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# Reconstruction of decayed particles based on the Kalman filter

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**Abstract**

The note describes a reconstruction package for fitting decayed particles. The package is based on the Kalman filter. The Kalman filter procedure has been modified in order to operate with an extended model of measurements and to filter by an optimum measurement.

The reconstructed decayed particle contains all necessary information about the particle both at the point of its generation and at the point of its decay. Therefore, the developed method is suitable both for the complete reconstruction of decayed particles and for the reconstruction of vertices only.

Results of tests of the reconstruction package are presented and discussed.

## 1 Introduction

In modern high-energy physics experiments the most interesting physics is often extracted from the properties of (short lived) decayed particles, which are not detected by the detector system and has to be reconstructed from its daughter particles.

Usually, the existing reconstruction packages [1, 2, 3, 4, 5, 6, 7, 8] are only focused on reconstruction of the production and decay vertices without direct estimation of the parameters of the decayed particle itself.

The goal of this paper is to develop a method of reconstruction of the decayed particle parameters and associated covariance matrix using a set of daughter tracks estimates and their covariance matrices.

The developed algorithm is based on the Kalman filter method [1]. The standard Kalman filter approach has been modified in order to operate with an extended model of measurements and to filter by an optimum measurement.

The algorithm uses a natural particle parametrization $\mathbf{r} = (x, y, z, p_x, p_y, p_z, E, s)^T$, which makes the algorithm independent on geometry of the detector system. After estimation of the parameters of the particle, additional physics parameters, which are not explicitly included into the state vector, the particle momentum $P$, the invariant mass $M$, the length of flight $L$ in the laboratory coordinate system, and the time of life of the particle $cT$ in its own coordinate system, are easily calculated.

The algorithm has been successfully tested on simulated data of the CBM experiment [9, 10].

## 2 The Kalman filter method

The Kalman filter method [1] is intended for finding the optimum estimation $\mathbf{r}$ of an unknown state vector $\mathbf{r}^t$ according to the measurements $\mathbf{m}_k,\ k = 1 \ldots n$ of the vector $\mathbf{r}^t$.

The Kalman filter starts with a certain initial approximation $\mathbf{r} = \mathbf{r}_0$ and refines the vector $\mathbf{r}$, consecutively adding one measurement after the other. The optimum value is obtained after the addition of the last measurement.

In the general case the unknown vector $\mathbf{r}^t$ can change from one measurement to the next:

$$\mathbf{r}^t_k = A_k \mathbf{r}^t_{k-1} + \boldsymbol{\nu}_k\,, \tag{1}$$

where $A_k$ — a linear opperator, $\boldsymbol{\nu}_k$ — a process noise between $(k-1)$-th and $k$-th measurements.

The measurement $\mathbf{m}_k$ linearly depends on $\mathbf{r}^t_k$:

$$\mathbf{m}_k = H_k \mathbf{r}^t_k + \boldsymbol{\eta}_k\,, \tag{2}$$

where $\boldsymbol{\eta}_k$ — an error of the $k$-th measurement.

It is assumed that measurement errors $\boldsymbol{\eta}_i$ and the process noise $\boldsymbol{\nu}_j$ are uncorrelated, unbiased ($<\boldsymbol{\eta}_i>=<\boldsymbol{\nu}_j>=\mathbf{0}$) and those covariance matrices $V_k$, $Q_k$ are known:

$$\begin{aligned} <\boldsymbol{\eta}_i \cdot \boldsymbol{\eta}_i^T> &\equiv V_i\,, \\ <\boldsymbol{\nu}_j \cdot \boldsymbol{\nu}_j^T> &\equiv Q_j\,. \end{aligned} \tag{3}$$

The algorithm of fitting consists of four stages:

1. Initialization step. Choose an approximate value of vector $\mathbf{r}_0$. Its covariance matrix is set to $C_0 = \mathrm{I} \cdot \inf^2$, where inf denotes a large positive number.

2. Prediction step. If it is known that the original vector $\mathbf{r}^t$ changes between $(k-1)$-th and $k$-th measurement (Eq.1), then upon transfer to the $k$-th measurement its current estimation $\mathbf{r}_{k-1}$ also changes in the same manner:

$$\begin{aligned} \widetilde{\mathbf{r}}_k &= A_k \mathbf{r}_{k-1}\,, \\ \widetilde{C}_k &= A_k C_{k-1} A_k^T + Q_k\,, \end{aligned} \tag{4}$$

   where $\widetilde{\mathbf{r}}_k$ — an optimal estimation of the vector $\mathbf{r}^t_k$ according to the first $k-1$ measurements. In contrast to the prediction operator $A_k$, describing deterministic changes of the vector $\mathbf{r}^t$ in time, the process noise $Q_k$ describes probabilistic deviations of the vector $\mathbf{r}^t$.

3. Filtration step. For each measurement $\mathbf{m}_k$ a vector $\mathbf{r}_k$, which is the optimum estimation of the vector $\mathbf{r}^t_k$ according to the first $k$ measurements, is calculated:

$$\begin{aligned} K_k &= \widetilde{C}_k H_k^T (V_k + H_k \widetilde{C}_k H_k^T)^{-1}\,, \\ \mathbf{r}_k &= \widetilde{\mathbf{r}}_k + K_k(\mathbf{m}_k - H_k \widetilde{\mathbf{r}}_k)\,, \\ C_k &= \widetilde{C}_k - K_k H_k \widetilde{C}_k\,, \\ \chi^2_k &= \chi^2_{k-1} + (\mathbf{m}_k - H_k \widetilde{\mathbf{r}}_k)^T (V_k + H_k \widetilde{C}_k H_k^T)^{-1} (\mathbf{m}_k - H_k \widetilde{\mathbf{r}}_k)\,. \end{aligned} \tag{5}$$

Here $\widetilde{\mathbf{r}}_k$, $\widetilde{C}_k$ — the optimum estimation, and its covariance matrix, obtained at the previous step and extrapolated to the $k$-th measurement; $\mathbf{m}_k$, $V_k$ — the $k$-th measurement and its covariance matrix; $\boldsymbol{\zeta}_k$ — residual; the matrix $H_k$ — the model of the measurement; the matrix $K_k$ is the so-called gain matrix; the value $\chi^2_k$ is the total $\chi^2$-deviation of the obtained estimation $\mathbf{r}_k$ from the measurements $\mathbf{m}_1, \ldots \mathbf{m}_k$.

The vector $\mathbf{r}_n$ obtained after the filtration of the last measurement is the desired optimal estimation with the covariance matrix $C_n$.

In the case the measurements $\mathbf{m}_k$ non-linearly depend on $\mathbf{r}^t_k$, it is necessary to linearize the model of measurement. As a point of linearization a certain vector $\mathbf{r}^0_k$ is taken:

$$\mathbf{m}_k(\mathbf{r}^t_k) \;=\; \mathbf{h}_k(\mathbf{r}^t_k) + \boldsymbol{\eta}_k \;\approx\; \mathbf{h}_k(\mathbf{r}^0_k) + H_k(\mathbf{r}^t_k - \mathbf{r}^0_k) + \boldsymbol{\eta}_k\,, \tag{6}$$

where $H_k$ is the Jacobian of $\mathbf{h}_k(\mathbf{r}_k)$ at $\mathbf{r}^0_k$:

$$H_{k\,(ij)} \;=\; \left.\frac{\partial \mathbf{h}_k(\mathbf{r}_k)_{\,(i)}}{\partial \mathbf{r}_{k\ (j)}}\right|_{\mathbf{r}_k=\mathbf{r}^0_k}. \tag{7}$$

The Kalman filter with non-linear measurement model is called the extended Kalman filter. Equations of filtration for the extended Kalman filter are the same as in the linear case (5) with an exception for the residual $\boldsymbol{\zeta}_k$, which is calculated according to the formula:

$$\boldsymbol{\zeta}_k \;=\; \mathbf{m}_k - \left(\mathbf{h}_k\left(\mathbf{r}^0_k\right) + H_k\left(\widetilde{\mathbf{r}}_k - \mathbf{r}^0_k\right)\right). \tag{8}$$

The result of the non-linear fit depends on the point of linearization $\mathbf{r}^0_k$. Taking the current solution $\widetilde{\mathbf{r}}_k$ as the point of linearization for $k$-th measurement, the extended Kalman filter (8) coincides with the usual one (5). But in order to get more robust and precise results, the fitting procedure must be repeated several times, using the optimal estimation $\mathbf{r}_n$ as the linearization point for all the measurements on the next iteration.

In this work two modifications of the conventional filtration procedure will be used: filtration with an extended model of measurement and filtration by an optimum measurement.

## 2.1 Filtration with an extended model of measurement

Here we extend the standard equations of filtration (5) for the case of a more general model of measurement.

Let the measurement $\mathbf{m}_k$ (2) is related to the state vector with a more general equation:

$$G_k\mathbf{m}^t_k \;=\; H_k\mathbf{r}^t_k\,, \tag{9}$$

where $\mathbf{m}^t_k$— the true value of the measurement.

Let us show that for the extended model of measurement (9) the equations of filtration (5) have the following form:

$$\begin{aligned}
K_k &= \widetilde{C}_k H_k^T\left(H_k\widetilde{C}_k H_k^T + G_k V_k G_k^T\right)^{-1},\\
\boldsymbol{\zeta}_k &= G_k\mathbf{m}_k - H_k\widetilde{\mathbf{r}}_k\,,\\
\mathbf{r}_k &= \widetilde{\mathbf{r}}_k + K_k\cdot\boldsymbol{\zeta}_k\,,\\
C_k &= \widetilde{C}_k - K_k H_k\widetilde{C}_k\,,\\
\chi^2_k &= \chi^2_{k-1} + \boldsymbol{\zeta}_k^T\cdot\left(H_k\widetilde{C}_k H_k^T + G_k V_k G_k^T\right)^{-1}\cdot\boldsymbol{\zeta}_k\,.
\end{aligned} \tag{10}$$

One can see that in the case of the standard measurement model ($G_k \equiv \mathrm{I}$) Eqs. (10) coincide with the conventional Kalman filter (5).

The equations of filtration (10) estimate the optimum value $\mathbf{r}_k$ of the state vector from all previous measurements and the measurement $\mathbf{m}_k$. However, the measurement $\mathbf{m}_k$ itself is random variable, and its optimum value $\mathbf{m}_k^f$ can also be estimated from all previous measurements:

$$\begin{aligned} K_k^m &= V_k G_k^T \left( H_k \widetilde{C}_k H_k^T + G_k V_k G_k^T \right)^{-1} , \\ \mathbf{m}_k^f &= \mathbf{m}_k - K_k^m \cdot \boldsymbol{\zeta} , \\ V_k^f &= V_k - K_k^m G_k V_k , \\ D_k^f &= K_k^m H_k \widetilde{C}_k , \end{aligned} \tag{11}$$

where $K_k^m$ — the measurement gain matrix; $\mathbf{m}_k^f$, $V_k^f$— the optimum values of the measurement $\mathbf{m}_k$ and its covariance matrix; $D_k^f$ — the matrix of covariances between $\mathbf{m}_k^f$ and $\mathbf{r}_k$:

$$D^f_{k\,(i,j)} = \mathrm{cov}(\mathbf{m}^f_{k\,(i)}, \mathbf{r}_{k\,(j)}) . \tag{12}$$

To proof the Eqs. (10, 11) let us group the state vector $\widetilde{\mathbf{r}}_k$ and the measurement $\mathbf{m}_k$ into a combined state vector $\widehat{\mathbf{r}}_k$:

$$\begin{aligned} \widehat{\mathbf{r}}_k &= \begin{pmatrix} \widetilde{\mathbf{r}}_k \\ \mathbf{m}_k \end{pmatrix} , \\ \widehat{C}_k &= \begin{pmatrix} \widetilde{C}_k & \mathrm{O} \\ \mathrm{O} & V_k \end{pmatrix} , \end{aligned} \tag{13}$$

and make an update of the combined state vector $\widehat{\mathbf{r}}_k$ according to the measurement model (9). From (9) follows:

$$\mathbf{0} = H_k \mathbf{r}_k^t - G_k \mathbf{m}_k^t \;\equiv\; (H_k,\; -G_k) \cdot \widehat{\mathbf{r}}_k^t . \tag{14}$$

Therefore, the update of the state vector $\widetilde{\mathbf{r}}_k$ by the measurement $\mathbf{m}_k$ is equivalent to the update of the combined state vector $\widehat{\mathbf{r}}_k$ by a zero measurement $\mathbf{0}$ with the null matrix of errors and the measurement model $\widehat{H}_k$:

$$\begin{aligned} \widehat{H}_k &= (H_k,\; -G_k) , \\ \mathbf{0} &= \widehat{H}_k \cdot \widehat{\mathbf{r}}_k^t . \end{aligned} \tag{15}$$

Now the combined state vector $\widehat{\mathbf{r}}_k$ with the measurement (15) can be updated by the conventional Kalman filter (5):

$$\begin{aligned} \widehat{K}_k &= \widehat{C}_k \widehat{H}_k^T \left( \mathrm{O} + \widehat{H}_k \widehat{C}_k \widehat{H}_k^T \right)^{-1} = \begin{pmatrix} K_k \\ -K_k^m \end{pmatrix} , \\ \boldsymbol{\zeta}_k &= \mathbf{0} - \widehat{H}_k \widehat{\mathbf{r}}_k \;=\; G_k \mathbf{m}_k - H_k \widetilde{\mathbf{r}}_k , \\ \begin{pmatrix} \mathbf{r}_k \\ \mathbf{m}_k^f \end{pmatrix} &= \widehat{\mathbf{r}}_k + \widehat{K}_k \boldsymbol{\zeta}_k \;=\; \begin{pmatrix} \widetilde{\mathbf{r}} + K_k \boldsymbol{\zeta}_k \\ \mathbf{m}_k - K_k^m \boldsymbol{\zeta}_k \end{pmatrix} , \\ \begin{pmatrix} C_k & D_k^{f\,T} \\ D_k^f & V_k^f \end{pmatrix} &= \widehat{C}_k - \widehat{K}_k \widehat{H}_k \widehat{C}_k \;=\; \begin{pmatrix} \widetilde{C}_k & \mathrm{O} \\ \mathrm{O} & V_k \end{pmatrix} - \begin{pmatrix} K_k \\ -K_k^m \end{pmatrix} \begin{pmatrix} H_k \widetilde{C}_k, & -G_k V_k \end{pmatrix} \\ &= \begin{pmatrix} \widetilde{C}_k - K_k H_k \widetilde{C}_k & \widetilde{C}_k H_k^T K_k^{m\,T} \\ K_k^m H_k \widetilde{C}_k & V_k - K_k^m G_k V_k \end{pmatrix} , \\ \chi_k^2 &= \chi_{k-1}^2 + \boldsymbol{\zeta}_k^T \left( \mathrm{O} + \widehat{H}_k \widehat{C}_k \widehat{H}_k^T \right)^{-1} \boldsymbol{\zeta}_k \\ &= \chi_{k-1}^2 + \boldsymbol{\zeta}_k^T \left( H_k \widetilde{C}_k H_k^T + G_k V_k G_k^T \right)^{-1} \boldsymbol{\zeta}_k . \end{aligned} \tag{16}$$

After taking the parts corresponding to the state vector $\mathbf{r}_k$ and the measurement $\mathbf{m}_k^f$ from $\widehat{\mathbf{r}}_k$ and $\widehat{C}_k$, we obtain the optimum estimation $\mathbf{r}_k$, $C_k$ of the vector $\mathbf{r}_k^t$ (Eqs. 10) and the optimum estimation $\mathbf{m}_k^f$, $V_k^f$ of the measurement $\mathbf{m}_k^t$ (Eqs. 11) from the first $k$ measurements.

In the case of a non-linear model of measurement, similarly to the conventional Kalman filter (8), only the residual $\boldsymbol{\zeta}_k$ is changed:

$$\boldsymbol{\zeta}_k = \mathbf{g}_k(\mathbf{m}_k^0) + G_k(\mathbf{m}_k - \mathbf{m}_k^0) - (\mathbf{h}_k(\mathbf{r}_k^0) + H_k(\mathbf{r}_k - \mathbf{r}_k^0)) \,. \tag{17}$$

## 2.2 Filtration of the state vector by an optimum measurement

Here we construct the optimum estimation $\mathbf{r}_k$, when it is more convenient to operate not with the measurement $\mathbf{m}_k$ itself, but with its optimum[1] value $\mathbf{m}_k^f$. In this case the equation of filtration (10) are transformed such that $\mathbf{r}_k \,, C_k \,, \chi^2_k$ are expressed using the optimum values $\mathbf{m}_k^f, V_k^f$:

$$\begin{array}{lcl} K_k^f &=& \widetilde{C}_k H_k^T \left(H_k \widetilde{C}_k H_k^T\right)^{-1} \,, \\ \boldsymbol{\zeta}_k^f &=& G_k \mathbf{m}_k^f - H_k \widetilde{\mathbf{r}}_k \,, \\ \mathbf{r}_k &=& \widetilde{\mathbf{r}}_k + K_k^f \,\boldsymbol{\zeta}_k^f \,, \\ C_k &=& \widetilde{C}_k - K_k^f \left(H_k \widetilde{C}_k H_k^T - G_k V_k^f G_k^T\right) K_k^{f\,T} \,, \\ \chi^2_k &=& \chi^2_{k-1} + \boldsymbol{\zeta}_k^{f\,T} \left(H_k \widetilde{C}_k H_k^T - G_k V_k^f G_k^T\right)^{-1} \boldsymbol{\zeta}_k^f \,. \end{array} \tag{18}$$

To proof Eqs. (18) let us introduce several temporary matrices:

$$\begin{array}{lcl} A &=& H_k \widetilde{C}_k H_k^T \,, \\ B &=& G_k V_k G_k^T \,, \\ S &=& (A+B)^{-1} \,. \end{array} \tag{19}$$

To exclude $\mathbf{m}_k, V_k$ from Eqs. (10) it is sufficient to express $S$ and $\boldsymbol{\zeta}_k$ using the optimum values $\mathbf{m}_k^f, V_k^f$. Transforming the expressions for $G_k \mathbf{m}_k^f$ and $G_k V_k^f G_k^T$ from Eqs. (11):

$$\begin{array}{lcl} G_k \mathbf{m}_k^f &=& G_k \mathbf{m}_k - B\,(A+B)^{-1} \boldsymbol{\zeta}_k \\ &=& G_k \mathbf{m}_k - (\mathrm{I} - A\,(A+B)^{-1}) \cdot (G_k \mathbf{m}_k - H_k \widetilde{\mathbf{r}}_k) \\ &=& H_k \widetilde{\mathbf{r}}_k + AS \cdot \boldsymbol{\zeta}_k \,, \end{array} \tag{20}$$

$$G_k V_k^f G_k^T = B - B\,(A+B)^{-1} B = A - A\,(A+B)^{-1} A = A - ASA \,,$$

we obtain the expressions for $\boldsymbol{\zeta}_k$ and $S$:

$$\begin{array}{lcl} \boldsymbol{\zeta}_k &=& S^{-1} A^{-1} \left(G_k \mathbf{m}_k^f - H_k \widetilde{\mathbf{r}}_k\right) \,, \\ S &=& A^{-1} \left(A - G_k V_k^f G_k^T\right) A^{-1} \,. \end{array} \tag{21}$$

Now we substitute $\boldsymbol{\zeta}_k$ and $S$ from (21) into (10):

$$\begin{array}{lcl} \mathbf{r}_k &=& \widetilde{\mathbf{r}}_k + \widetilde{C}_k H_k^T S \boldsymbol{\zeta}_k \;=\; \widetilde{\mathbf{r}}_k + \widetilde{C}_k H_k^T A^{-1} \left(G_k \mathbf{m}_k^f - H_k \widetilde{\mathbf{r}}_k\right) \\ C_k &=& \widetilde{C}_k - \widetilde{C}_k H_k^T S H_k \widetilde{C}_k \;=\; \widetilde{C}_k - \widetilde{C}_k H_k^T A^{-1} \left(A - G_k V_k^f G_k^T\right) A^{-1} H_k \widetilde{C}_k \,, \\ \chi^2_k &=& \chi^2_{k-1} + \boldsymbol{\zeta}_k^T S \boldsymbol{\zeta}_k \;=\; \left(G_k \mathbf{m}_k^f - H_k \widetilde{\mathbf{r}}_k\right)^T A^{-1} (A+B)\, A^{-1} \left(G_k \mathbf{m}_k^f - H_k \widetilde{\mathbf{r}}_k\right) \\ &=& \left(G_k \mathbf{m}_k^f - H_k \widetilde{\mathbf{r}}_k\right)^T \left(A - G_k V_k^f G_k^T\right)^{-1} \left(\mathbf{m}_k^f - H_k \widetilde{\mathbf{r}}_k\right) \,. \end{array} \tag{22}$$

[1]Such problem appears, for instance, during fitting a particle to the already reconstructed vertex.

After introducing a matrix $K_k^f$ and a vector $\boldsymbol{\zeta}_k^f$ (by analogy to $K_k$ and $\boldsymbol{\zeta}_k$ in Eqs. (10)) and substituting them into Eqs. (22), we obtain the required equations of filtration (18).

## 3 Construction of the mother particle at the decay vertex

The first task in the reconstruction of the mother particle is determination by the modified Kalman filter (10) of its position, momentum and energy at the decay point, using the estimates of the daughter particles obtained after the track fit.

Let the mother particle decayed into $n$ daughter particles. We arrange the parameters of the mother particle reconstructed from the first $k$ daughter particles in a 7-dimensional state vector $\mathbf{r}_k$:

$$\mathbf{r}_k \equiv (x,\, y,\, z,\, p_x,\, p_y,\, p_z,\, E)^T \;, \tag{23}$$

or

$$\mathbf{r}_k \equiv \begin{pmatrix} \mathbf{v}_k \\ \underline{\mathbf{p}}_k \end{pmatrix} \,, \tag{24}$$

where $\mathbf{v}_k$ — the coordinate of the particle at the decay point, and $\underline{\mathbf{p}}_k$ — its 4-momentum. Let us denote the covariance matrix of the state vector as $C_k$ and the assumed position of the decay point, used for the linearization of equations, as $\mathbf{v}^0$. Let us transport all daughter particles into the region of $\mathbf{v}^0$.

Let the parameters of the $k$-th daughter particle are denoted $\mathbf{r}_k^d$

$$\mathbf{r}_k^d \equiv \begin{pmatrix} \mathbf{v}_k^d \\ \underline{\mathbf{p}}_k^d \end{pmatrix} \tag{25}$$

and the covariance matrix $C_k^d$.

For measuring the mother particle it is necessary to transport a daughter particle along its trajectory into the decay point. The parameters of a daughter particle at the decay point are $\mathbf{m}_k$:

$$\begin{aligned} \mathbf{m}_k &= \mathbf{r}_k^d + \begin{pmatrix} \mathbf{p}_k^d \\ \mathbf{p}_k^d \times B \cdot q_k \\ 0 \end{pmatrix} \cdot s_k^d + O\left(s_k^{d\,2}\right) \,, \\ < s_k^d > &= 0\,, \\ \sigma^2_{s_k^d} &= \inf\,, \end{aligned} \tag{26}$$

where $s_k^d = l_k^d/p_k^d$ — the unknown length of the trajectory $l_k^d$ from the parametrization point of the daughter particle $\mathbf{v}_k^d$ to the decay point $\mathbf{v}_k$, normalized by the momentum of the daughter particle; $\sigma^2_{s_k^d}$ — the error of the parameter $s_k^d$; $B$— the magnetic field value at the point $\mathbf{v}_k^d$; $q_k$ — charge of the daughter particle; the term $O\left(s_k^{d\,2}\right)$ describes the higher order deviations of the daughter particle trajectory from a straight line in a magnetic field (see details in [11]).

Linearizing (26) at $s_k^d = 0$, we obtain the measurement of the daughter particle parameters at the decay point:

$$\begin{aligned} \mathbf{m}_k &= \mathbf{r}_k^d\,, \\ V_k &= C_k^d + \begin{pmatrix} \mathbf{p}^d \\ \mathbf{p}^d \times B \cdot q_k \\ 0 \end{pmatrix} \begin{pmatrix} \mathbf{p}^d \\ \mathbf{p}^d \times B \cdot q_k \\ 0 \end{pmatrix}^T \cdot \sigma^2_{s_k^d}\,, \end{aligned} \tag{27}$$

where $V_k$ — the covariance matrix of the daughter particle parameters at the decay point.

The mother and daughter particles are related to each other:

$$(I,\mathrm{O})\,\mathbf{m}_k^t \;=\; (I,\mathrm{O})\,\mathbf{r}_{k-1}^t\,, \tag{28}$$

which is filtered by the modified Kalman filter (10) substituting:

$$\begin{array}{lcl} \widetilde{\mathbf{r}}_k & \equiv & \mathbf{r}_{k-1}\,, \\ \widetilde{C}_k & \equiv & C_{k-1}\,, \\ \mathbf{m}_k & \equiv & \mathbf{r}_k^d\,, \\ G_k & = & H_k \;\equiv\; (I,\mathrm{O})\;. \\ \mathbf{r}_k & \equiv & \mathbf{r}_{k-1}^f\,, \\ C_k & \equiv & C_{k-1}^f\,. \end{array} \tag{29}$$

Let us give the equations of filtration in detail. In order to simplify the calculations, the covariance matrices is split into the coordinate and momentum parts:

$$C_{k-1} \;\equiv\; \begin{pmatrix} C_{k-1}^v & C_{k-1}^{vp\,T} \\ C_{k-1}^{vp} & C_{k-1}^p \end{pmatrix}, \qquad V_k \;\equiv\; \begin{pmatrix} V_k^v & V_k^{vp\,T} \\ V_k^{vp} & V_k^p \end{pmatrix} \tag{30}$$

and also a temporary matrix $S_k$ is introduced:

$$S_k \;=\; \left(C_{k-1}^v + V_k^v\right)^{-1}\,. \tag{31}$$

In these notations the equations of filtration (10, 11) can be written as:

$$\begin{array}{lcl} K_k & = & \begin{pmatrix} C_{k-1}^v \\ C_{k-1}^{vp} \end{pmatrix} S_k\,, \qquad K_k^m \;=\; \begin{pmatrix} V_k^v \\ V_k^{vp} \end{pmatrix} S_k\,, \\ \boldsymbol{\zeta}_k & = & \mathbf{m}_k^v - \mathbf{v}_k\,, \\ \mathbf{r}_{k-1}^f & = & \mathbf{r}_{k-1} + K_k\boldsymbol{\zeta}_k \;=\; \begin{pmatrix} \mathbf{v}_{k-1} + C_{k-1}^v S_k\boldsymbol{\zeta}_k \\ \underline{\mathbf{p}}_{k-1} + C_{k-1}^{vp} S_k\boldsymbol{\zeta}_k \end{pmatrix}, \\ \mathbf{m}_k^f & = & \mathbf{m}_k - K_k^m\boldsymbol{\zeta}_k \;=\; \begin{pmatrix} \mathbf{v}_k^d - V_k^v S_k\boldsymbol{\zeta}_k \\ \underline{\mathbf{p}}_k^d - V_k^{vp} S_k\boldsymbol{\zeta}_k \end{pmatrix}, \\ C_{k-1}^f & = & C_{k-1} - K_k\left(C_{k-1}^v,\; C_{k-1}^{vp\,T}\right) \;=\; \begin{pmatrix} C_{k-1}^v - C_{k-1}^v S_k C_{k-1}^v & C_{k-1}^{vp\,T} - C_{k-1}^v S_k C_{k-1}^{vp\,T} \\ C_{k-1}^{vp} - C_{k-1}^{vp} S_k C_{k-1}^v & C_{k-1}^p - C_{k-1}^{vp} S_k C_{k-1}^{vp\,T} \end{pmatrix}, \\ V_k^f & = & V_k - K_k^m\left(V_k^v,\; V_k^{vp\,T}\right) \;=\; \begin{pmatrix} V_k^v - V_k^v S_k V_k^v & V_k^{vp\,T} - V_k^v S_k V_k^{vp\,T} \\ V_k^{vp} - V_k^{vp} S_k V_k^v & V_k^p - V_k^{vp} S_k V_k^{vp\,T} \end{pmatrix}, \\ D_k^f & = & K_k^m\left(C_{k-1}^v,\; C_{k-1}^{vp\,T}\right) \;=\; \begin{pmatrix} V_k^v S_k C_{k-1}^v & V_k^v S_k C_{k-1}^{vp\,T} \\ V_k^{vp} S_k C_{k-1}^v & V_k^{vp} S_k C_{k-1}^{vp\,T} \end{pmatrix}, \\ \chi_k^2 & = & \chi_{k-1}^2 + \boldsymbol{\zeta}_k^T S_k\boldsymbol{\zeta}_k\,, \\ \mathrm{ndf}_k & = & \mathrm{ndf}_{k-1} + 2\,. \end{array} \tag{32}$$

After the filtration the 4-momentum of the daughter particle is added to the 4-momentum of the mother particle:

$$\begin{array}{lcl} \mathbf{r}_k & = & \mathbf{r}_{k-1}^f + A_k\mathbf{m}_k^f\,, \\ C_k & = & C_{k-1}^f + A_k D_k^f + D_k^{f\,T} A_k^T + A_k V_k^f A_k^T\,, \end{array} \tag{33}$$

where the matrix $A_k$:

$$A_k \;=\; \begin{pmatrix} \mathrm{O} & \mathrm{O} \\ \mathrm{O} & \mathrm{I} \end{pmatrix}. \tag{34}$$

After substituting the expressions for $\mathbf{r}^f_{k-1}, \mathbf{m}^f_k, C^f_{k-1}, V^f_k$ and $D^f_k$ from (32) into (33), we obtain the final equations for the update of the state vector of the mother particle by the $k$-th daughter particle:

$$
\begin{aligned}
S_k &= \left(C^v_{k-1} + V^v_k\right)^{-1} . \\
\mathbf{r}_k &= \begin{pmatrix} \mathbf{v}_{k-1} + C^v_{k-1} S_k \left(\mathbf{v}^d_k - \mathbf{v}_{k-1}\right) \\ \underline{\mathbf{p}}_{k-1} + \underline{\mathbf{p}}^d_k + \left(C^{vp}_{k-1} - V^{vp}_k\right) S_k \left(\mathbf{v}^d_k - \mathbf{v}_{k-1}\right) \end{pmatrix} , \\
C_k &= \begin{pmatrix} C^v_{k-1} - C^v_{k-1} S_k C^v_{k-1} & C^{vp\,T}_{k-1} - C^v_{k-1} S_k \left(C^{vp}_{k-1} - V^{vp}_k\right)^T \\ C^{vp}_{k-1} - \left(C^{vp}_{k-1} - V^{vp}_k\right) S_k C^v_{k-1} & C^p_{k-1} + V^p_k - \left(C^{vp}_{k-1} - V^{vp}_k\right) S_k \left(C^{vp}_{k-1} - V^{vp}_k\right)^T \end{pmatrix} , \\
\chi^2_k &= \chi^2_{k-1} + \left(\mathbf{v}^d_k - \mathbf{v}_{k-1}\right)^T S_k \left(\mathbf{v}^d_k - \mathbf{v}_{k-1}\right) , \\
\mathrm{ndf}_k &= \mathrm{ndf}_{k-1} + 2 .
\end{aligned}
\tag{35}
$$

For more accurate linearization of the measurement $\mathbf{m}_k$ (27), the filtration is accomplished twice: first, according to (32) an approximate momentum $\mathbf{p}^{d\,0}$ of the daughter particle is calculated:

$$
\underline{\mathbf{p}}^{d\,0}_k = \underline{\mathbf{p}}^d_k - V^{vp}_k \left(V^v_k\right)^{-1} \left(\mathbf{v}^d_k - \mathbf{v}^0\right) , \tag{36}
$$

then $\underline{\mathbf{p}}^{d\,0}$ is substituted into the matrix $V_k$ (27), after which the filtration (35) is carried out.

If it is necessary in addition to select daughter tracks, then according to (32) the $\chi^2$ probability of the fact that the $k$-th particle $\mathbf{r}^d_k$ is daughter particle is calculated:

$$
\chi^2_d = \left(\mathbf{v}^d_k - \mathbf{v}^0\right)^T \left(C^{\mathbf{v}^0} + V^v_k\right)^{-1} \left(\mathbf{v}^d_k - \mathbf{v}^0\right) , \tag{37}
$$

where $C^{\mathbf{v}^0}$ — the assumed error of the initial approximation $\mathbf{v}^0$. Then, only the particles passing the $\chi^2$ cut are added to the mother track.

# 4 Measurement by the production vertex

After the particle is reconstructed at the decay point, into the state vector can be added a new parameter $s$, which is equal to the particle path length from its production point to the decay point, normalized to the particle momentum[2]:

$$
s = \frac{l}{p} , \tag{38}
$$

where $l$ — the length of the trajectory in the laboratory coordinate system, $p$ — the particle momentum.

The parameter $s$ is initially taken equal to the approximate value $s^0$, and the corresponding element of the covariance matrix is initialized by the value $\sigma^2_s = \mathrm{inf}$:

$$
\begin{aligned}
\mathbf{r} &\longrightarrow \begin{pmatrix} \mathbf{r} \\ s^0 \end{pmatrix} , \\
C &\longrightarrow \begin{pmatrix} C & \mathbf{0} \\ \mathbf{0} & \sigma^2_s \end{pmatrix}
\end{aligned}
\tag{39}
$$

All components of the state vector are now:

$$
\mathbf{r} = (x, y, z, p_x, p_y, p_z, E, s)^T . \tag{40}
$$

[2]Such normalization is convenient, since in the used parametrization the direction of the particle motion is assigned to the momentum vector.

After adding the parameter $s$, all parameters of the particle are transported from the decay point into the production point, where they are measured by the production vertex.

Let us denote an operator of the transport of the particle parameters into the production point as $f$:

$$f(\mathbf{r}) \;\equiv\; f(\mathbf{v}, \underline{\mathbf{p}}, s) \;=\; \mathbf{r} - \begin{pmatrix} \mathbf{p} \\ \mathbf{p} \times B \cdot q \\ 0 \end{pmatrix} \cdot s + O\left(s^2\right) . \tag{41}$$

We linearize the operator $f$ with respect to the parameter $s$:

$$f(\mathbf{r}) \;=\; f(\mathbf{v}, \underline{\mathbf{p}}, s^0) - \begin{pmatrix} \mathbf{p} \\ \mathbf{p} \times B \cdot q \\ 0 \end{pmatrix} \cdot \left(s - s^0\right) . \tag{42}$$

It is convenient to split the transport into two steps, corresponding to the terms in (42): the transport of the particle position on the fixed value $s = s^0$ and the subsequent correction of the covariance matrix taking into account the error of the parameter $s$.

Since the transport on the fixed value $s^0$ will be done only when the parameter $s$ is either already optimum or when it is equal to $s^0$, then the linearization is always done at the current value $s^0 = s$.

In the general case the transport is done in magnetic field [11]. Here we illustrate the transport in a special case, when the mother particle is not charged, or there is no magnetic field, and the transport is accomplished along a straight line. The transported particle position is:

$$\widehat{\mathbf{v}} \;=\; \mathbf{v} - s^0 \cdot \mathbf{p} . \tag{43}$$

The other components of the state vector do not change. The Jacobian $F_t$ of the transport along a straight line is:

$$F_t \;=\; \begin{pmatrix} 1 & 0 & 0 & -s^0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -s^0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -s^0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} , \tag{44}$$

and the transported particle $\widehat{\mathbf{r}}$ and its covariance matrix $\widehat{C}$ are:

$$\begin{aligned} \widehat{\mathbf{r}} \;&=\; F_t \mathbf{r} , \\ \widehat{C} \;&=\; F_t C F_t^T . \end{aligned} \tag{45}$$

Since $s^0 = s$, then the state vector does not change during the correction. However, since $s$ has error, the covariance matrix will change. Here we will give the general case of the operator $f$ for the charged particle in a magnetic field:

$$\begin{aligned} \widetilde{\mathbf{r}} \;&=\; \widehat{\mathbf{r}} - \begin{pmatrix} \widehat{\mathbf{p}}^0 \\ \widehat{\mathbf{p}}^0 \times B \cdot q \\ 0 \\ 0 \end{pmatrix} \cdot \left(s - s^0\right) , \\ s^0 \;&=\; s . \end{aligned} \tag{46}$$

The Jacobian $F_c$ of the correction is:

$$F_c = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & -\widehat{p}_x^0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\widehat{p}_y^0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -\widehat{p}_z^0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -(\widehat{p}_y^0 B_z - \widehat{p}_z^0 B_y)\cdot q \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -(\widehat{p}_z^0 B_x - \widehat{p}_x^0 B_z)\cdot q \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -(\widehat{p}_x^0 B_y - \widehat{p}_y^0 B_x)\cdot q \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \tag{47}$$

where $B$ — the magnetic field at the production vertex, $q$ — the particle charge. The corrected values of the state vector and of the covariance matrix are:

$$\begin{aligned} \widetilde{\mathbf{r}} &= \widehat{\mathbf{r}}, \\ \widetilde{C} &= F_c \widehat{C} F_c^T. \end{aligned} \tag{48}$$

After the transport of the particle into the production point, its position is measured by the production vertex. We assume that the optimum position $\mathbf{v}^p$ of the production vertex is already known, and it does not change when fitting the particle to the vertex[3]. Since the optimum value of the vertex is given, the filtration is accomplished by the modified Kalman filter (18), where the production vertex $\mathbf{v}^p$ is considered as measurement with the measurement model $H^p$:

$$\begin{aligned} \mathbf{m}^f &\equiv \mathbf{v}^p, \\ V^f &\equiv C^p, \\ H^p &= (\mathrm{I}, \mathrm{O}). \end{aligned} \tag{49}$$

The measurement of the production point completes the reconstruction procedure.

# 5 Complete reconstruction scheme

Let us give the complete scheme of reconstruction of the particle parameters

$$\mathbf{r} = (x, y, z, p_x, p_y, p_z, E, s)^T \tag{50}$$

and its covariance matrix according to the daughter particles $\mathbf{r}_k^d$, $k = 1 \ldots n$.

Since the problem is nonlinear, the complete procedure of reconstruction is processed several times, where each iteration consists of the following steps:

1. Choice of the initial approximation $\mathbf{v}^0$, initialization of $\chi_0^2 = 0$ and $\mathrm{ndf}_0 = -3$.
2. Transport of the $k$-th daughter particle $\mathbf{r}_k^d$, $C_k^d$ into the initial vertex position $\mathbf{v}^0$, construction of the parameters $\mathbf{m}_k$ of the daughter particle at the decay point:

$$\begin{aligned} \mathbf{m}_k &\equiv \begin{pmatrix} \mathbf{v}_k^d \\ \underline{\mathbf{p}}_k^d \end{pmatrix} = \mathbf{r}_k^d, \\ V_k &\equiv \begin{pmatrix} V_k^v & V_k^{vp\,T} \\ V_k^{vp} & V_k^p \end{pmatrix} = C_k^d + \begin{pmatrix} \mathbf{p}_k^d \\ \mathbf{p}_k^d \times B \cdot q_k \\ 0 \end{pmatrix} \begin{pmatrix} \mathbf{p}_k^d \\ \mathbf{p}_k^d \times B \cdot q_k \\ 0 \end{pmatrix}^T \cdot \sigma_s^2. \end{aligned} \tag{51}$$

[3] Either this is the event primary vertex or, in the case of a decay chain, the production vertex is first fitted using the particle, and then the particle is fitted to the reconstructed vertex.

It is sufficient to take as $\sigma_s$ the 10-times larger distance between $\mathbf{v}^0$ and $\mathbf{v}_k^d$ divided by the momentum $p_k^d$.

3. If it is necessary to select daughter tracks, then the $\chi^2$ probability of the fact that the $k$-th particle $\mathbf{r}_k^d$ is daughter particle is calculated:

$$\chi_d^2 \;=\; \left(\mathbf{v}_k^d - \mathbf{v}^0\right)^T \left(C^{v^0} + V_k^v\right)^{-1} \left(\mathbf{v}_k^d - \mathbf{v}^0\right) . \tag{52}$$

4. Calculation of the approximated momentum $\underline{\mathbf{p}}_k^{d\,0}$ of the daughter particle:

$$\underline{\mathbf{p}}_k^{d\,0} \;=\; \underline{\mathbf{p}}_k^d - V_k^{vp} \left(V_k^v\right)^{-1} \left(\mathbf{v}_k^d - \mathbf{v}^0\right) , \tag{53}$$

and refinement of the matrix $V_k$:

$$V_k \;=\; C_k^d + \begin{pmatrix} \mathbf{p}_k^{d\,0} \\ \mathbf{p}_k^{d\,0} \times B \cdot q_k \\ 0 \end{pmatrix} \begin{pmatrix} \mathbf{p}_k^{d\,0} \\ \mathbf{p}_k^{d\,0} \times B \cdot q_k \\ 0 \end{pmatrix}^T \cdot \sigma_s^2 . \tag{54}$$

5. Measurement of the state vector $\mathbf{r}_{k-1}$ by the daughter particle $\mathbf{m}_k$ adding the 4-momentum of the daughter particle to the 4-momentum of the mother particle:

$$\begin{aligned}
S_k &= \left(C_{k-1}^v + V_k^v\right)^{-1} . \\
\mathbf{r}_k &= \begin{pmatrix} \mathbf{v}_{k-1} + C_{k-1}^v S_k \left(\mathbf{v}_k^d - \mathbf{v}_{k-1}\right) \\ \underline{\mathbf{p}}_{k-1} + \underline{\mathbf{p}}_k^d + \left(C_{k-1}^{vp} - V_k^{vp}\right) S_k \left(\mathbf{v}_k^d - \mathbf{v}_{k-1}\right) \end{pmatrix} , \\
C_k &= \begin{pmatrix} C_{k-1}^v - C_{k-1}^v S_k C_{k-1}^v & C_{k-1}^{vp\,T} - C_{k-1}^v S_k \left(C_{k-1}^{vp} - V_k^{vp}\right)^T \\ C_{k-1}^{vp} - \left(C_{k-1}^{vp} - V_k^{vp}\right) S_k C_{k-1}^v & C_{k-1}^p + V_k^p - \left(C_{k-1}^{vp} - V_k^{vp}\right) S_k \left(C_{k-1}^{vp} - V_k^{vp}\right)^T \end{pmatrix} , \\
\chi_k^2 &= \chi_{k-1}^2 + \left(\mathbf{v}_k^d - \mathbf{v}_{k-1}\right)^T S_k \left(\mathbf{v}_k^d - \mathbf{v}_{k-1}\right) , \\
\mathrm{ndf}_k &= \mathrm{ndf}_{k-1} + 2 .
\end{aligned} \tag{55}$$

Since at the first measurement the parameters of the mother particle are not yet determined, the equations of filtration (55) are simplified and the measurement $\mathbf{m}_1$ is directly copied into the state vector $\mathbf{r}_1$:

$$\begin{aligned}
\mathbf{r}_1 &= \mathbf{m}_1 , \\
C_1 &= V_1 , \\
\chi_1^2 &= 0 , \\
\mathrm{ndf}_1 &= -1 .
\end{aligned} \tag{56}$$

6. Repeat from the step 2 for the next daughter particle, until all the daughters are treated.

7. Precision of the particle parameters obtained after the fit can be improved in the case of invariant mass $M$ of the particle is known:

$$M^2 \;=\; E^2 - \left(p_x^2 + p_y^2 + p_z^2\right) . \tag{57}$$

In this case the parameters of the particle are measured by the one-dimensional measurement with the measured value $M^2$, the null error and the measurement matrix $H_{M^2}$

$$H_{M^2} \;=\; (0, 0, 0, -p_x, -p_y, -p_z, E, 0) \tag{58}$$

using the conventional Kalman filter (5).

8. If the production vertex is given, then the constructed mother particle is transported into the production point and then is measured by the production vertex:

$$\begin{aligned}
\mathbf{r} &= \begin{pmatrix} \mathbf{v}_p \\ \underline{\widetilde{\mathbf{p}}} + \widetilde{C}^{vp}\left(\widetilde{C}^{v}\right)^{-1}(\mathbf{v}_p - \widetilde{\mathbf{v}}) \end{pmatrix}, \\
C &= \begin{pmatrix} C_p & C_p\left(\widetilde{C}^{v}\right)^{-1}\widetilde{C}^{vp\,T} \\ \widetilde{C}^{vp}\left(\widetilde{C}^{v}\right)^{-1}C_p & \widetilde{C}^{p} - \widetilde{C}^{vp}\left(\widetilde{C}^{v}\right)^{-1}\left(\widetilde{C}^{v} - C_p\right)\left(\widetilde{C}^{v}\right)^{-1}\widetilde{C}^{vp\,T} \end{pmatrix}, \\
\Delta\chi^2 &= (\mathbf{v}_p - \widetilde{\mathbf{v}})^T\left(\widetilde{C}^{v} - C_p\right)^{-1}(\mathbf{v}_p - \widetilde{\mathbf{v}}), \\
\Delta\mathrm{ndf} &= 2\,.
\end{aligned} \tag{59}$$

In all iterations, except the last one, the particle is transported back into the decay point by changing $-s$ to $s$ in (47, 48) in order to determine the linearization point $\mathbf{v}^0$ for the next iteration.

The reconstructed state vector and its covariance matrix contain all necessary information about the particle both at the production point and at the decay point. Therefore, after reconstruction the parameters of the particle can be transported into the decay point or into the production point, as it is described in Sec. 4.

After estimation of the parameters of the particle, additional physics parameters, which are not explicitly included into the state vector, the particle momentum $P$, the invariant mass $M$, the length of flight $L$ in the laboratory coordinate system, and the time of life of the particle $cT$ in its own coordinate system, can be easily calculated as:

$$\begin{aligned}
P &= \sqrt{p_x^2 + p_y^2 + p_z^2}\,, & \sigma_P^2 &= H_P C H_P^T\,, \\
M &= \sqrt{E^2 - P^2}\,, & \sigma_M^2 &= H_M C H_M^T\,, \\
L &= s \cdot P\,, & \sigma_L^2 &= H_L C H_L^T\,, \\
cT &= s \cdot M\,, & \sigma_{cT}^2 &= H_{cT} C H_{cT}^T\,,
\end{aligned} \tag{60}$$

where

$$\begin{aligned}
H_P &= (\ 0,\ 0,\ 0,\ p_x,\ p_y,\ p_z,\ 0,\ 0\ )/P\,, \\
H_M &= (\ 0,\ 0,\ 0,\ -p_x,\ -p_y,\ -p_z,\ E,\ 0\ )/M\,, \\
H_L &= (\ 0,\ 0,\ 0,\ sp_x,\ sp_y,\ sp_z,\ 0,\ P^2\ )/P\,, \\
H_{cT} &= (\ 0,\ 0,\ 0,\ -sp_x,\ -sp_y,\ -sp_z,\ sE,\ M^2\ )/M\,.
\end{aligned} \tag{61}$$

# 6 Results and discussion

The algorithm has been implemented for the CBM experiment [9, 10]. For these studies central Au+Au collisions at 25 $A$GeV have been simulated. In the simulations we used the main tracking detector of 7 silicon pixel stations positioned at 10, 20, 30, 40, 60, 80 and 100 cm from the target. The first 2 stations have thickness of 150 $\mu$m, while others — 400 $\mu$m. All detectors have idealized response (no fake hits, efficiency losses, pile-up etc.). The non-homogeneous active magnetic field has been used to trace particles through the detector.

For tests of the developed algorithm we have reconstructed $D^0$ mesons, which are generated in the event production vertex and then decayed into $\pi^+$ and $K^-$ particles. Since $D^0$ meson has a very short life time, it is not detected by the detector system, while its daughter particles are well within the detector acceptance.

The ideal[4] track finder has been used to collect hits into track groups. The track fitting routine realises the Kalman filter in its conventional approach. The default $\pi$ particle hypothesis has been used for all tracks. For the $\pi^+$ and $K^-$ daughter particles the correct particle hypothesis have been used during the track fit in order to properly account for multiple scattering effects, and in the reconstruction procedure to calculate the $\pi^+$ and $K^-$ energy.

| | Production Vertex$_{[\mu m]}$ | | | Decay Vertex$_{[\mu m]}$ | | | Physical Parameters | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | x | y | z | x | y | z | $P_{[\%]}$ | $M_{[MeV/c]}$ | $L_{[\mu m]}$ | $cT_{[\mu m]}$ |
| Accuracy | 0.81 | 0.73 | 5.50 | 2.64 | 2.64 | 63.88 | 0.79 | 11.34 | 64.10 | 9.81 |
| Pull | 1.14 | 1.10 | 1.11 | 1.13 | 1.13 | 1.10 | 1.20 | 1.19 | 1.11 | 1.11 |

Table 1: Resolutions and pulls of the decayed particle parameters obtained from $10^4$ $D^0$ decays in central Au+Au collisions at 25 $A$GeV

In the tests the algorithm first reconstructs the event production vertex using all reconstructed tracks, then $D^0$ meson is reconstructed from its two daughter particles $\pi^+$ and $K^-$ using the event production vertex as the production point.

The chosen parametrization of the decayed particle contains all necessary information about the particle both at the point of its generation and at the point of its decay. Therefore, the developed method is suitable both for the complete reconstruction of decayed particles and for the reconstruction of vertices only. In the second case, the state vector can be reduced to $\mathbf{v}$, and all operations with $\underline{\mathbf{p}}$, $s$ are removed, after that the algorithm is similar to the standard approach (see, for instance, [6]).

Table 1 shows, that the algorithm provides a very high accuracy for the event vertex: the resolutions of the $x$ and $y$ positions of the $D^0$ production vertex are less than 1 $\mu$m, and the $z$ position is reconstructed with an accuracy 5.5 $\mu$m. The resolution of the $D^0$ decay vertex is 2.64 $\mu$m for $x$ and $y$, and 63.88 $\mu$m for $z$. The normalized residuals (pulls) are close to unity, thus showing that all parameters are well estimated.

The choosen parametrization is also physically natural and, therefore, is convenient for further physics analysis. Table 1 shows resolutions and pulls of $D^0$ physical parameters reconstructed by the algorithm. The algorithm provides, for instance, estimations of the time of life of the particle and the decay length together with the corresponding errors. Here the time of life $cT$ is reconstructed with an accuracy 9.8 $\mu$m, showing that the reconstructed $D^0$ particles are well separated from the event production vertex.

In addition, Figure 1 gives distributions of residuals and normalised residuals (pulls) of the $D^0$ physical parameters. The RMS of the Gaussian fits to the residual and normalized residual distributions are also given. A measure of the reliability of the fit are the pull distributions of the fitted parameters. All pulls are centered at zero indicating that there is no systematic shift in the reconstructed track parameter values. The distributions are well fitted using Gaussian functions with small tails caused by the various non-Gaussian contributions to the fit.

Figure 2 gives the distribution of the $D^0$ life time with the fitted mean life (122.1 ± 2.2) $\mu$m, which is close to the $D^0$ mean life $c\tau_{D^0} = 122.9$ $\mu$m [12] used in the simulations.

[4]It uses the Monte-Carlo information.

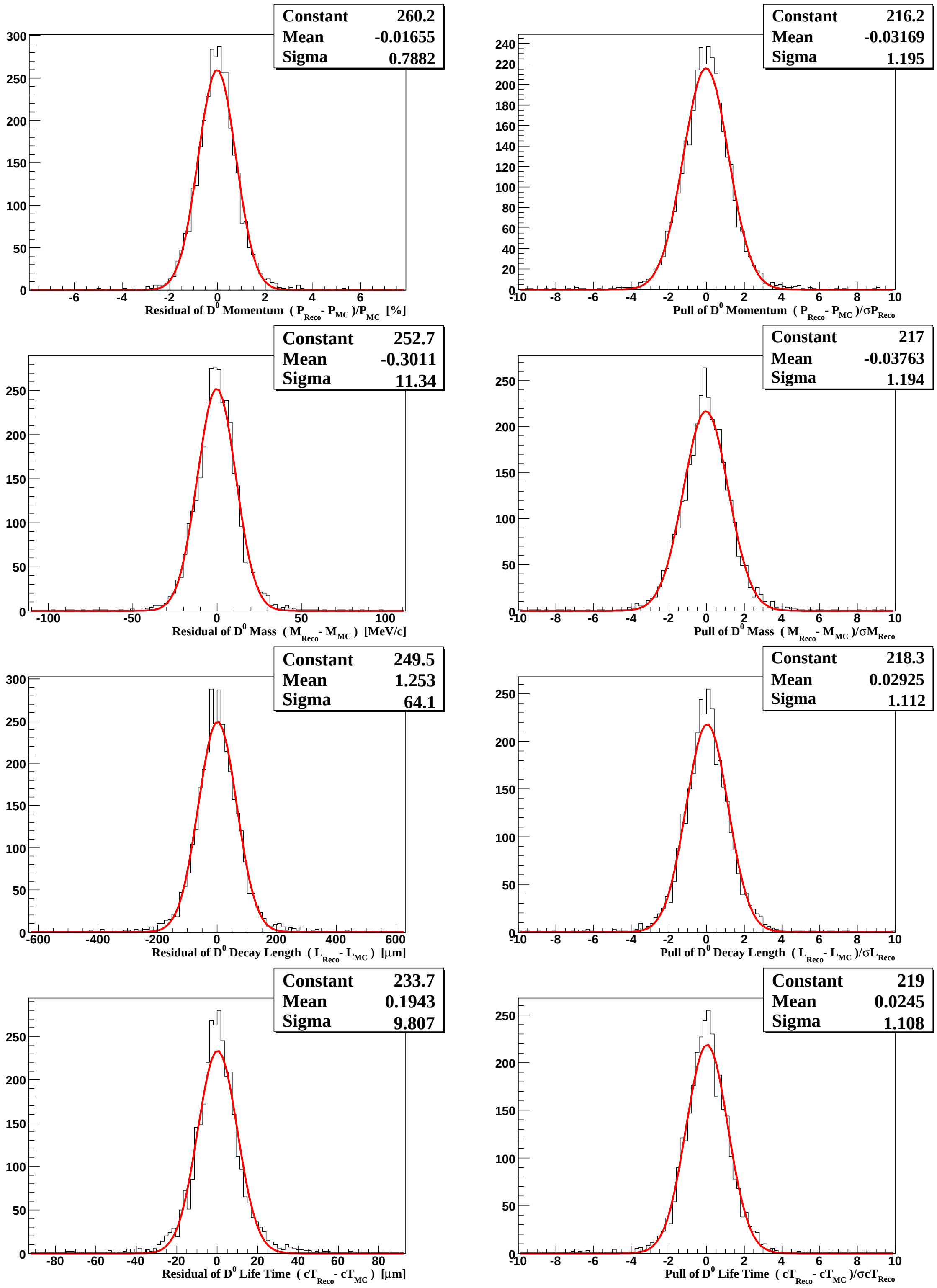


Figure 1: Residuals and normalized residuals (pulls) of the $D^0$ physical parameters $P$, $M$, $L$ and $cT$

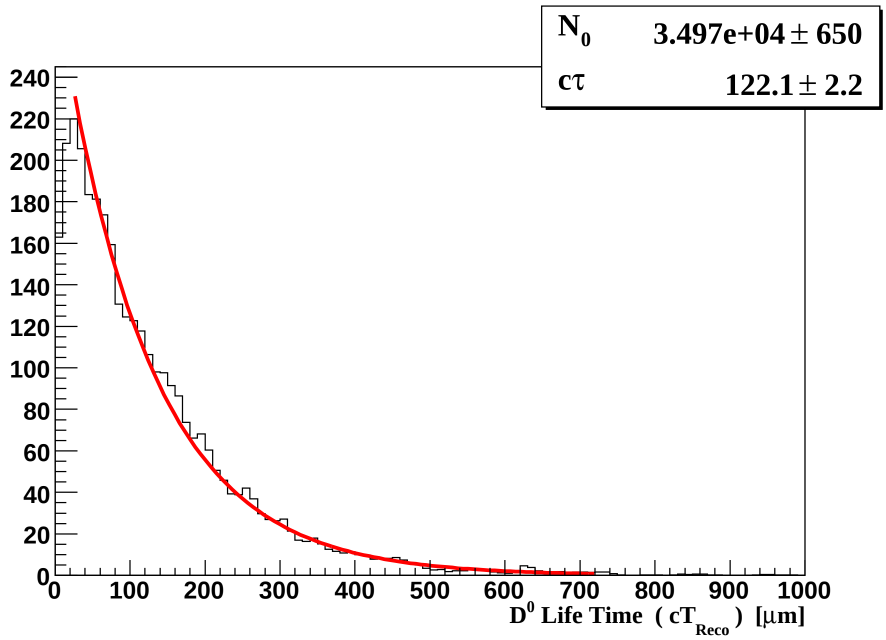


Figure 2: Distribution of the $D^0$ life time

The developed algorithm has significantly reduced amount of calculations comparing to the standard approach of vertex fitting. The state vector has the fixed size and does not grow if the number of daughter particles increases. There is no inversion of $5 \times 5$ matrices in the modified equations of filtration, thus improving the robustness of the covariance computations with respect to round-off errors.

The algorithm extrapolates the track estimates $\mathbf{r}_k^d$ to the point $\mathbf{v}^0$ of the vertex linearization. As a result the measurement model $H_k$ (29) is trivial and does not require matrix operations. The linearization of all measurements remains correct also in presence of magnetic field.

There is no filtration of the first daughter track, that in the case of two-prong decays reduces twice the calculations and, furthermore, avoids large initial values in the covariance matrix making the algorithm numerically stable.

Having the mother particle fully reconstructed the algorithm can operate with it like with an ordinary reconstructed track. For instance, the algorithm is able to transport the charged mother particle in magnetic field. It is also possible to add measurements to the reconstructed mother particle, that is important, when the decay had been occurred in a considerable distance from the production vertex, and the mother particle itself has been registered by the detector system.

# 7 Conclusion

A decayed particles reconstruction package based on the Kalman filter method has been developed for the CBM experiment. The algorithm shows a high accuracy and reliability. The presented algorithm of the decayed particles reconstruction provides the optimum estimation of the parameters of the mother particle in the production and decay vertices.

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# References

[1] R. Frühwirth et al., Data analysis techniques for high-energy physics. Second edition, Cambridge Univ. Press (2000).

[2] P. Billoir and S. Qian, Fast vertex fitting with a local parametrization of tracks. Nucl. Instr. and Meth. A311 (1992) 139-150.

[3] R. Frühwirth, P. Kubinec, W. Mitaroff and M. Regler, Vertex reconstruction and track bundling at the LEP collider using robust algorithms. Comp. Phys. Commun. 96 (1996) 189-208.

[4] D. Emeliyanov, I. Kisel, S. Masciocchi, M. Sang and Yu. Vassiliev, Primary vertex reconstruction by Rover. HERA-B note 00-139, 2000.

[5] V. Eiges, D. Emeliyanov, V. Kekelidze, I. Kisel and Yu. Vassiliev, Test of vertex reconstruction and fitting algorithms on $J/\psi \to \mu^+\mu^-$ data. HERA-B note 00-182, 2000.

[6] S. Gorbunov and I. Kisel, Primary vertex fit based on the Kalman filter. CBM-SOFT-note-2006-001, GSI, Darmstadt, 9 January 2006.

[7] S. Gorbunov and I. Kisel, Secondary vertex fit based on the Kalman filter. CBM-SOFT-note-2006-002, GSI, Darmstadt, 14 September 2006.

[8] W. Hulsbergen, Decay chain fitting with a Kalman filter. Nucl. Instr. and Meth. A 552 (2005) 566-575.

[9] CBM Collaboration, Compressed Baryonic Matter Experiment. Technical Status Report. GSI, Darmstadt, 2005; 2006 Update (`http://www.gsi.de/documents/DOC-2006-Feb-108-1.pdf`).

[10] I. Kisel, Event reconstruction in the CBM experiment. Nucl. Instr. and Meth. A566 (2006) 85-88.

[11] S. Gorbunov and I. Kisel, Analytic formula for track extrapolation in non-homogeneous magnetic field. Nucl. Instr. and Meth. A559 (2006) 148-152.

[12] W.-M. Yao et al. (Particle Data Group), Review of Particle Physics, J. Phys. G: Nucl. Part. Phys. 33 (2006) 1-1232.