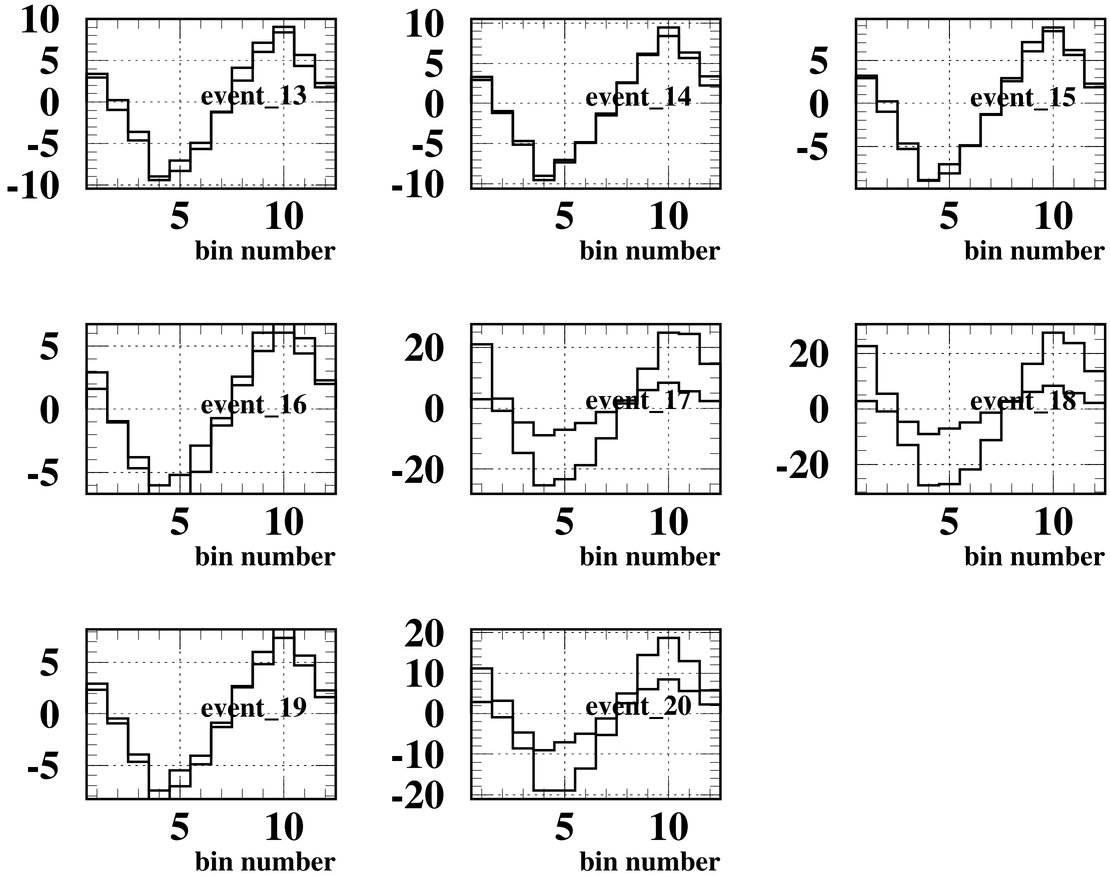

Figure 2: Comparison of autocorrelation and null hypothesis, 12 bins.

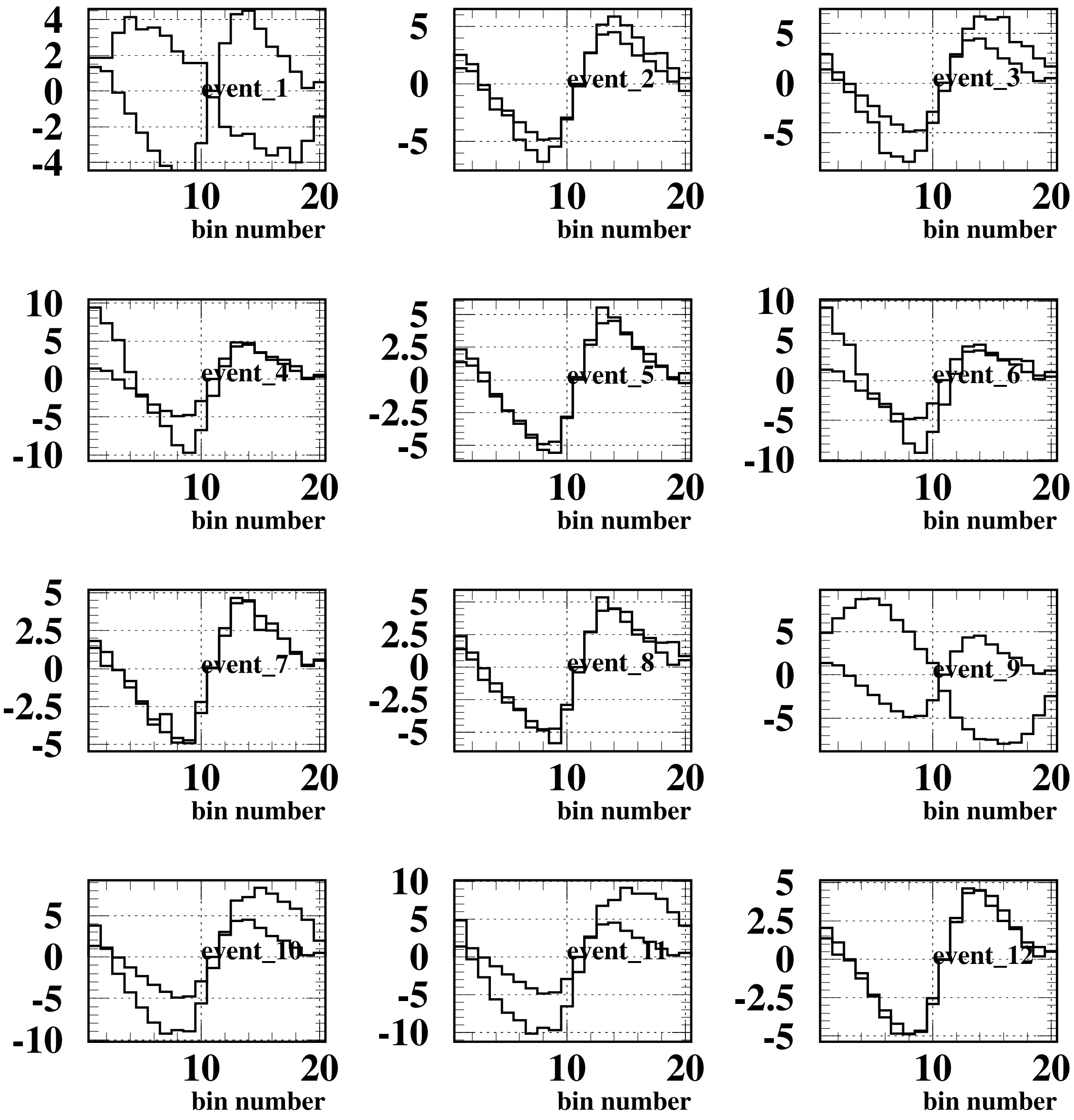

Figure 3: Comparison of autocorrelation and null hypothesis, 20 bins.

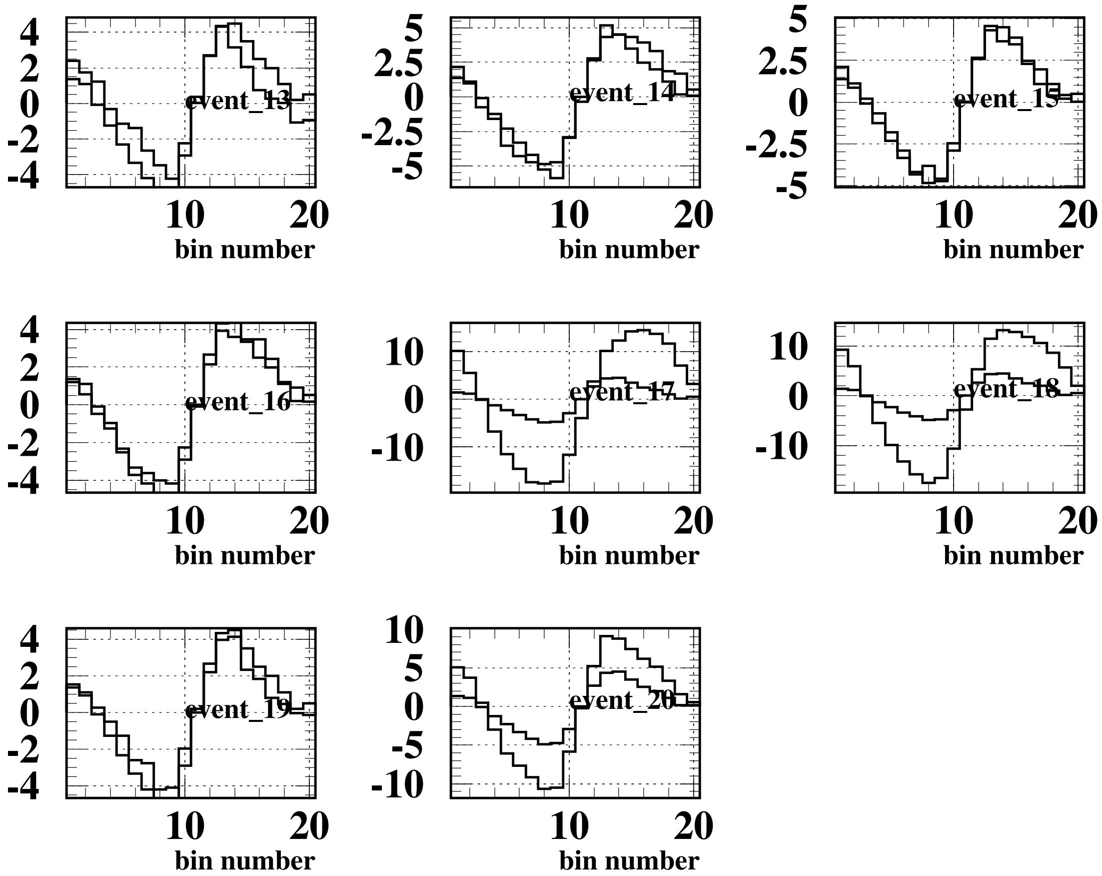

Figure 4: Comparison of autocorrelation and null hypothesis, 20 bins.

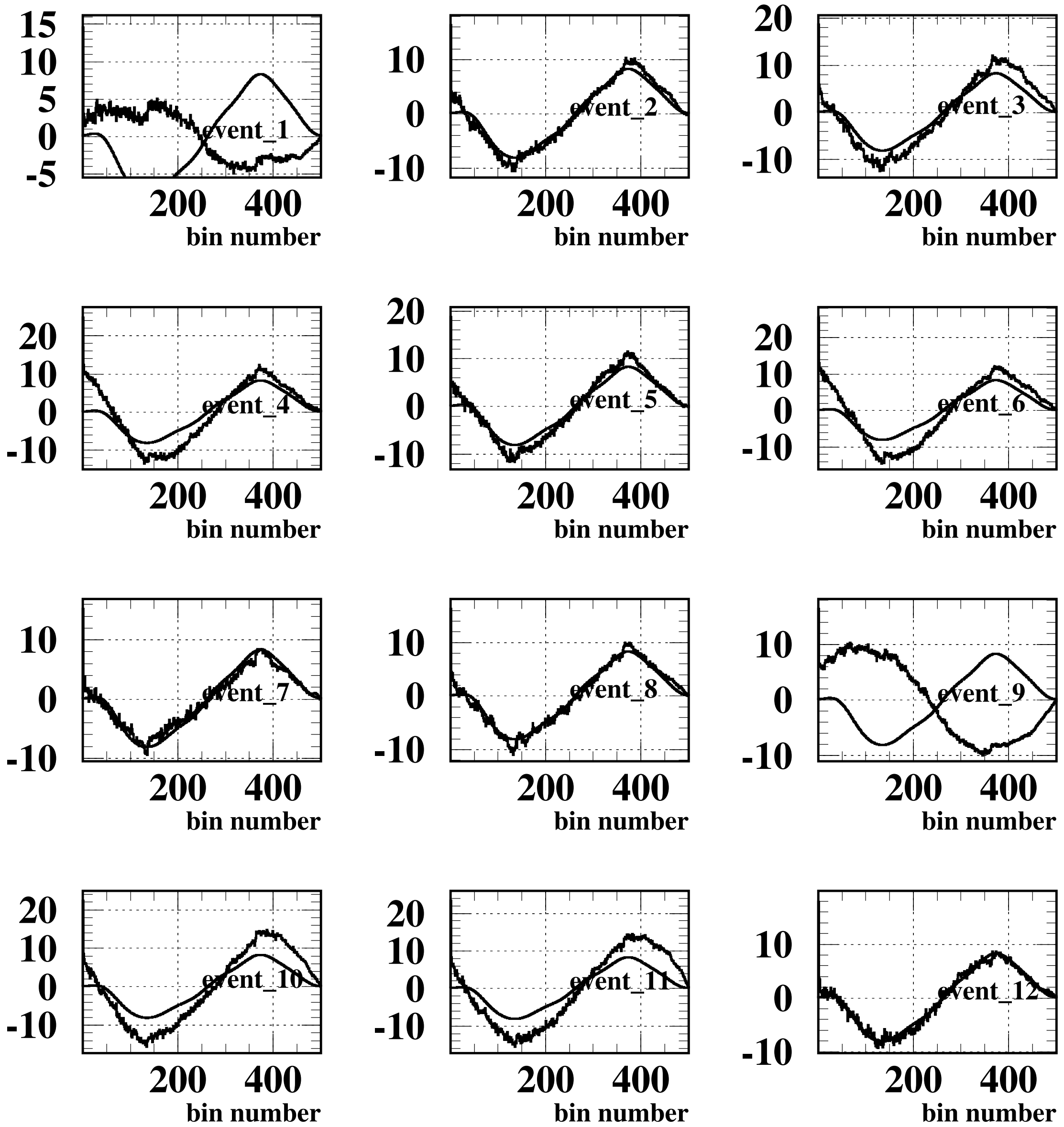

Figure 5: Comparison of autocorrelation and null hypothesis, 500 bins.

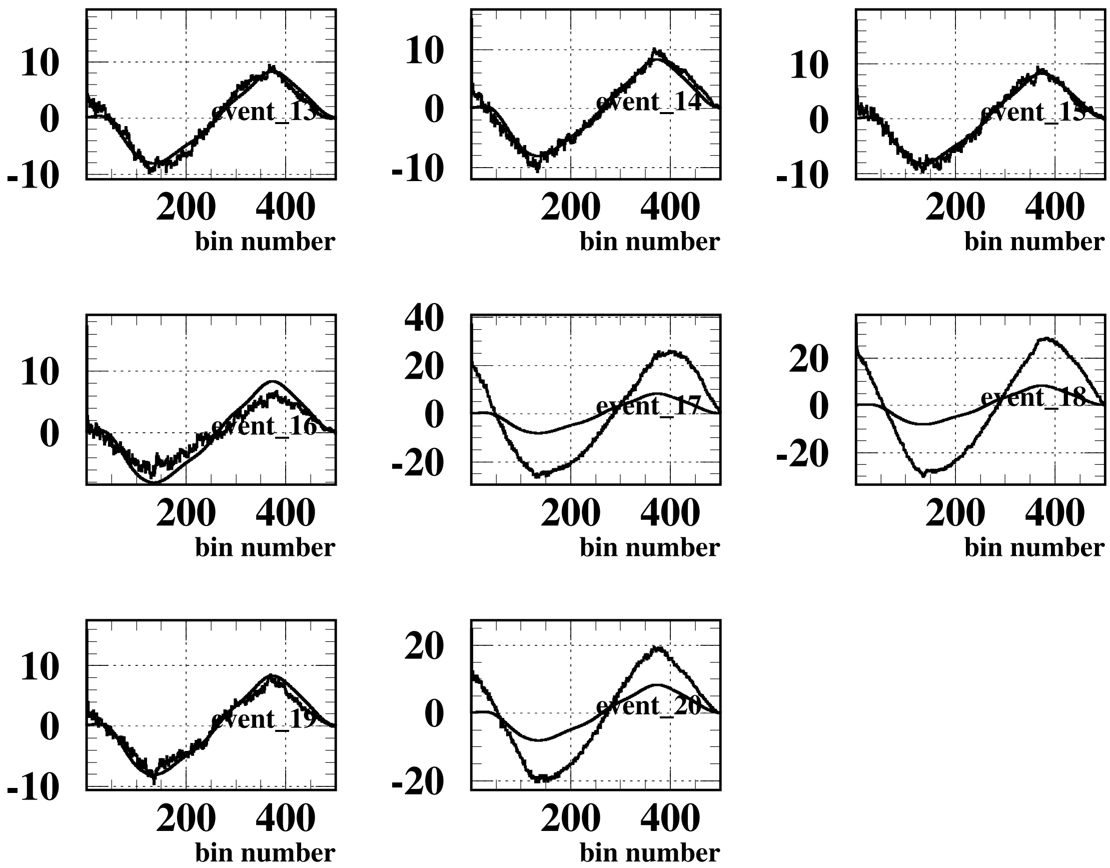

Figure 6: Comparison of autocorrelation and null hypothesis, 500 bins.

3.3 Null Hypothesis

I show here in graphical form the null hypotheses for each of the four binning models studied. The first six sine coefficients are shown on the left and cosine coefficients on the right for each binning model as denoted on the plot. The null hypothesis for the 20-bin model differs significantly from the others because the central four bins in this model are four times wider in η than the outer bins. The other three models have uniform binning. Note that at most only the first four coefficients are significant for any binning model.

These null hypotheses are determined by an average over the entire population of 'normal' events. An alternave hypothesis could be determined by selecting any one event and using its Fourier representation as the null hypothesis.

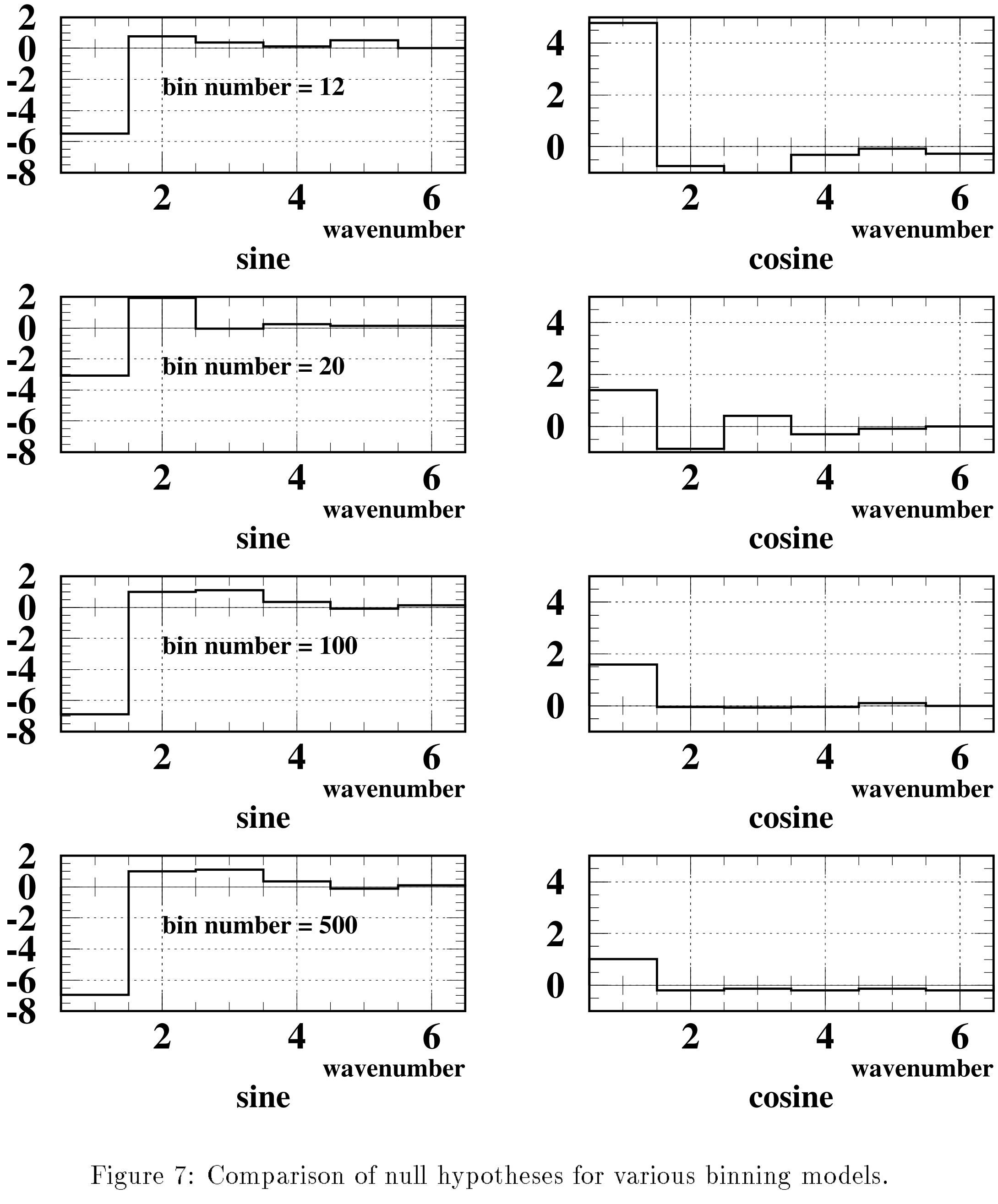

Figure 7: Comparison of null hypotheses for various binning models.

3.4 Multiplicity Autocorrelations

The 20-bin autocorrelation is included for reference with the 100-bin analysis in terms of the detailed structure of the autocorrelations.

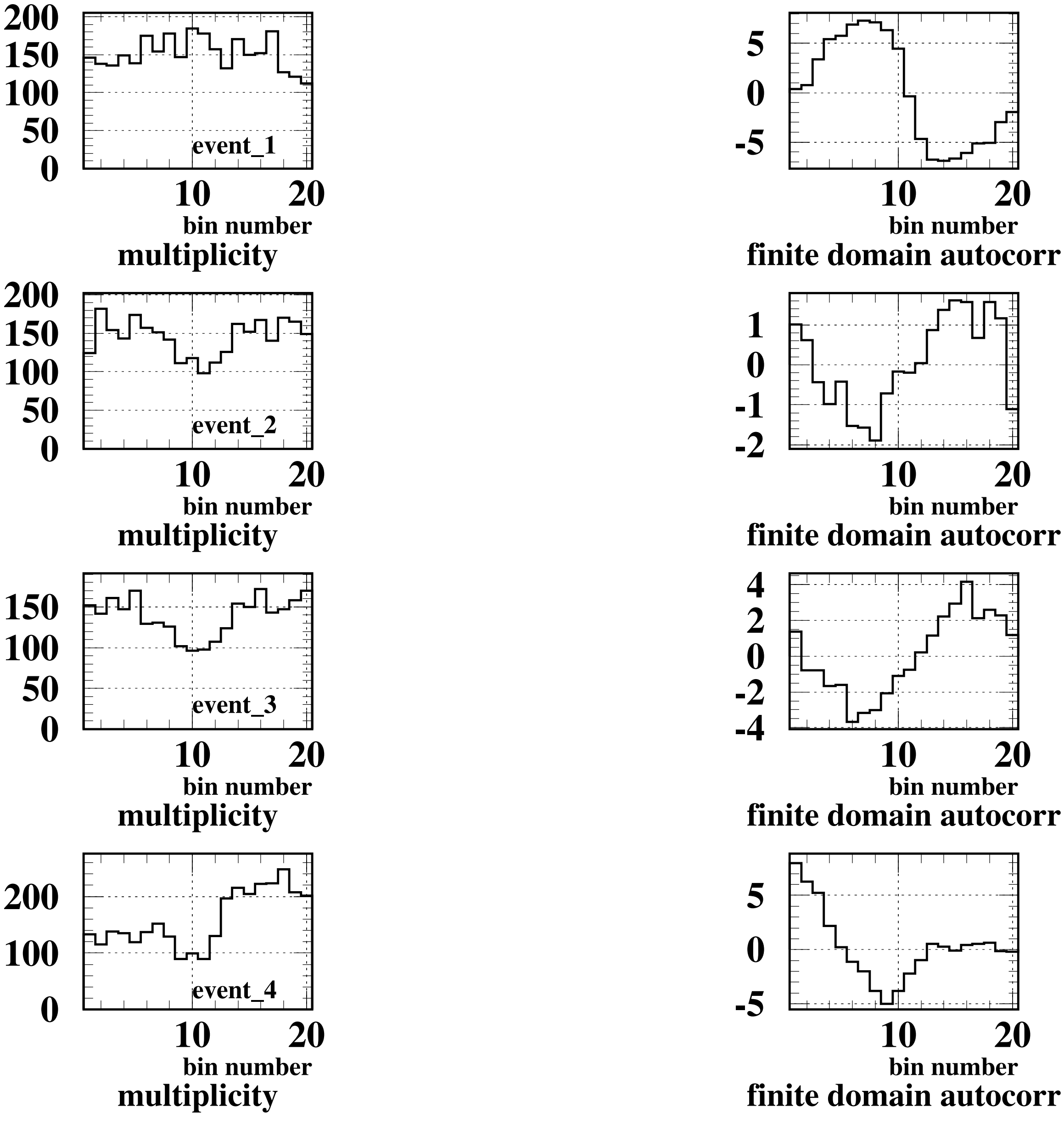

Figure 8: Multiplicity autocorrelations.

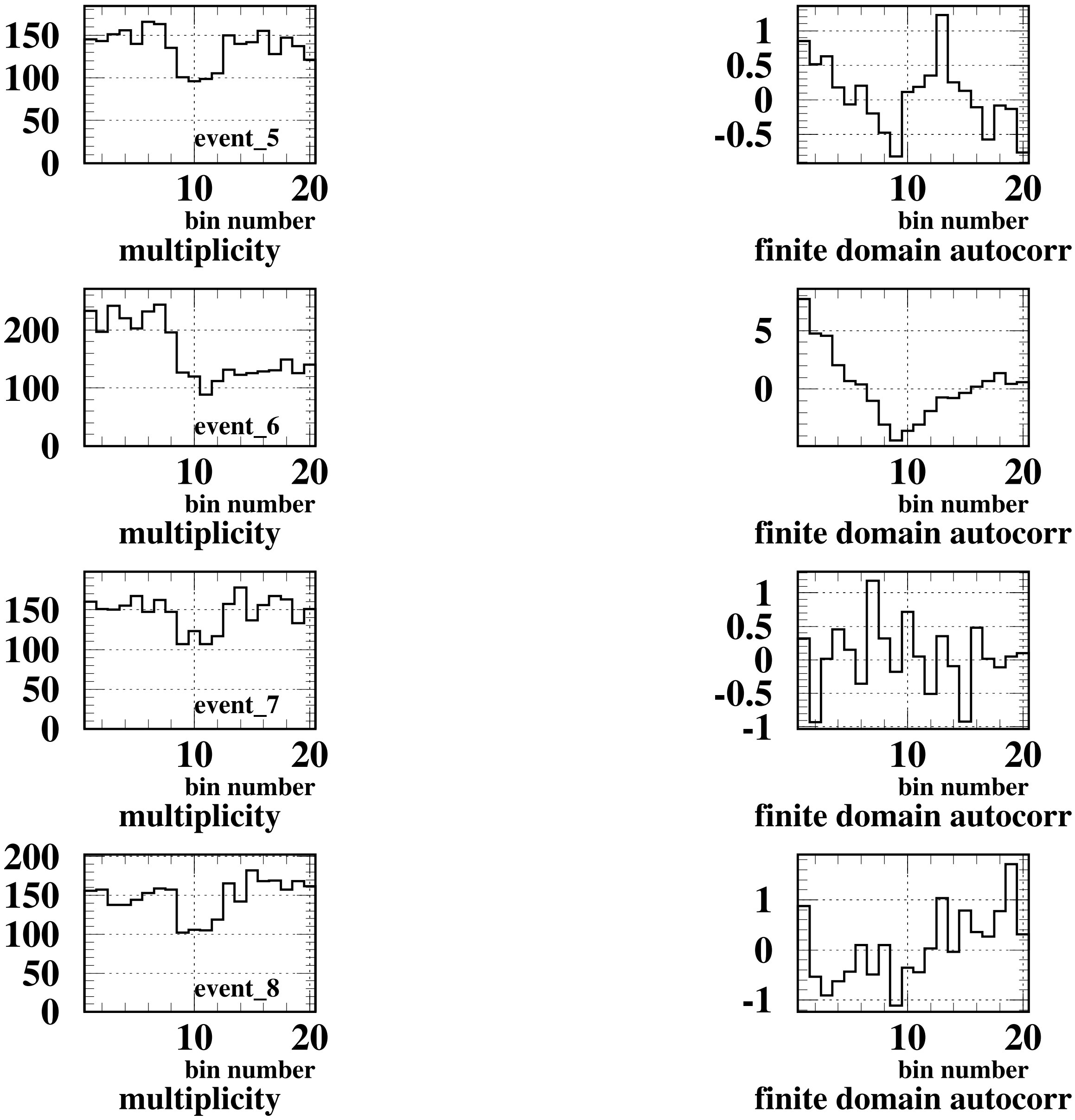

Figure 9: Multiplicity autocorrelations.

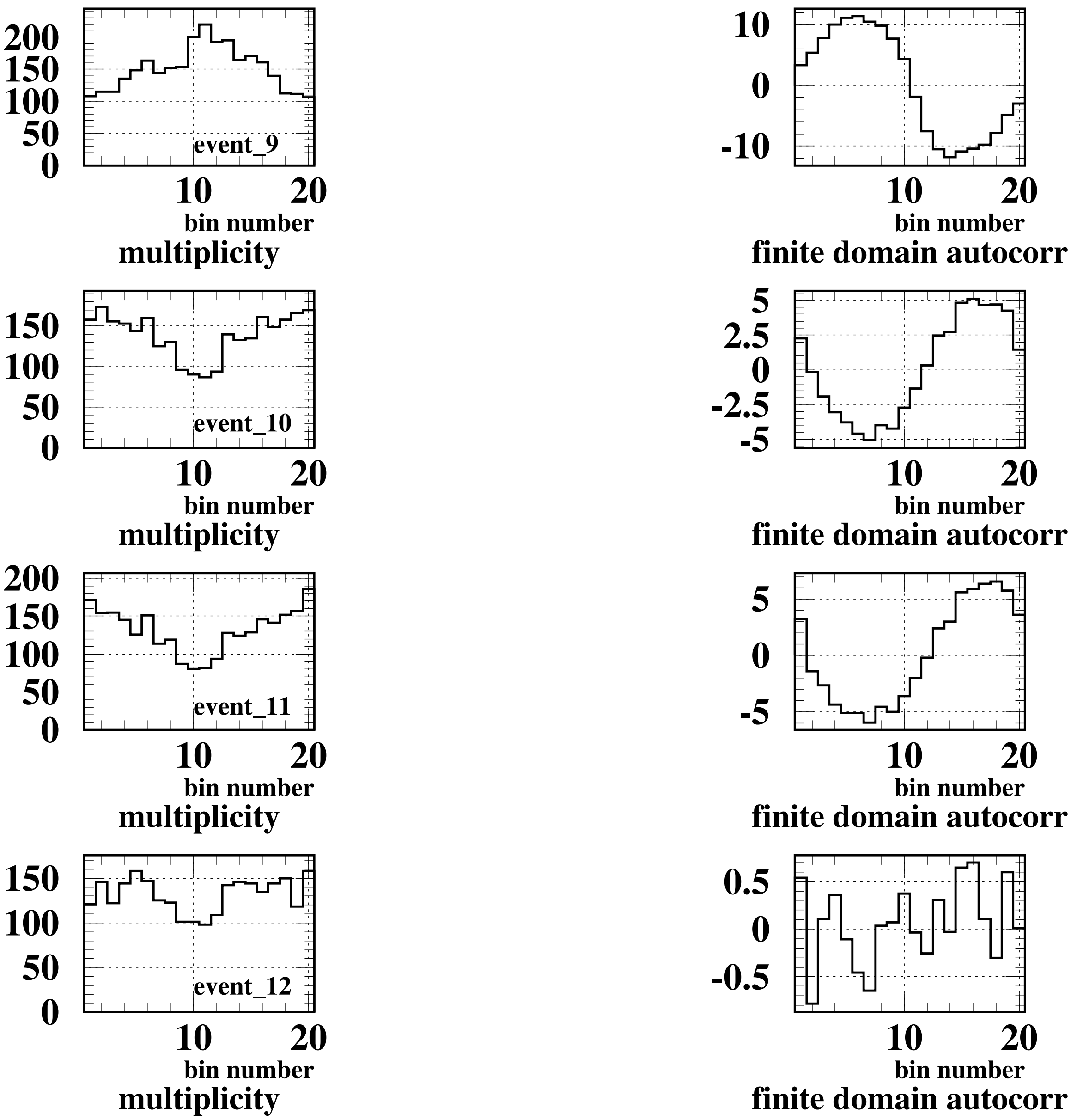

Figure 10: Multiplicity autocorrelations.

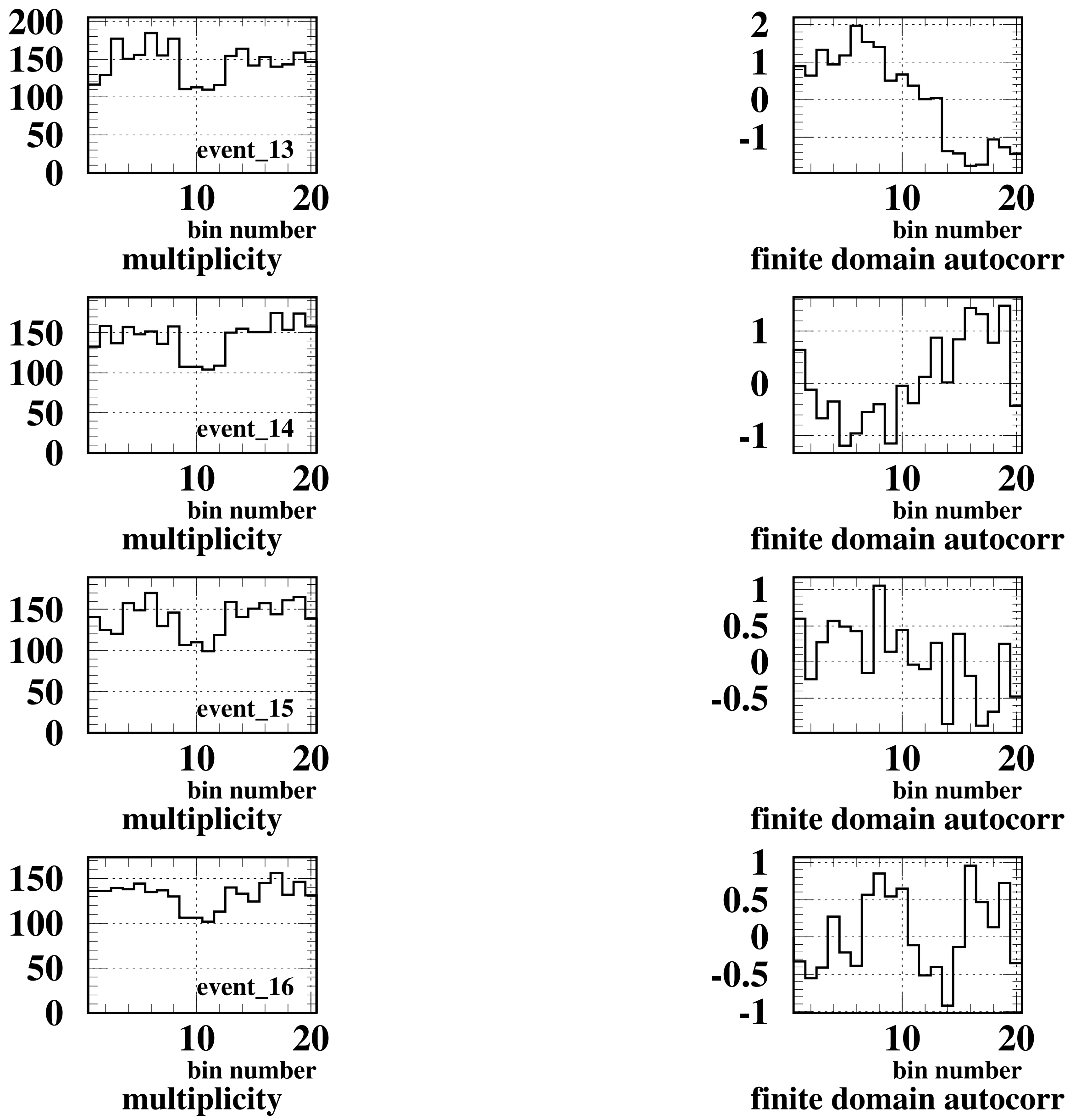

Figure 11: Multiplicity autocorrelations.

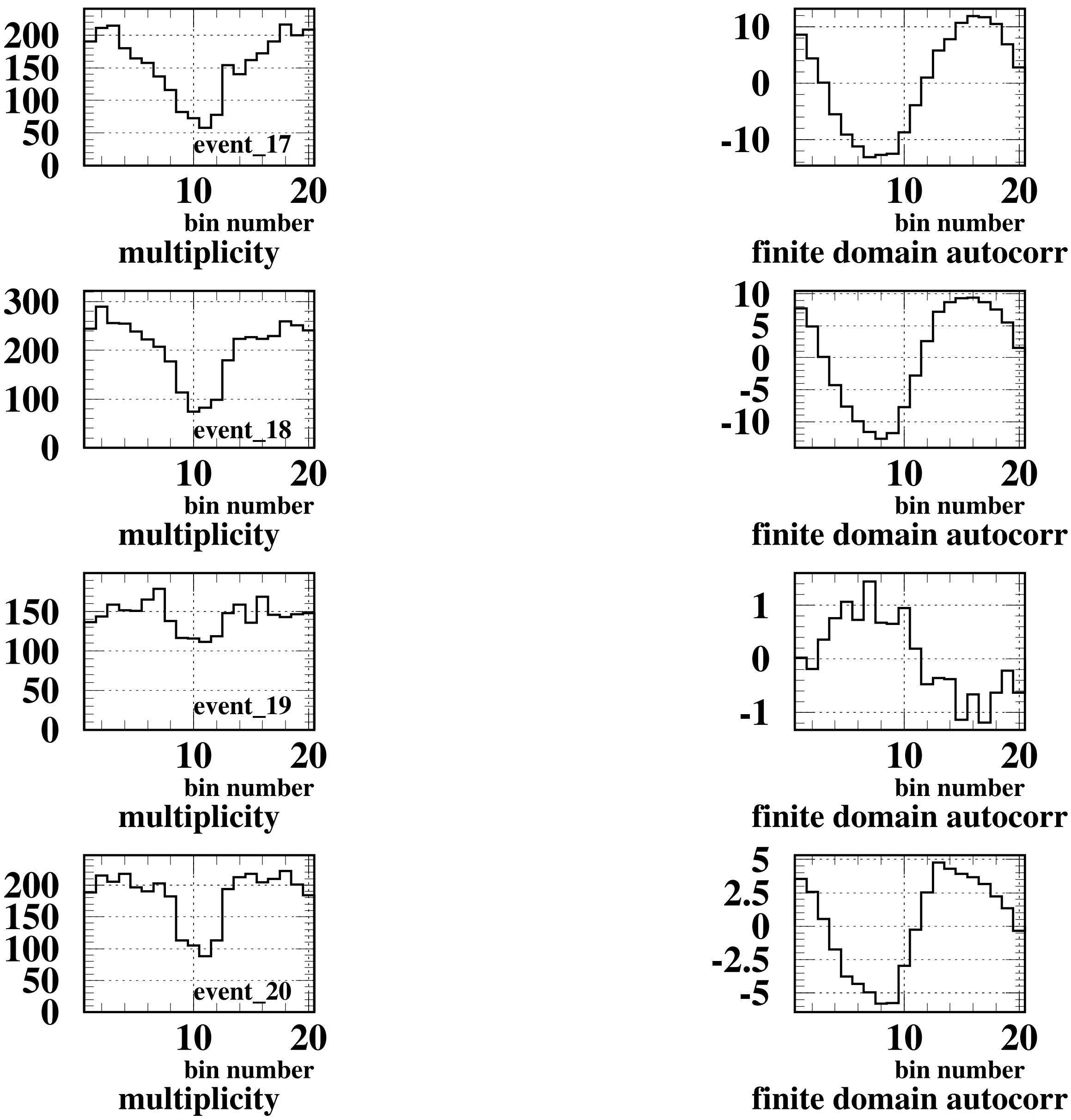

Figure 12: Multiplicity autocorrelations.

3.5 Power/Phase Densities

These figures correspond to the optimum null hypothesis for minimum power in the normal events. One can make detailed event-by-event comparisons with the 100-bin analysis. Note that the 'bandwidth' required to represent the differential autocorrelations is certainly no more than 5 bins in the Fourier transform, and usually no more than 2-3 bins. This is true regardless of the binning model for 12 or more bins. This reflects the lack of fine structure in the anomaly models.

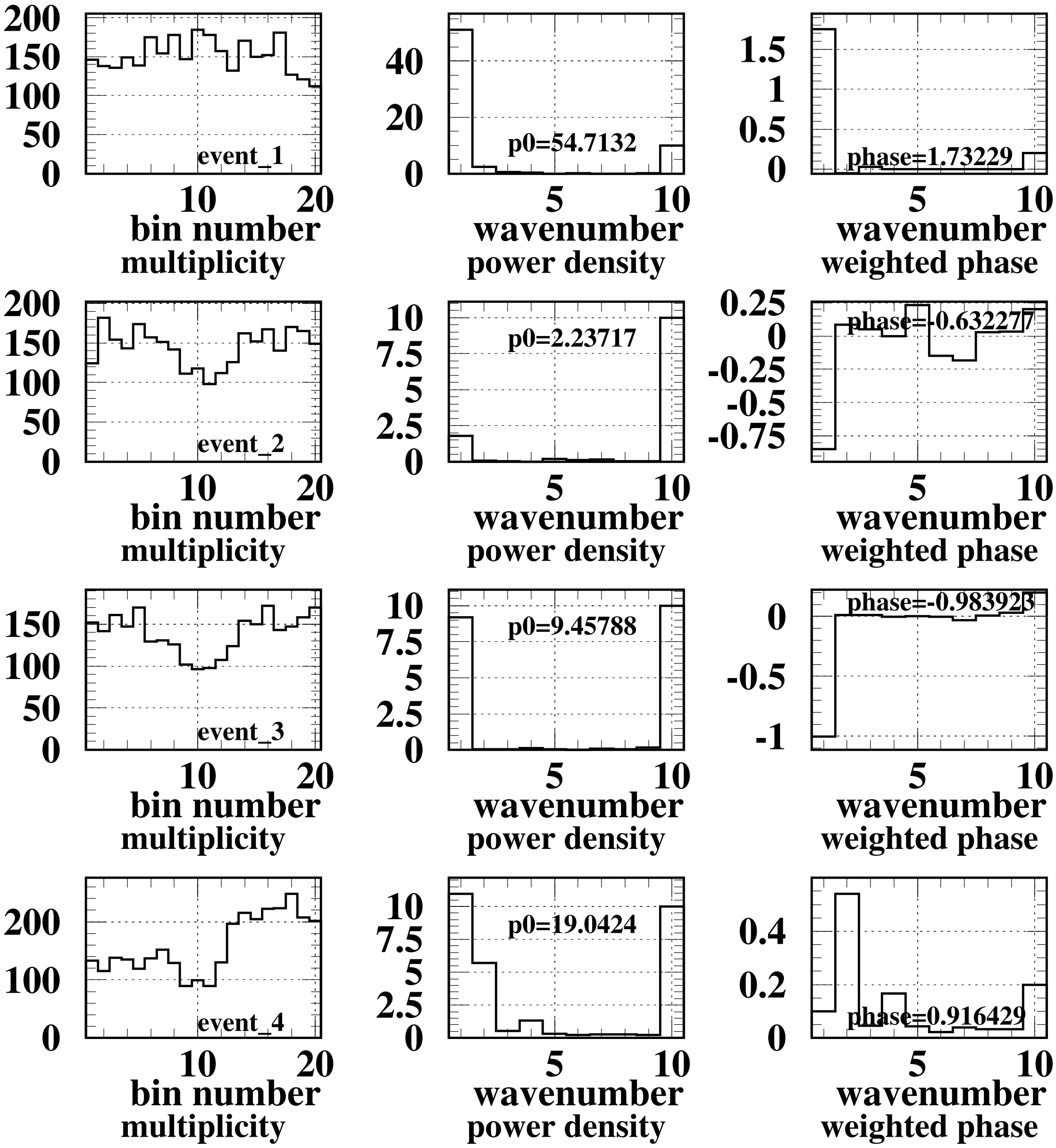

Figure 13: Power and phase spectra.

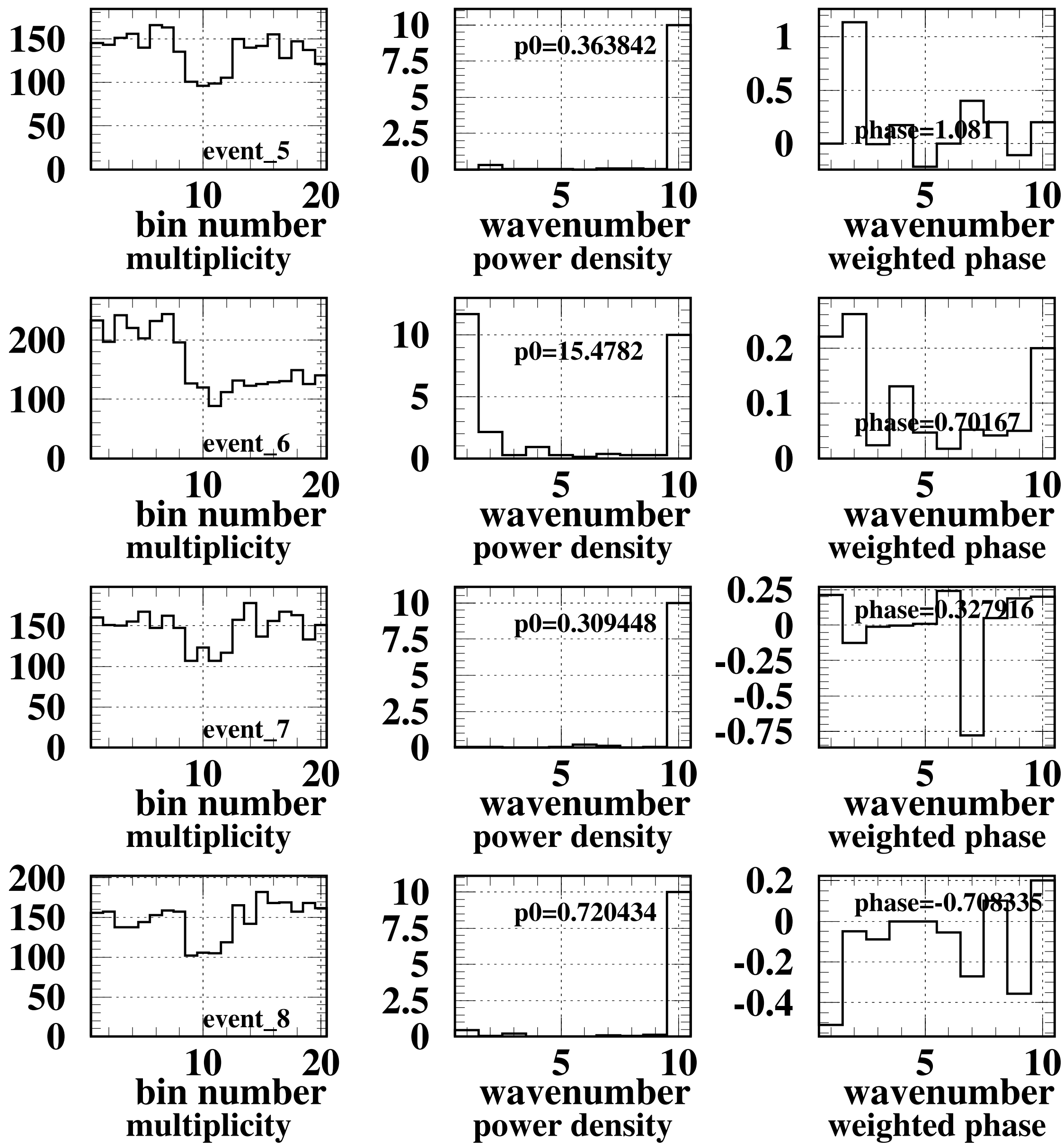

Figure 14: Power and phase spectra.

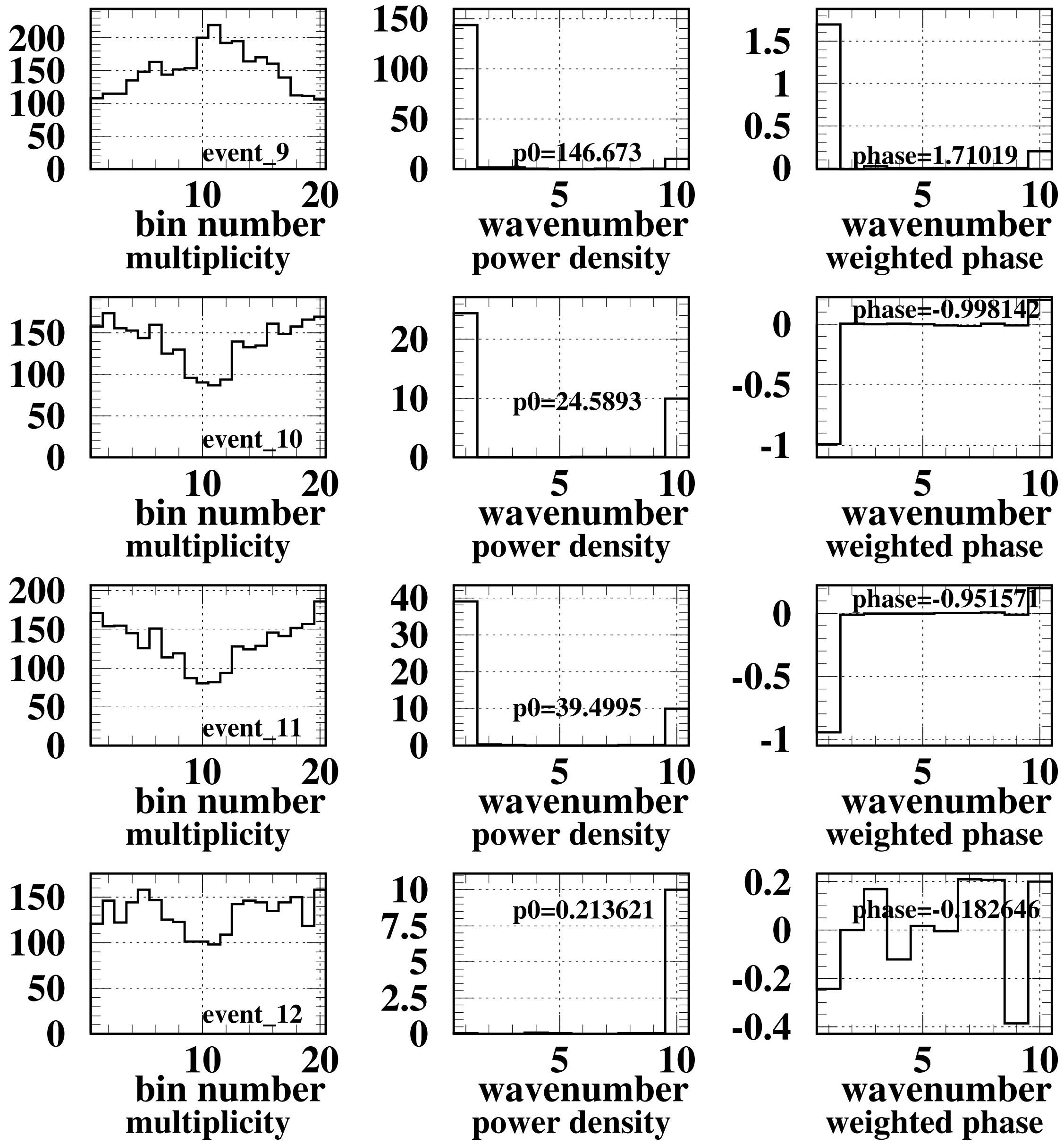

Figure 15: Power and phase spectra.

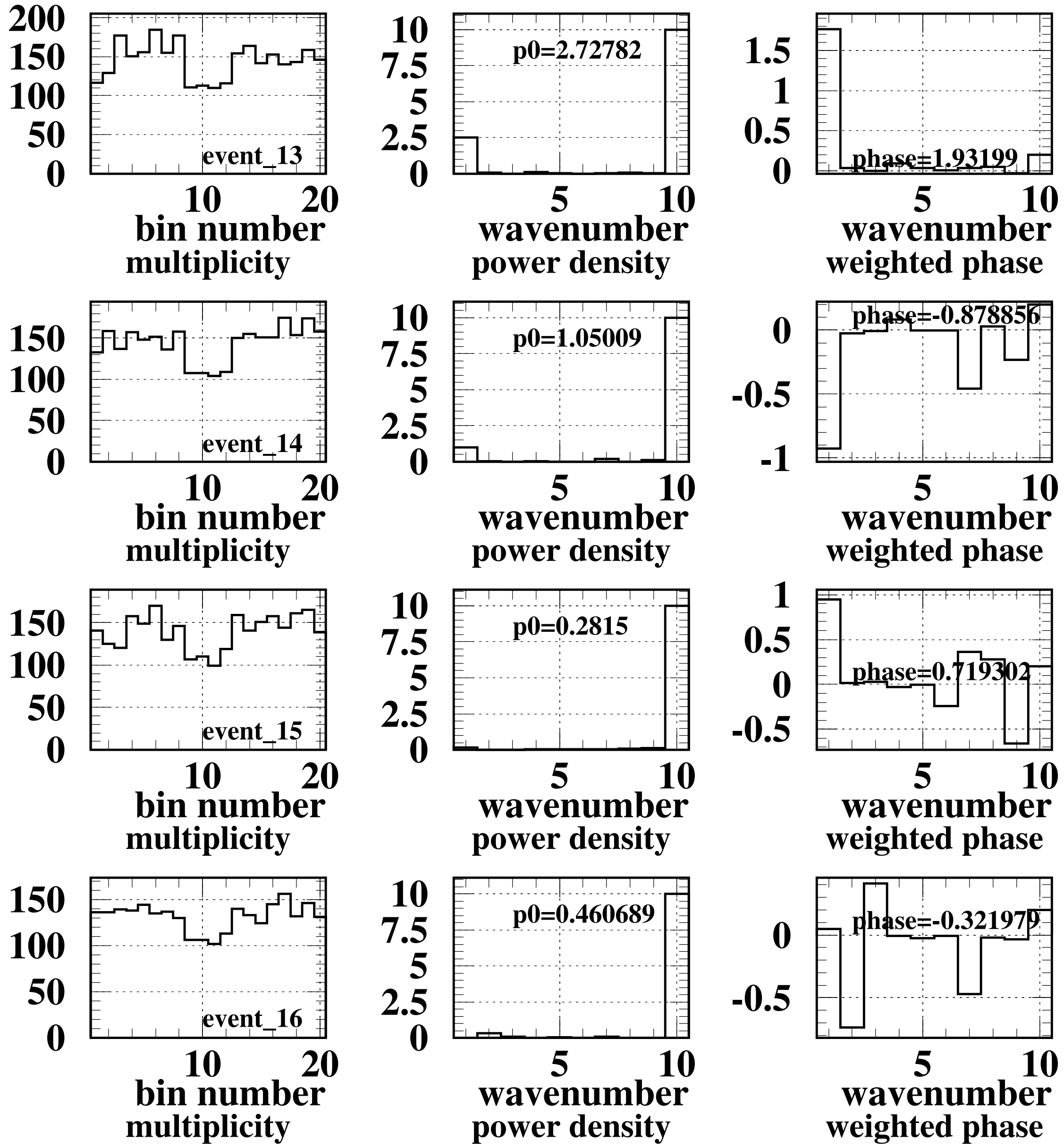

Figure 16: Power and phase spectra.

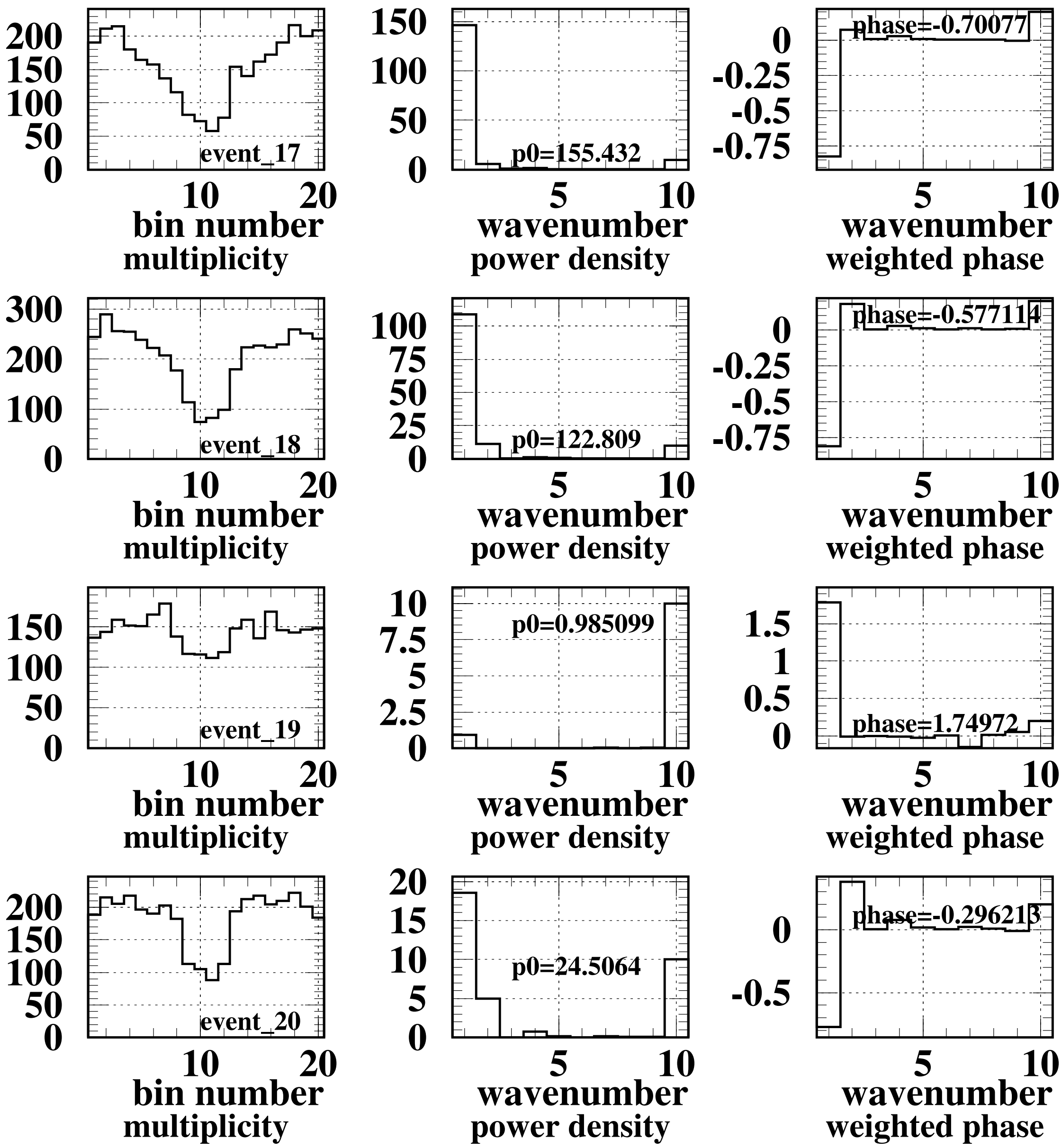

Figure 17: Power and phase spectra.

3.6 Pattern Spaces

Figure 18 shows pattern spaces for four different choices of null hypothesis for the 20-bin model. Note that by varying the null hypothesis the normal events can be moved to minimum power, or increased compactness, or maximum separation from anomalous events. In an on-line situation the system should be able to accommodate adjustment of the null hypothesis while events are analyzed to determine experimentally what is the optimum. The set of normal events should be considered a set of *training* events for the trigger analysis chain.

Figure 20 shows the 20-bin minimum power pattern space (upper left in Figure 18). Figure 23 shows the result of the analysis with the null hypothesis turned off for comparison. Obviously, the selection ability is greatly reduced, if not eliminated.

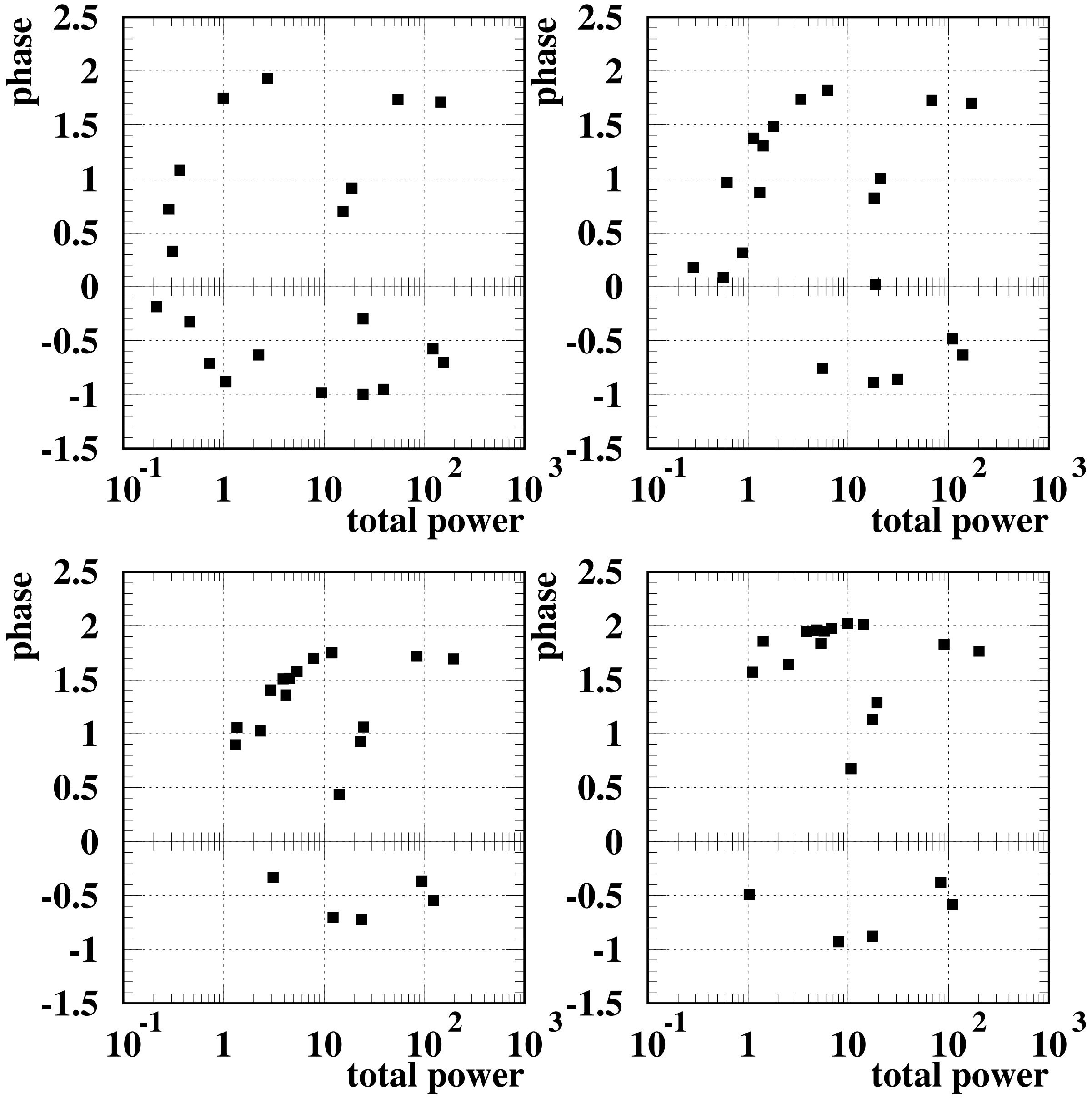

Figure 18: Pattern space with various choices of null hypothesis.

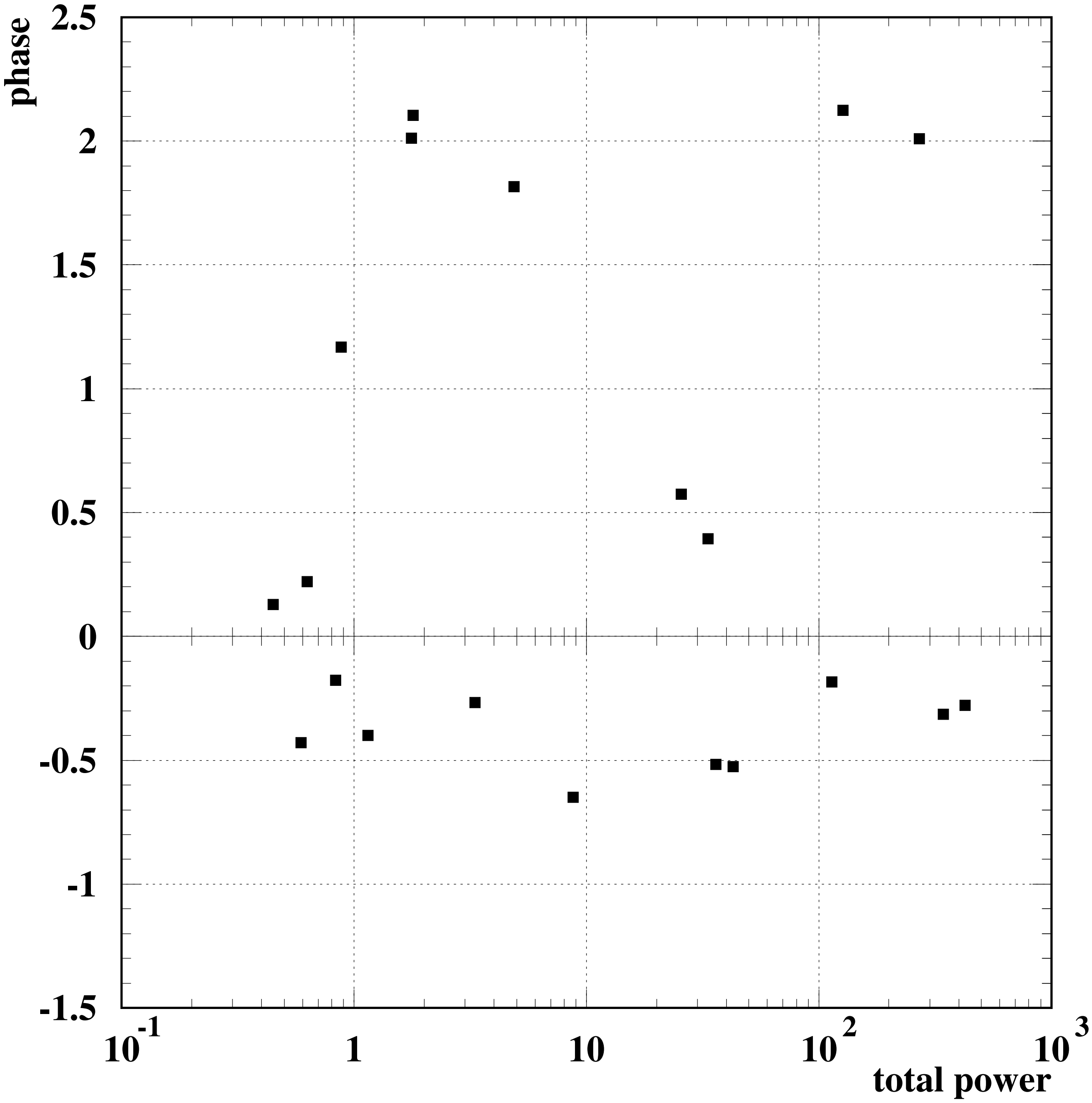

Figure 19: 12-bin pattern space with minimum power normal events.

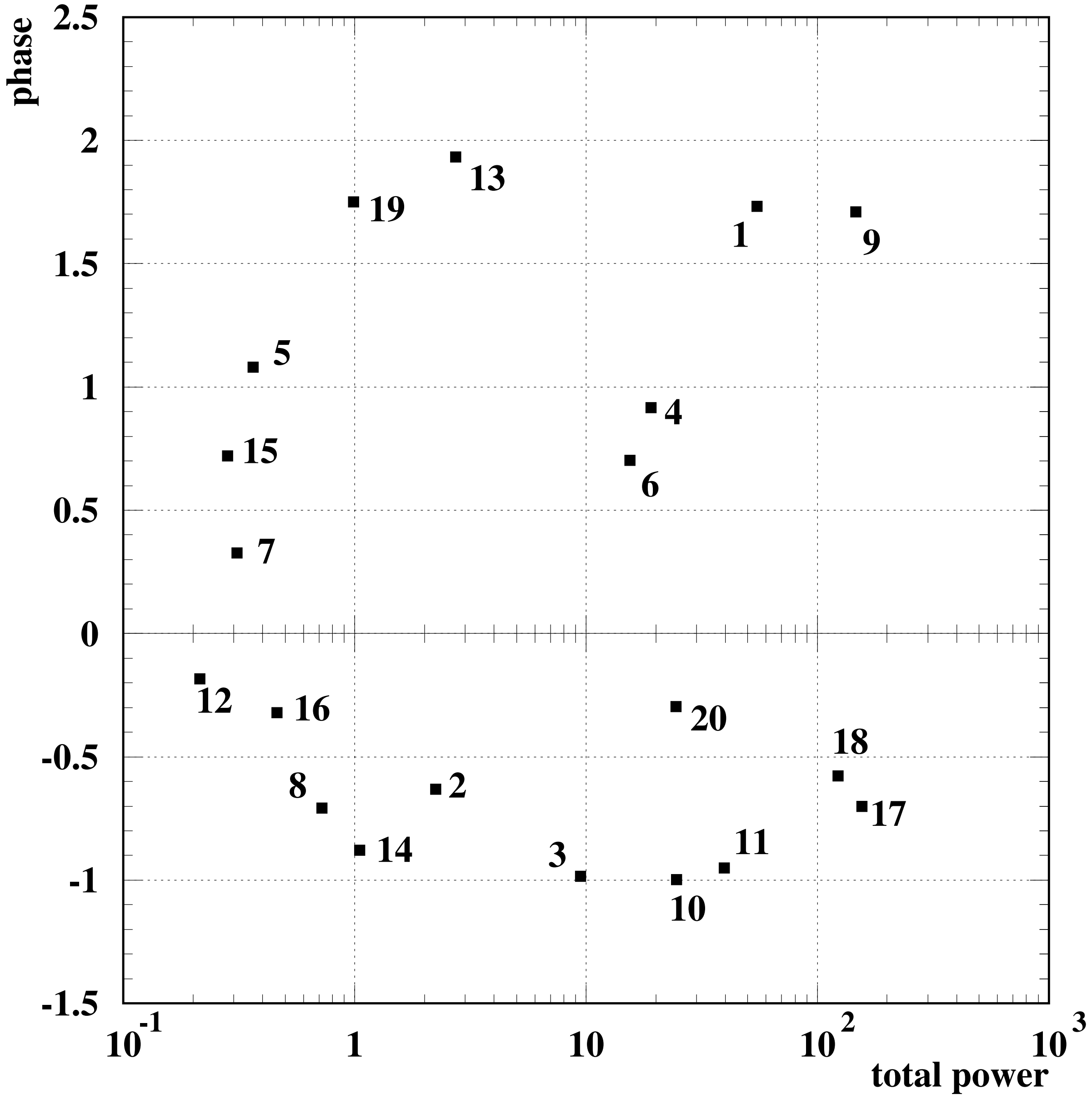

Figure 20: 20-bin pattern space with minimum power normal events.

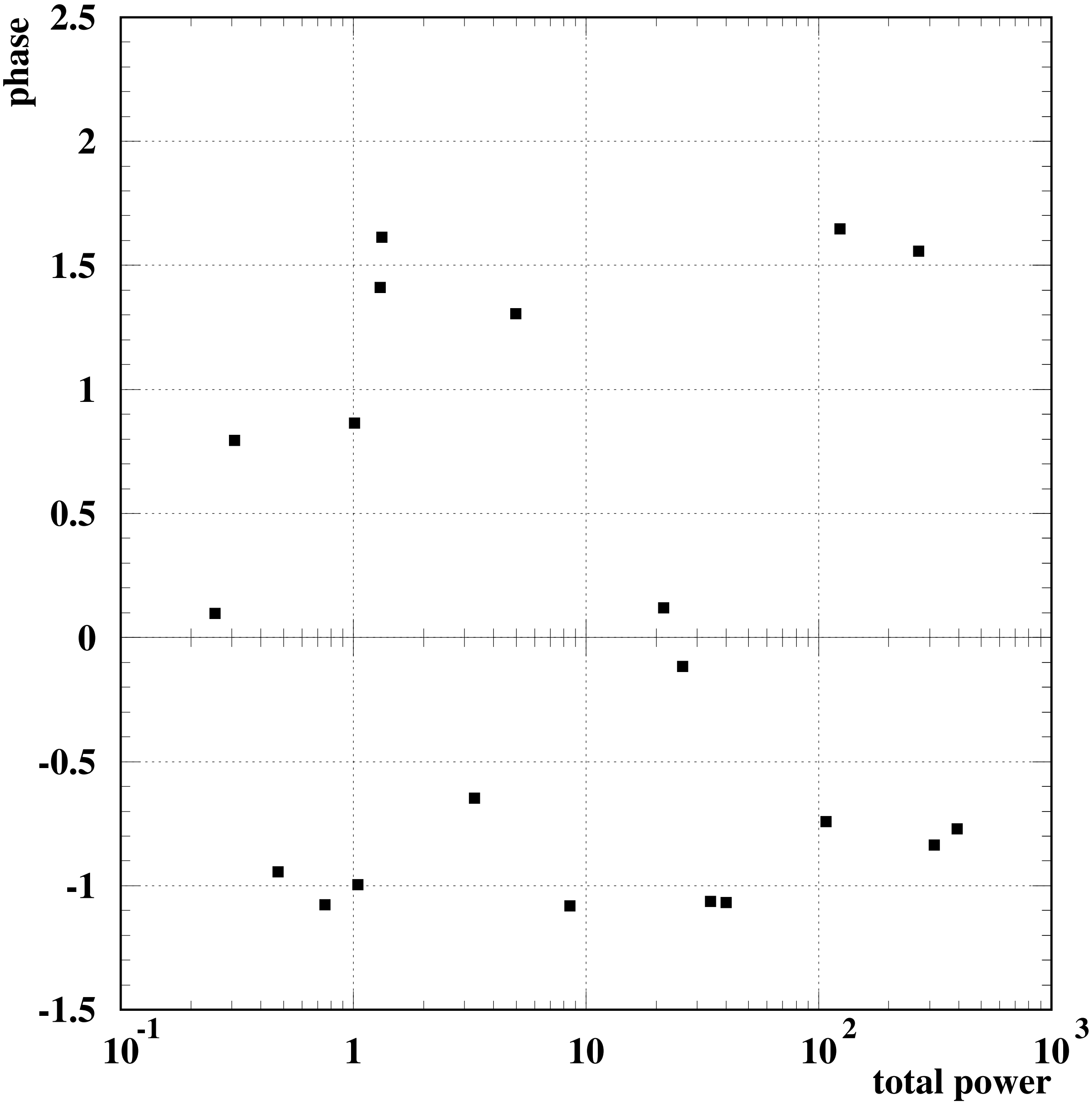

Figure 21: 100-bin pattern space with minimum power normal events.

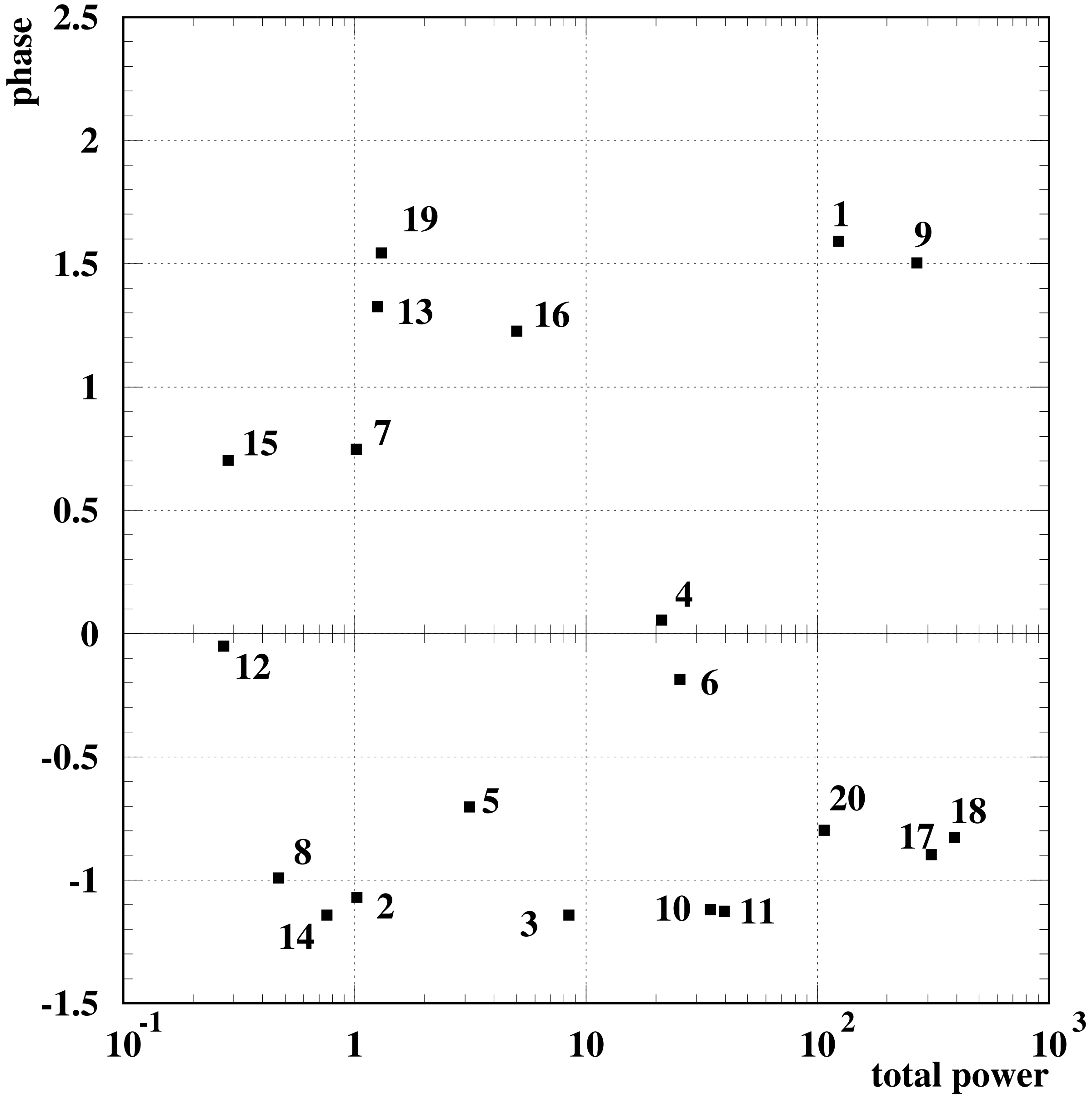

Figure 22: 500-bin pattern space with minimum power normal events.

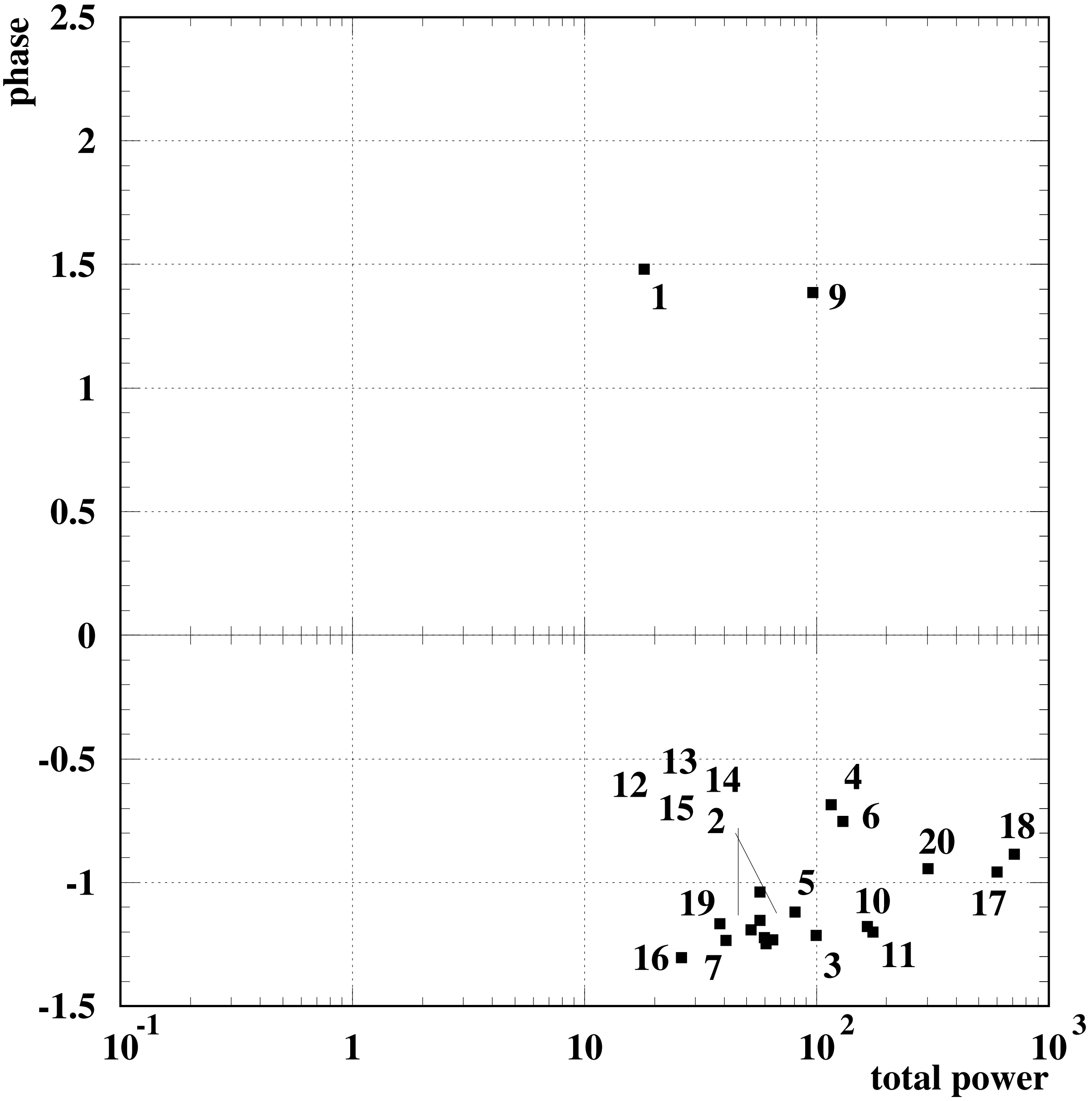

Figure 23: 100-bin pattern space with null hypothesis turned off.

4 Conclusion

By comparing the pattern space results for the various bin models we can conclude that reduction in bin number to as small as 12 bins does not significantly reduce the selection ability for this particular event set. This is perfectly understandable because the 'bandwidth' needed to represent these anomalous events is small compared to that available with even a 12-bin binning. So we just confirm here what one would conclude from a general power spectrum analysis of the additional information contained in the anomalous events.

We also see that the nonuniform bin size for the 20-bin model is easily accommodated by the null hypothesis technique, as was the high yield to the MWC segments in both analyses.

The limitation posed by low bin numbers (12, 20) is primarily the inability to sense structure in the multiplicity at scales below 0.2-0.4 units of η. For the level 1 and 2 triggers this may not be a problem, and is certainly not a problem with current models of anomalous stopping. Correlation structures at scales below this level can be determined either post upgrade with the finer resolution of the EM calorimeter at level 1 or 2, or at startup with the level 3 trigger when hit resolution approaches the two-track resolution of the TPC at the outer diameter. In any case the present study presents a favorable result for the baseline level 1 and 2 trigger detector system.