Method For Determining Relative Luminosity From Detection Probabilities (Run9 200 GeV)

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1 Introduction

As instantaneous luminosity continues to increase at RHIC, multiple collisions in one beam crossing become more likely, and the use of a detector coincidence (such as the "AND" of the BBC arms) as a luminosity monitor becomes less and less valid. This is because a simple detector coincidence is a "binary" result that conveys no direct information about how many times the detector was hit by particles from the collision. However, by modeling the *probability* of firing the detector over many crossings of the beam, we can indirectly extract this information. In effect, we change our assumption from 0 = no collision, 1 = 1 collision to 0 = no collision, 1 = ($\geq$ 1 collision) and find the true hit rate in the detectors from this information.

Once we have done this, we will look at the implications for relative luminosity in Run 09 $\sqrt{s} = 200$ GeV, specifically focusing on comparisons between the ZDC and BBC (which are traditionally used to quantify systematic uncertainty on the relative luminosity). We will also try to study the dependence of a certain class of events (events capable of triggering only one arm of the detector) on rate.

2 Extracting Rate From Detection Probabilities

2.1 Measuring Collisions with a Single-Sided Detector

In this section and the next, it is assumed there are only events capable of causing a real (not accidental) coincidence in the two detectors, i.e. double-sided events. This includes, for instance, double-diffractive events or a single particle traveling through both detectors. Single-sided events will be taken into account later.

First, we consider the probability distribution for the *true* number of collisions in one crossing, regardless of whether or not they registered in one or both of the detectors. If there are of order $\sim 10^{11}$ protons in each bunch, the number of possible collisions in one crossing is of order $\sim 10^{22}$. Since we know from experience that the number of collisions actually occurring in each crossing is much less than $\sim 10^{22}$ (typically of order 1), we can safely assume the number of collisions follows a Poisson distribution with an average number λ,

$$P_{DS}(i) = \frac{\lambda^i e^{-\lambda}}{i!}, \tag{1}$$

where i is the number of collisions.

For clarity let us suppose we are working with the "South" detector and give it the label S. Now the probability for the detector to be hit at k_S distinct times (it doesn't matter if we can't distinguish them in practice since we won't be counting anyway) should be given by

$$P_{DS}(k_S) = \sum_{i=k_S}^{\infty} \binom{i}{k_S} \epsilon_S^{k_S} \left(1-\epsilon_S\right)^{i-k_S} P_{DS}(i). \tag{2}$$

Here ϵ_S is the "efficiency" of the detector, which here we take to mean the probability for detecting a collision. There are appropriate factors for detecting k_S collisions and not detecting $i - k_S$ others, along with a factor $\binom{i}{k_S}$ for the number of ways this can happen. The sum starts at k_S since if there are k_S distinct hits detected (neglecting random noise), we can be sure there were at least k_S collisions.

By pulling k_S factors of λ out of the sum and re-indexing, one can show that the probability for k_S collisions also a Poisson distribution:

$$\begin{aligned} P_{DS}(k_S) &= \sum_{i=k_S}^{\infty} \frac{i!}{k_S!(i-k_S)} \epsilon_S^{k_S} \left(1-\epsilon_S\right)^{i-k_S} \frac{\lambda^i e^{-\lambda}}{i!} \\ &= \frac{1}{k_S!} \epsilon_S^{k_S} \lambda_S^{k} e^{-\lambda} \sum_{i=k_S}^{\infty} \frac{1}{(i-k_S)!} \left(1-\epsilon_S\right)^{i-k_S} \lambda^{i-k_S} \\ &= \frac{1}{k_S!} \epsilon_S^{k_S} \lambda_S^{k} e^{-\lambda} e^{(1-\epsilon_S)\lambda} \\ &= \frac{\left(\epsilon_S \lambda\right)^{k_S} e^{-\epsilon_S \lambda}}{k_S!}. \end{aligned} \tag{3}$$

2.2 A Two-Sided Detector

We can derive the probability $P_{DS}(k_S, k_N)$ for a two-sided detector from the one-sided detector result. First, note that

$$P_{DS}(k_S, k_N) = P_{DS}(k_N|k_S) P_{DS}(k_S) = \left(\sum_{i=k_N}^{\infty} \binom{i}{k_N} \epsilon_N^{k_N} \left(1-\epsilon_N\right)^{i-k_N} P_{DS}(i|k_S) \right) P_{DS}(k_S). \tag{4}$$

$P(i|k_S)$ is the probability that there were i collisions given that we measured k_S hits. Using Bayes' theorem, we can express

$$P_{DS}(i|k_S) = \frac{P_{DS}(k_S|i) P_{DS}(i)}{P_{DS}(k_S)}. \tag{5}$$

$P_{DS}(k_S|i)$ is just

$$P_{DS}(k_S|i) = \binom{i}{k_S} \epsilon_S^{k_S} \left(1-\epsilon_S\right)^{i-k_S} \Theta(i-k_S), \tag{6}$$

where the step function ensures we do not see more distinct hits then there are collisions.

Substituting Equations 5 and 6 into 4, we arrive at

$$P_{DS}(k_S, k_N) = \sum_{i=\max(k_S,k_N)}^{\infty} \binom{i}{k_S} \binom{i}{k_N} \epsilon_S^{k_S} \epsilon_N^{k_N} \left(1-\epsilon_S\right)^{i-k_S} \left(1-\epsilon_N\right)^{i-k_N} P_{DS}(i). \tag{7}$$

Note that the step function, along with the original lower limit on the sum of k_S, can be accounted for by setting the lower limit to $\max(k_S, k_N)$. For the special case $P_{DS}(k_S = 0, k_N = 0)$ this formula reduces to

$$P_{DS}(k_S = 0, k_N = 0) = e^{\epsilon_S \epsilon_N \lambda - \epsilon_S \lambda - \epsilon_N \lambda} = e^{-\epsilon_N (1-\epsilon_S) \lambda} e^{-\epsilon_S \lambda}. \tag{8}$$

2.3 Allowing Single-Sided Events

We now consider single-sided events, which by our definition are only capable of triggering one detector. The distributions describing the number of such collisions in a crossing are again Poisson, this time with average numbers λ_S and λ_N. The probability of seeing k_S distinct time hits in the south detector, for instance, is the same as before:

$$P_{SS}(k_S) = \frac{\left(\epsilon_S \lambda_S\right)^{k_S} e^{-\epsilon_S \lambda_S}}{k_S!}. \tag{9}$$

Most importantly, the probability for seeing zero hits is $e^{-\epsilon_S \lambda_S}$. The probability of detecting a total of zero hits in the south detector, both from single and double-sided events, is then

$$P(k_S = 0) = P_{DS}(k_S = 0) P_{SS}(k_S = 0) = e^{-\epsilon_S(\lambda + \lambda_S)}, \tag{10}$$

where we may, for notational convenience, absorb any difference in ϵ_S^{SS} for single-sided events into λ_S itself. Note that the samples these two distributions apply to do not overlap and so the (non-)events are independent. Similarly, the probability of detecting zero collisions in the two-sided detector is given by

$$P(k_S = 0, k_N = 0) = P_{DS}(k_S = 0, k_N = 0) P_{SS}(k_S = 0) P_{SS}(k_N = 0) = e^{\epsilon_S \epsilon_N \lambda - \epsilon_S(\lambda + \lambda_S) - \epsilon_N(\lambda + \lambda_N)}. \tag{11}$$

2.4 Removing Single-Sided Events

The utility of these three separate probability distributions becomes apparent when we consider the function

$$ln(P(k_S = 0, k_N = 0)) - ln(P(k_S = 0)) - ln(P(k_N = 0)) = \epsilon_N \epsilon_S \lambda, \tag{12}$$

or, since $P(k_S = 0) = 1 - P_S$, $P(k_N = 0) = 1 - P_N$, and $P(k_S = 0, k_N = 0) = 1 - P_{OR}$, where P_S, P_N, and P_{OR} are the probabilities of south, north, and OR triggers,

$$ln(1 - P_{OR}) - ln(1 - P_S) - ln(1 - P_N) = \epsilon_N \epsilon_S \lambda. \tag{13}$$

Only events and backgrounds capable of causing true coincidence in the two detectors contribute to this quantity. Furthermore, it completely takes into account the effects of multiple collisions. If ϵ_S and ϵ_N are not spin dependent (which is an assumption of the present method for calculating relative luminosity) this quantity can be used to calculate the relative luminosity, although it will not include a vertex cut requirement.

2.5 Measurement and Statistical Uncertainty

As is always the case in experiments, it is not possible to know our true parameters exactly; we must estimate P_N, P_S, and P_{OR} from the data. The way to do this is to look at a set number of unbiased events (i.e. clock events) and count the number of triggers within that sample. Since the probability of a trigger in any given crossing is significant, we can not approximate with a Poisson distribution and must use the full binomial distribution. The probability distribution for the number of OR triggers in N_{clock} events, for example, is

$$P(N_{OR} = n) = \binom{N_{clock}}{n} P_{OR}^n (1 - P_{OR})^{N_{clock} - n}. \tag{14}$$

The standard deviation in this case is $\sqrt{N_{clock} P_{OR}(1 - P_{OR})}$, and the mean is $N_{clock} P_{OR}$. We can see that estimating the sample mean by the actual number of OR triggers, N_{OR}, is equivalent to estimating P_{OR} as N_{OR}/N_{clock}. The estimate for the standard deviation of N_{OR} is then $\sqrt{N_{OR}(1 - \frac{N_{OR}}{N_{clock}})}$.

In propagating our errors through Equation 13, we should be careful to include correlations between our samples. We use the general uncertainty propagation formula

$$\begin{aligned}(\sigma_{f(x,y,z)})^2 &= \left(\frac{\partial f}{\partial x}\right)^2 (\sigma_x)^2 + \left(\frac{\partial f}{\partial y}\right)^2 (\sigma_y)^2 + \left(\frac{\partial f}{\partial z}\right)^2 (\sigma_z)^2 \\ &+ 2\left(\frac{\partial f}{\partial x}\right)\left(\frac{\partial f}{\partial y}\right)\rho_{x,y}\sigma_x\sigma_y + 2\left(\frac{\partial f}{\partial x}\right)\left(\frac{\partial f}{\partial z}\right)\rho_{x,z}\sigma_x\sigma_z + 2\left(\frac{\partial f}{\partial y}\right)\left(\frac{\partial f}{\partial z}\right)\rho_{y,z}\sigma_y\sigma_z\end{aligned} \tag{15}$$

where ρ's are the Pearson correlation coefficients. With $f = ln(1 - \frac{N_{OR}}{N_{clock}}) - ln(1 - \frac{N_S}{N_{clock}}) - ln(1 - \frac{N_N}{N_{clock}})$, we need to consider the correlations between the number of S, N, and OR triggered events.

To see how the terms in the uncertainty formula behave, we can go a step further and insert the partial derivatives

$$\begin{aligned}
\frac{\partial f}{\partial N_{OR}} &= -\frac{\frac{1}{N_{clock}}}{\left(1-\frac{N_{OR}}{N_{clock}}\right)} \\
\frac{\partial f}{\partial N_{S}} &= \frac{\frac{1}{N_{clock}}}{\left(1-\frac{N_{S}}{N_{clock}}\right)} \\
\frac{\partial f}{\partial N_{N}} &= \frac{\frac{1}{N_{clock}}}{\left(1-\frac{N_{S}}{N_{clock}}\right)}
\end{aligned} \tag{16}$$

(note the different sign for the OR term) and standard deviations

$$\begin{aligned}
\sigma_{OR} &= \sqrt{N_{OR}\left(1-\frac{N_{OR}}{N_{clock}}\right)} \\
\sigma_{S} &= \sqrt{N_{S}\left(1-\frac{N_{S}}{N_{clock}}\right)} \\
\sigma_{N} &= \sqrt{N_{N}\left(1-\frac{N_{N}}{N_{clock}}\right)}.
\end{aligned} \tag{17}$$

Then, in terms of the various counts and correlation coefficients, the uncertainty becomes

$$\begin{aligned}
\left(\sigma_{f(N_{OR},N_N,N_S)}\right)^2 &= \left(\frac{1}{N_{clock}}\right) * \{ \frac{\frac{N_{OR}}{N_{clock}}}{1-\frac{N_{OR}}{N_{clock}}} + \frac{\frac{N_{S}}{N_{clock}}}{1-\frac{N_{S}}{N_{clock}}} + \frac{\frac{N_{N}}{N_{clock}}}{1-\frac{N_{N}}{N_{clock}}} \\
&- 2\sqrt{\left(\frac{\frac{N_{OR}}{N_{clock}}}{1-\frac{N_{OR}}{N_{clock}}}\right)\left(\frac{\frac{N_{S}}{N_{clock}}}{1-\frac{N_{S}}{N_{clock}}}\right)}\rho_{N_{OR},N_S} \\
&- 2\sqrt{\left(\frac{\frac{N_{OR}}{N_{clock}}}{1-\frac{N_{OR}}{N_{clock}}}\right)\left(\frac{\frac{N_{N}}{N_{clock}}}{1-\frac{N_{N}}{N_{clock}}}\right)}\rho_{N_{OR},N_N} \\
&+ 2\sqrt{\left(\frac{\frac{N_{S}}{N_{clock}}}{1-\frac{N_{S}}{N_{clock}}}\right)\left(\frac{\frac{N_{N}}{N_{clock}}}{1-\frac{N_{N}}{N_{clock}}}\right)}\rho_{N_S,N_N} \}
\end{aligned} \tag{18}$$

Note that there are two negative correlation terms involving N_{OR}, which serve to reduce the uncertainty, and one positive term involving N_N and N_S which would increase it.

3 Possible Extensions

3.1 Three-Sided Detector

For an "OR" coincidence between a three-sided detector, the probability for zero hits detected is described by

$$\begin{aligned}
ln(P(k_a=0,k_b=0,k_c=0)) &= ln\left(1-\frac{N_{OR}}{N_{clock}}\right) = -\epsilon_a\epsilon_b\epsilon_c\lambda + \epsilon_a\epsilon_b\lambda + \epsilon_a\epsilon_c\lambda + \epsilon_b\epsilon_c\lambda \\
&- \epsilon_a(\lambda+\lambda_a) - \epsilon_b(\lambda+\lambda_b) - \epsilon_c(\lambda+\lambda_c).
\end{aligned} \tag{19}$$

This, along with the three possible two-arm results, allows for the extraction of $\epsilon_a\epsilon_b\epsilon_c\lambda$. In certain instances, this result could be divided by the two-arm results to extract each of the "efficiencies" separately, and, ultimately, the raw rate λ. This, however, would require that the effective "efficiencies" for each two-arm detector pair be the same "efficiencies" that show up in the quantity $\epsilon_a\epsilon_b\epsilon_c\lambda$. This amounts to a requirement that the "efficiency" of detector c be constant over all sub-classes of events with differing product $\epsilon_a\epsilon_b$. For then

$$\frac{\sum_{i\in \text{ sub-classes}}(\epsilon_a\epsilon_b)^i\epsilon_c^i\lambda_i}{\sum_{i\in \text{ sub-classes}}(\epsilon_a\epsilon_b)^i} = \frac{\epsilon_c\sum_{i\in \text{ sub-classes}}(\epsilon_a\epsilon_b)^i\lambda_i}{\sum_{i\in \text{ sub-classes}}(\epsilon_a\epsilon_b)^i} = \epsilon_c \quad (20)$$

To achieve this goal, the three detectors should be as similar as possible.

Obviously this method would not work with a configuration of, say, two arms of the BBC and one of the ZDC, since there are certainly different classes of double-sided collisions for the BBC with different ZDC efficiencies (for example, events with forward neutrons versus events without). To use the above extension for a reasonable estimate of the "efficiencies", then, it would be necessary to do something like divide the BBC tubes on each arm into two groups, effectively making the BBC a four-arm detector with each arm having similar properties.

4 Star Scalers/GL1p Counts QA

Runs were removed in the cases where there were no Star Scalers or GL1p data, no BBC raw recorded data, obvious inconsistencies between the Star Scalers and GL1p counts, or where the Star Scalers were not properly timed in Crossings were not removed from the crossing block analysis, but were in the bunch fitting analysis when there were no Star Scaler or GL1p counts, a bad spin pattern given by the Spin database, or an asymmetry between Star Scaler and GL1p counts of greater than 95%. Some bunches were also explicitly removed: bunch 20, the steering bunch; bunches 38-39, 78-79, and 111-119, the gaps; and bunch 1, where the GL1p boards don't count.

5 Results of Crossing Block Analysis

5.1 Introduction

In order to study rate effects with high statistical accuracy run-by-run, a "crossing block" analysis was done using data recorded by the Star Scalers during Run 09 200 GeV. Existing Star Scaler counts for hits in the N and S detectors, as well as counts complied from the files for the "OR" and other measures of coincidence, were summed into "crossing blocks," in this case, blocks of counts between all empty bunches (0:0-37, 1:40-77, and 2:80-110). In addition to increasing the statistical accuracy for the study, another benefit of the "crossing block" analysis is that it washes out any effects from Star Scaler counts spilling from one crossing to the next, which was possible in Run 09 200 GeV since they weren't properly timed. From these summed counts, the quantity $\epsilon_N\epsilon_S\lambda$ was calculated for the BBC and ZDC.

5.2 Results and Conclusions

Results are shown versus both rate proxies and versus runnumber, in Figures 1 through 4. From the plots versus rate proxies, it is clear that the ratio $\frac{(\epsilon_N\epsilon_S\lambda)_{ZDC}}{(\epsilon_N\epsilon_S\lambda)_{BBC}}$ does not suffer the same rate effect as the traditional ratio $\frac{N_{ZDC}}{N_{BBC}}$. However, the spread in the values is quite large. This can be attributed to some run or time-depended effect, as shown in the plots versus runnumber. The values tend to rise over time, which points to some effect equivalent to a rise in the ratio $\frac{(\epsilon_N\epsilon_S)_{ZDC}}{(\epsilon_N\epsilon_S)_{BBC}}$

We can also measure how much we undercount or overcount using simple detector coincidence (with no vertex cut), by comparing the "AND" of the two arms with $\epsilon_N\epsilon_S\lambda$. Figures 5 and 6 show this comparison for the BBC and ZDC.

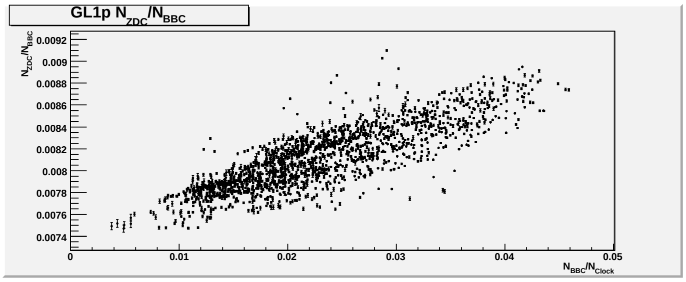


Figure 1:

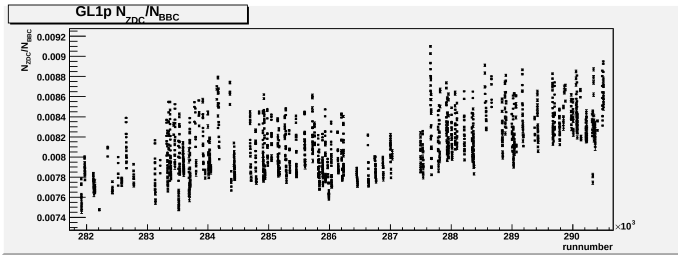


Figure 2:

It is interesting that the BBC coincidence undercounts, while the ZDC overcounts. This is likely due to the high single-sided event rate in the ZDC, which (see Section 7), is proportional to λ.

6 Results of Bunch-Fitting Analysis

6.1 Technique

In past Runs, in order to look for a possible bias in measuring Relative Luminosity (R) in the BBC, a comparison was made to the ZDC, the idea being that since the two detectors sit at different rapidity and are sensitive to different physics, they are unlikely to be biased in the same way. The raw asymmetry

$$\epsilon_{LL} \equiv \frac{r^{++} - r^{+-}}{r^{++} + r^{+-}}, r = \frac{N_{ZDC}}{N_{BBC}} \tag{21}$$

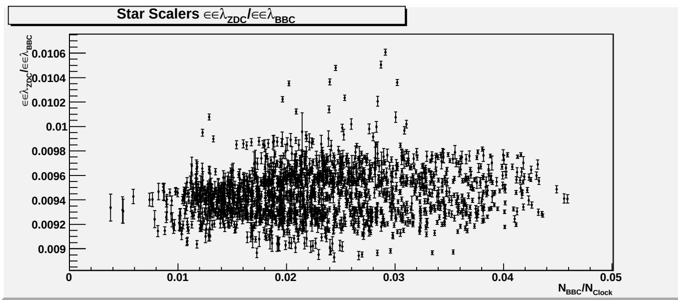


Figure 3:

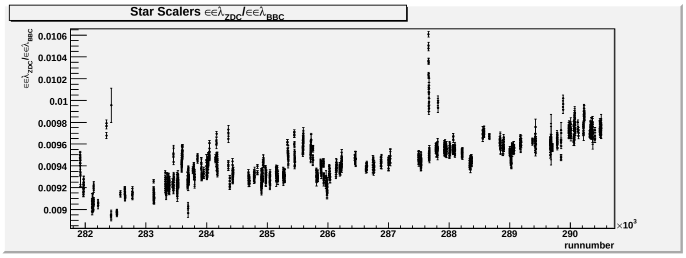


Figure 4:

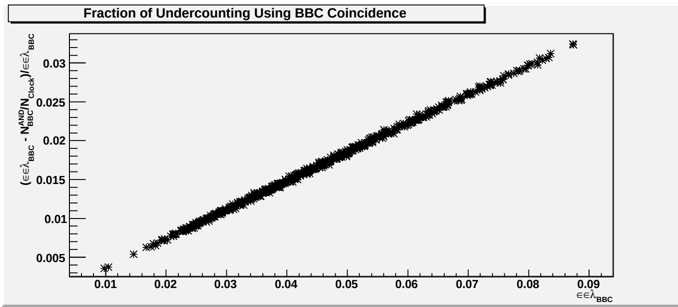


Figure 5: Measure of the fraction of "undercounting" in the BBC when using the simple coincidence of the two arms.

is measured by fitting across bunches in individual runs or fills, and then these values are combined to give an overall asymmetry. This value is scaled up by the polarizations of the beams to reflect the fact that it may be due to physics. There are four possible scenarios in which to draw conclusions from the final result:

- The BBC is unbiased.

  In this case, we can re-express the raw asymmetry in the form

$$\epsilon_{LL} = \frac{N^{++}_{ZDC} - \frac{N^{++}_{BBC}}{N^{+-}_{BBC}} N^{+-}_{ZDC}}{N^{++}_{ZDC} + \frac{N^{++}_{BBC}}{N^{+-}_{BBC}} N^{+-}_{ZDC}} \tag{22}$$

  Then the BBC measurement serves as the Relative Luminosity, and our result gives us the $++/+-$ asymmetry (physics or otherwise) for detection by the ZDC.

- The ZDC is unbiased.

  Similar to the first case, here we get the asymmetry for detection by the BBC.

- Both detectors are biased in the same direction.

  In this case, our measure of the bias reflects neither the BBC or ZDC asymmetry, and in fact is smaller than either bias alone. Recall that our assumption is that both detectors are unlikely to be biased in the same way. Assuming the detectors not to be biased in the same direction is a stronger assumption.

- Both detectors are biased in the opposite direction.

  In this case, our result is greater than the actual bias of either detector.

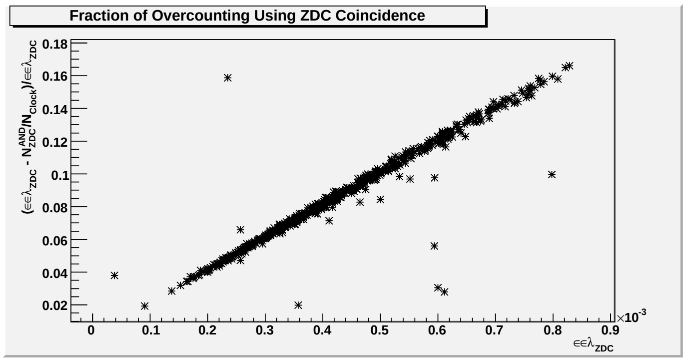


Figure 6: Measure of the fraction of "overcounting" in the ZDC when using the simple coincidence of the two arms.

6.2 Chi-Squared Bunch Fitting

We do a chi-squared fit to the equation

$$\left(\frac{N_{ZDC}}{N_{BBC}}\right)_i \equiv r_i^{\pm} = c(1 \pm \epsilon_{LL}), \tag{23}$$

where the sign $\pm$ depends on the helicity of the bunches ($\{++\} = +, \{+-\} = -$). This procedure gives ϵ_{LL} equivalent to Equation 21.

Chi-squared is given by

$$\sum_{i^+} \frac{(c(1+\epsilon_{LL}) - r_i^+)^2}{\sigma_{r_i^+}} + \sum_{i^-} \frac{(c(1-\epsilon_{LL}) - r_i^-)^2}{\sigma_{r_i^-}}. \tag{24}$$

Taking the derivatives with respect to c and ϵ_{LL} and setting them to zero results in

$$\begin{aligned} \frac{\partial \chi^2}{\partial \epsilon_{LL}} &= 0 = & \sum_{i^+} \frac{2c(c(1+\epsilon_{LL}) - r_i^+)}{\sigma_{r_i^+}} - \sum_{i^-} \frac{2c(c(1-\epsilon_{LL}) - r_i^-)}{\sigma_{r_i^-}}, \\ \frac{\partial \chi^2}{\partial c} &= 0 = & \sum_{i^+} \frac{2(1+\epsilon_{LL})(c(1+\epsilon_{LL}) - r_i^+)}{\sigma_{r_i^+}} + \sum_{i^-} \frac{2(1-\epsilon_{LL})(c(1-\epsilon_{LL}) - r_i^-)}{\sigma_{r_i^-}} \end{aligned}$$

These equations can be conveniently rewritten using the following definitions:

$$\begin{aligned} S^+ &= \sum_{i^+} \frac{1}{\sigma^2_{r_i^+}}, \\ S^- &= \sum_{i^-} \frac{1}{\sigma^2_{r_i^-}}, \\ S_r^+ &= \sum_{i^+} \frac{r_i^+}{\sigma^2_{r_i^+}}, \\ S_r^+ &= \sum_{i^-} \frac{r_i^-}{\sigma^2_{r_i^-}}. \end{aligned}$$

With these definitions we have (dividing out the common factors of 2 and $2c$)

$$\begin{aligned} 0 &= c(1+\epsilon_{LL})S^+ - c(1-\epsilon_{LL})S^- - S_r^+ + S_r^-, \\ 0 &= c(1+\epsilon_{LL})^2 S^+ + c(1-\epsilon_{LL})^2 S^- - (1+\epsilon_{LL})S_r^+ - (1-\epsilon_{LL})S_r^-. \end{aligned} \tag{25}$$

We now solve both of these equations for c. We get

$$\begin{aligned} c &= \frac{(1+\epsilon_{LL})S_r^+ + (1-\epsilon_{LL})S_r^-}{(1+\epsilon_{LL})^2 S^+ + (1-\epsilon_{LL})^2 S^-}, \\ c &= \frac{S_r^+ - S_r^-}{(1+\epsilon_{LL})S^+ - (1-\epsilon_{LL})S^-}. \end{aligned} \tag{26}$$

Setting the two equations equal, we can solve for ϵ_{LL}:

$$\epsilon_{LL} = \frac{S_r^+ S^- - S_r^- S^+}{S_r^+ S^- + S_r^- S^+} \tag{27}$$

which when substituted into the above equation for c yields:

$$c = \frac{S_r^+ S^- + S_r^- S^+}{2S^+ S^-} \tag{28}$$

We now calculate the uncertainty in each of these parameters. To do this, note that derivatives with respect to r_i^+ only affect S_r^+, and r_i^- S_r^-. Also, by the chain rule,

$$\frac{\partial S_r^+}{\partial r_i^+} = \frac{1}{\sigma_{r_i^+}^2}, \tag{29}$$

and similarly for $-$. Using this, we find

$$\sigma_c^2 = \frac{1}{4}\left(\frac{1}{S^+} + \frac{1}{S^-}\right) \tag{30}$$

$$\sigma_{\epsilon_{LL}}^2 = \frac{4(S^+S^-)^2}{(S_r^+ S^- + S_r^- S^+)^4}\left((S_r^+)^2 S^- + (S_r^-)^2 S^+\right). \tag{31}$$

6.3 Results and Conclusions

Figures 7 and 8 show the results of the bunch fitting analysis, performed both in the usual way with GL1p scaler data, and also with the results $\epsilon_N \epsilon_S \lambda$ using Star Scalers data. For the parameter c, we see a clear rise over the Run in both cases, which amounts to a rise in the ratio $\frac{(\epsilon_N\epsilon_S)_{ZDC}}{(\epsilon_N\epsilon_S)_{BBC}}$.

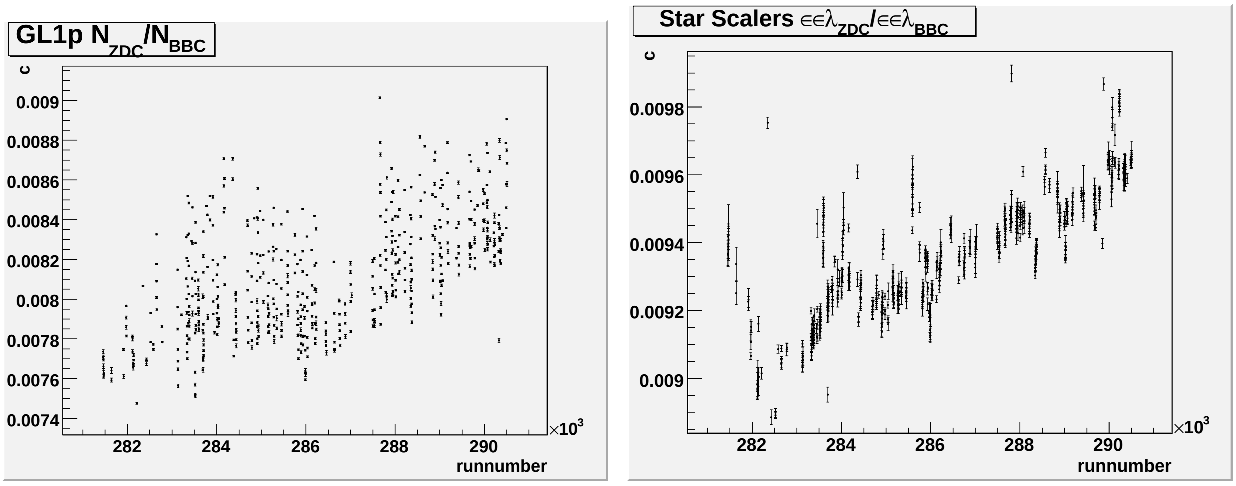
In both ϵ_{LL} plots we see evidence of positive asymmetries of comparable size. This result should be taken with a grain of salt since both chi-squareds are unreasonable. However, it should be noted that in the case of a full analysis that takes into account the effect of ZDC vertex-smearing (see AN 881), the GL1p result is essentially the same. Also, the chi-squared of the Star Scalers analysis is artificially low due to not accounting for correlation between N, S, and OR counts (which should reduce the uncertainty and increase chi-squared). Furthermore, since the Star Scalers analysis has no vertex cut, it should not suffer a ZDC vertex-smearing effect. For these reasons, we might take the ϵ_{LL} results as evidence that the ZDC-BBC asymmetry is not affected by rate.

7 Behavior Single Sided Event Rates

7.1 Fraction of Accidental Coincidences

The probability for the "AND" of a two-armed detector can be expressed as

$$\begin{aligned} P_{AND} &= 1 - P(k_N = 0, k_S = 0) - P(k_N > 0, k_S = 0) - P(k_N = 0, k_S > 0) \\ &= 1 + P(k_N = 0, k_S = 0) - P(k_N = 0) - P(k_S = 0) \\ &= 1 + e^{\epsilon_N\epsilon_S\lambda - \epsilon_N(\lambda+\lambda_N) - \epsilon_S(\lambda+\lambda_S)} - e^{\epsilon_N(\lambda+\lambda_N)} - e^{\epsilon_S(\lambda+\lambda_S)}. \end{aligned} \tag{32}$$



(a) (b)

Figure 7: Results for the constant term in the bunch fitting described in the text.

(Here we again write $\epsilon^{SS} = \epsilon$, although as in the above the results are equally valid without this assumption, since any difference may be absorbed into λ_N and λ_S.)

If we expand to first order in λ's and second order in ϵ's, we get simply

$$P_{AND} \approx \epsilon_N \epsilon_S \lambda. \tag{33}$$

Expanding to second order in λ's and second order in ϵ's gives an additional term:

$$\begin{aligned} P_{AND} &\approx \epsilon_N \epsilon_S \lambda + \epsilon_N (\lambda + \lambda_N) \epsilon_S (\lambda + \lambda_S) \\ &= \epsilon_N \epsilon_S \lambda + \epsilon_N \epsilon_S \lambda^2 + \epsilon_N \epsilon_S \lambda \lambda_N + \epsilon_N \epsilon_S \lambda \lambda_S + \epsilon_N \epsilon_S \lambda_N \lambda_S. \end{aligned} \tag{34}$$

From this equation we can pick out a contribution from accidental coincidence with, or involving only, single-sided events, which we define as P^{SS}_{AND}:

$$P^{SS}_{AND} \approx \epsilon_N \epsilon_S \lambda \lambda_N + \epsilon_N \epsilon_S \lambda \lambda_S + \epsilon_N \epsilon_S \lambda_N \lambda_S. \tag{35}$$

What we would like to estimate is

$$f \equiv \frac{P^{SS}_{AND}}{P_{AND}}, \tag{36}$$

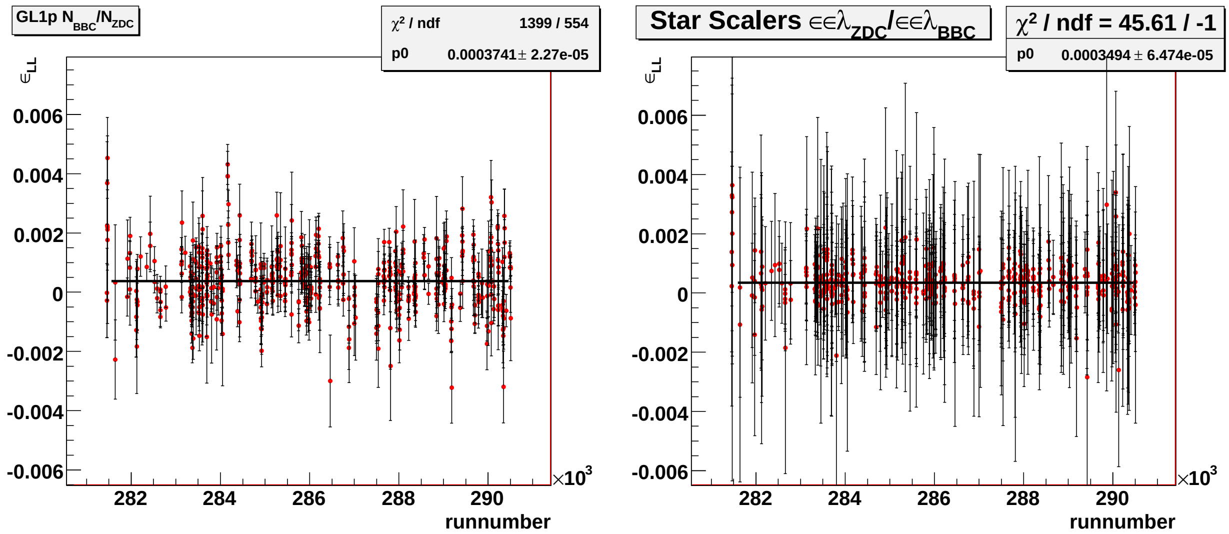
the fractional contribution of single-sided events to the total coincidence rate.

7.2 Dependence of Single-Sided Rates on the Double-Sided Rate

Other quantities of interest would be

$$\begin{aligned} d_N &\equiv \frac{\lambda_N}{\lambda} \\ d_S &\equiv \frac{\lambda_S}{\lambda}, \end{aligned} \tag{37}$$

since they tell us how the single-sided events behave as a function of rate.



(a) (b)

Figure 8: Results ϵ_{LL} from bunch fitting.

7.3 Estimating f and d

We can express the quantity f of Equation 36 in terms of the unknown quantity $\epsilon_N\epsilon_S$. To do this first we make the definitions

$$\begin{aligned} R_2 &\equiv \epsilon_N\epsilon_S\lambda \\ R_N &\equiv \epsilon_N\lambda + \epsilon_N\lambda_N = ln(1 - \frac{N_N}{N_{clock}}) \\ R_S &\equiv \epsilon_S\lambda + \epsilon_N\lambda_S = ln(1 - \frac{N_S}{N_{clock}}) \end{aligned} \tag{38}$$

Then we note that

$$P^{SS}_{AND} = R_N R_S - \epsilon_N\epsilon_S\lambda^2 = R_N R_S - \frac{R_2^2}{\epsilon_N\epsilon_S}, \tag{39}$$

in which case

$$f = \frac{N_{clock}}{N_{AND}} * \left(R_N R_S - \frac{R_2^2}{\epsilon_N\epsilon_S}\right) \tag{40}$$

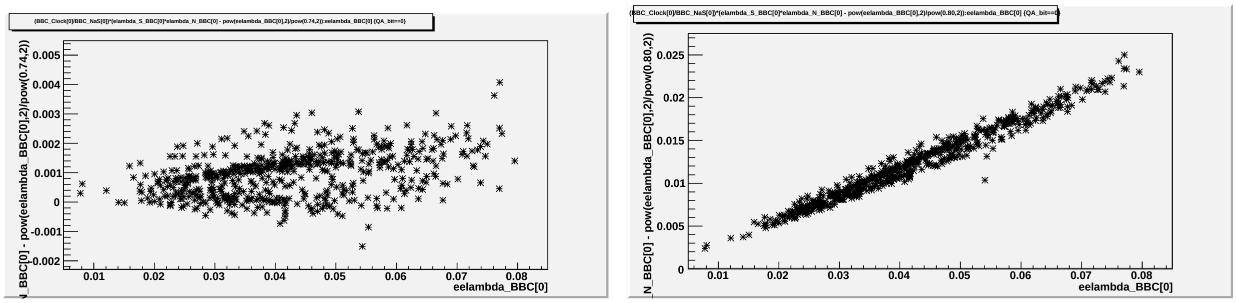
In order to estimate d, if we make the quantity

$$\epsilon_S * \frac{R_N}{R_2} - 1 = \epsilon_S\frac{\epsilon_N\lambda + \epsilon_N\lambda_N}{\epsilon_N\epsilon_S\lambda} - 1 = \frac{\lambda_N}{\lambda}, \tag{41}$$

we can isolate the ratio $\frac{\lambda_N}{\lambda}$ if we know ϵ_S. A similar expression holds for the South detector

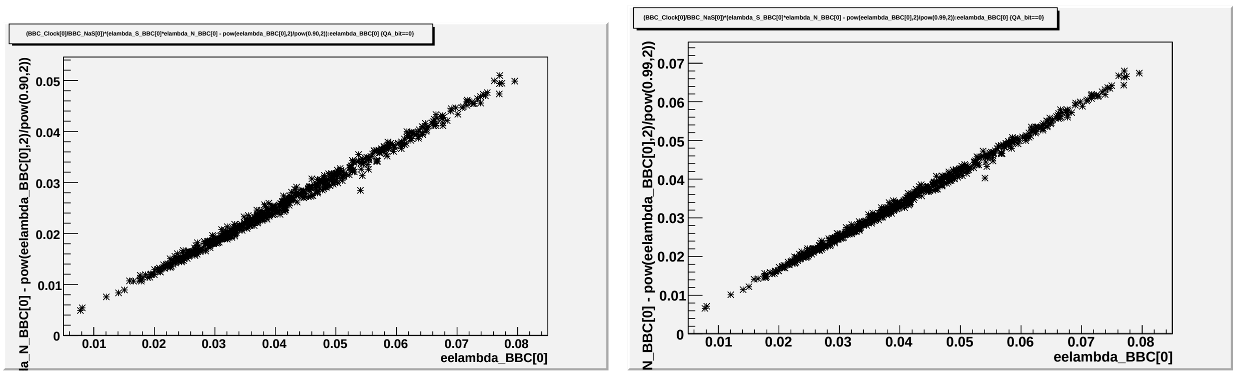
Thus, in order to estimate f and d, we must have estimates for the ϵ's. Figures 9 through 20 show what the quantities would be for different guesses of ϵ, assuming when necessary $\epsilon_N = \epsilon_S$.

One important conclusion that can be drawn from these plots is that $\frac{\lambda_N}{\lambda}$ is constant. This means that the rate of single-sided events is proportional to the rate of double-sided events, which implies that most of the background is beam-related.



(a) **(b)**

Figure 9: f for the BBC (see text) for assumptions (a) $\epsilon = 0.74$ and (b) $\epsilon = 0.80$.



(a) **(b)**

Figure 10: f for the BBC (see text) for assumptions (a) $\epsilon = 0.90$ and (b) $\epsilon = 0.99$.

8 Other Rate-Related Issues

8.1 Multiple Collisions and Z-vertex determination

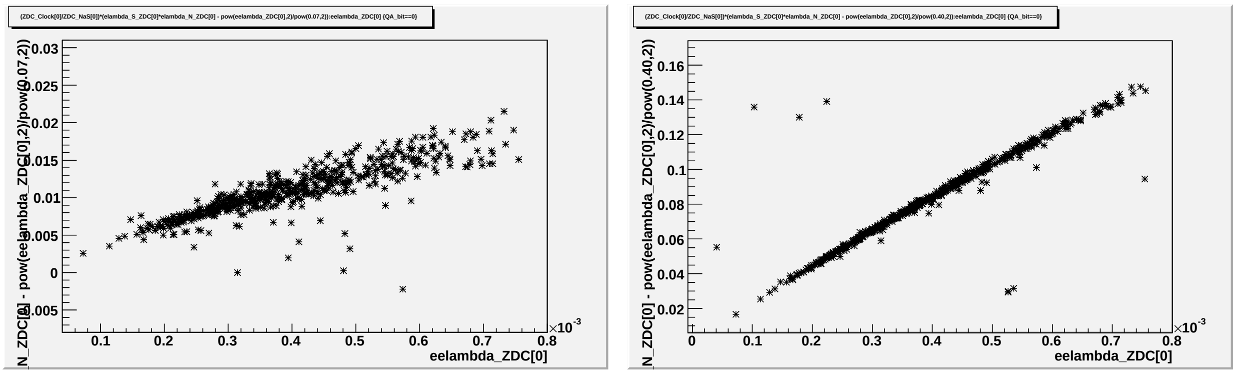
Here we derive the z-vertex distribution arising from incorrectly-paired multiple collisions: In general, for the arrival time of light signals in the N and S detectors, we have

$$\begin{aligned} t_N &= (l - z_N) + t_{0,N} \\ t_S &= (l + z_S) + t_{0,S}, \end{aligned} \tag{42}$$

where l is the distance between the detectors, z is the location along the beam line of the collision, and t_0 is the collision time relative to the beam clock tick. Here we have set the speed of light $c = 1$.

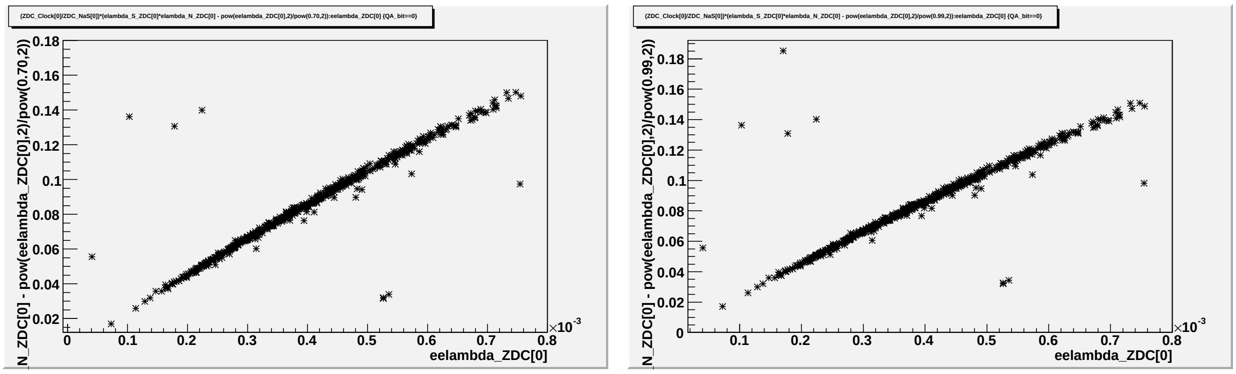
In the case where both signals come from the same vertex, the z's and t_0's are equal, and we can find z from $(t_S - t_N)/2$. In the case of signals from different vertices, however, if we try to construct z in the same way we get $z_{fake} = (z_S + z_N + (t_{0,S} - t_{0,N}))/2$. Whether the z_{fake} distribution is wider or narrower than the z_{real} distribution depends on the relative size of σ_z and σ_{t_0}. In fact, if z and t_0 are Gaussian distributed random variables, we have

$$\begin{aligned} \mu_{z_{fake}} &= \frac{\mu_{z_N} + \mu_{z_S} + (\mu_{t_{0,S}} - \mu_{t_{0,N}})}{2} \\ \sigma^2_{z_{fake}} &= (\frac{\sigma_{z_N}}{2})^2 + (\frac{\sigma_{z_S}}{2})^2 + (\frac{\sigma_{t_{0,N}}}{2})^2 + (\frac{\sigma_{t_{0,S}}}{2})^2. \end{aligned}$$



(a) **(b)**

Figure 11: f for the ZDC (see text) for assumptions (a) $\epsilon = 0.07$ and (b) $\epsilon = 0.40$.



(a) **(b)**

Figure 12: f for the ZDC (see text) for assumptions (a) $\epsilon = 0.70$ and (b) $\epsilon = 0.99$.

Taking the means and variances for the z's, as well as for the t_0's, to be equal results in

$$\mu_{z_{fake}} = \mu_{z_{real}} \tag{43}$$

$$\sigma^2_{z_{fake}} = \frac{\sigma^2_{z_{real}} + \sigma^2_{t_0}}{2}. \tag{44}$$

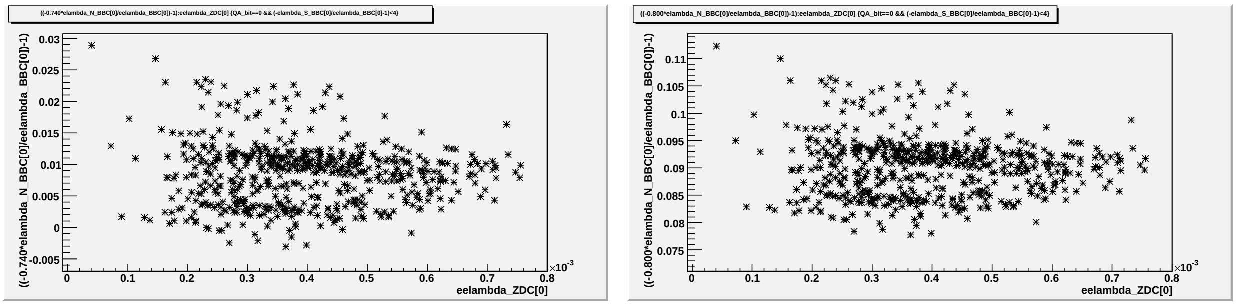
9 Loose Ends

There are two important points for further effort in the future

- Correlations

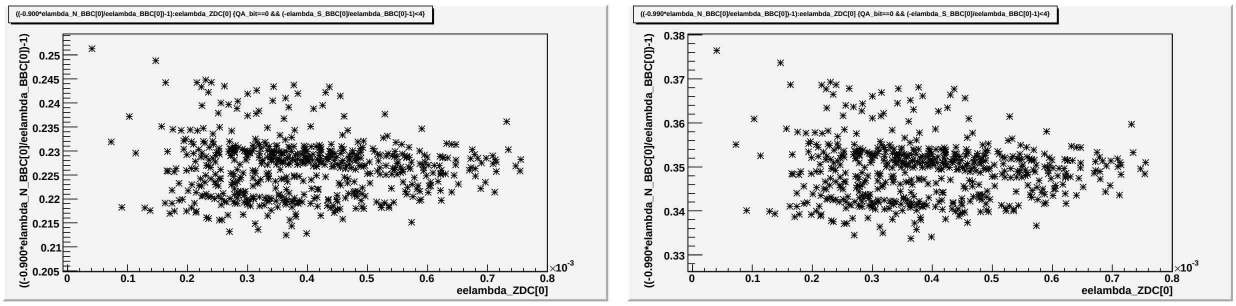
  In order to make the $\epsilon_N \epsilon_S \lambda$ calculations useful for determining/correcting relative luminosity, correlations should be properly taken into account so that chi-squared results are sensible. To measure the correlation experimentally, it would be necessary to bin results according to both some rate proxy and N_{clock}, since results should come from similar underlying probability distributions to make sense. Alternatively, the correlations might be calculated using Monte-Carlo techniques.

- Z-vertex cut



(a) **(b)**

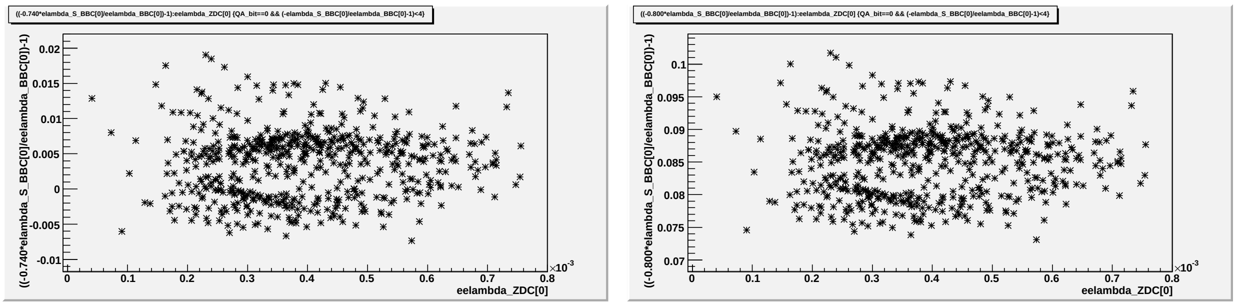
Figure 13: d for the BBC_N (see text) for assumptions (a) $\epsilon_S = 0.74$ and (b) $\epsilon_S = 0.80$.



(a) **(b)**

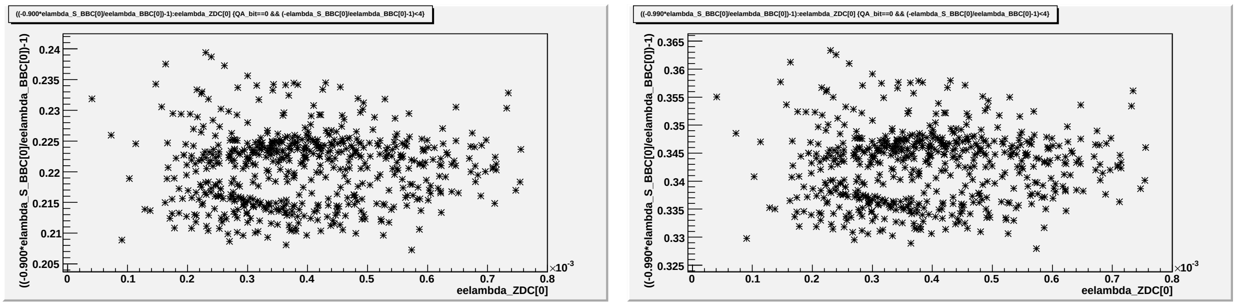
Figure 14: d for the BBC_N (see text) for assumptions (a) $\epsilon_S = 0.90$ and (b) $\epsilon_S = 0.99$.

To make any rate correction on GL1p scaler counts, or to make the $\epsilon_S \epsilon_N \lambda$ relative luminosity correspond to measurements with a z-vertex cut, we must determine the fraction of events within a specified range in z. This could be done in an unbiased way using clock data, although the uncertainty might be unfeasibly large. Also, as shown in Section 8.1, multiple collisions change the observed z-vertex, although since this effect is contained in multiple collisions, it might effectively be a correction on a correction.



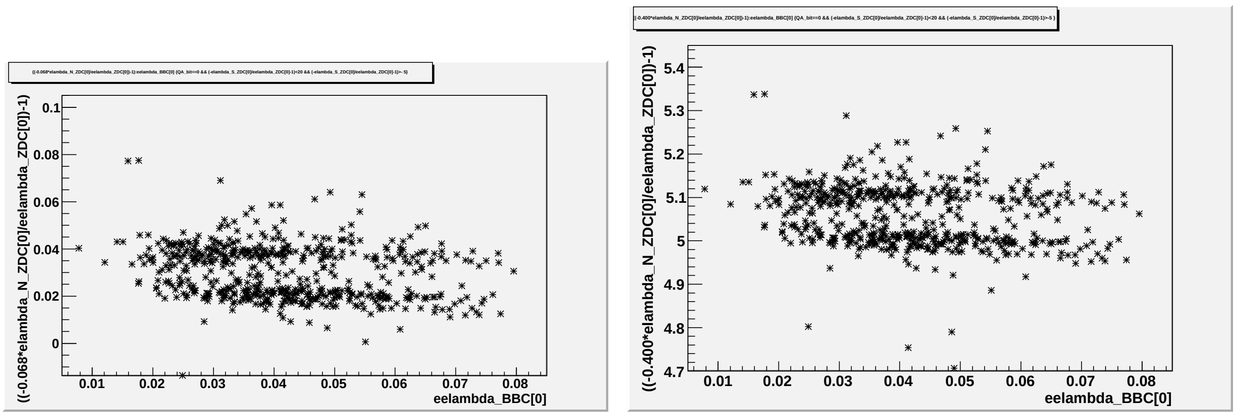
(a) **(b)**

Figure 15: d for the BBC_S (see text) for assumptions (a) $\epsilon_N = 0.74$ and (b) $\epsilon_N = 0.80$.



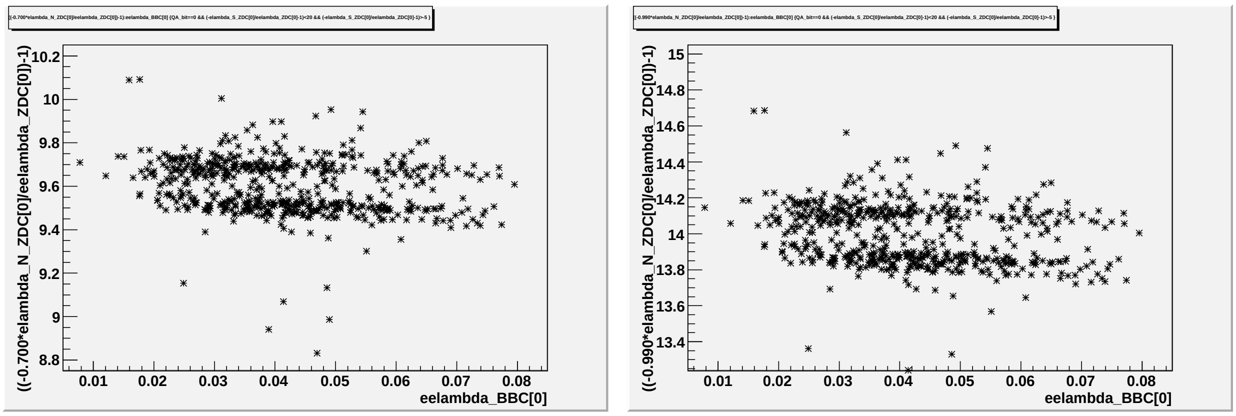
(a) **(b)**

Figure 16: d for the BBC_S (see text) for assumptions (a) $\epsilon_N = 0.90$ and (b) $\epsilon_N = 0.99$.



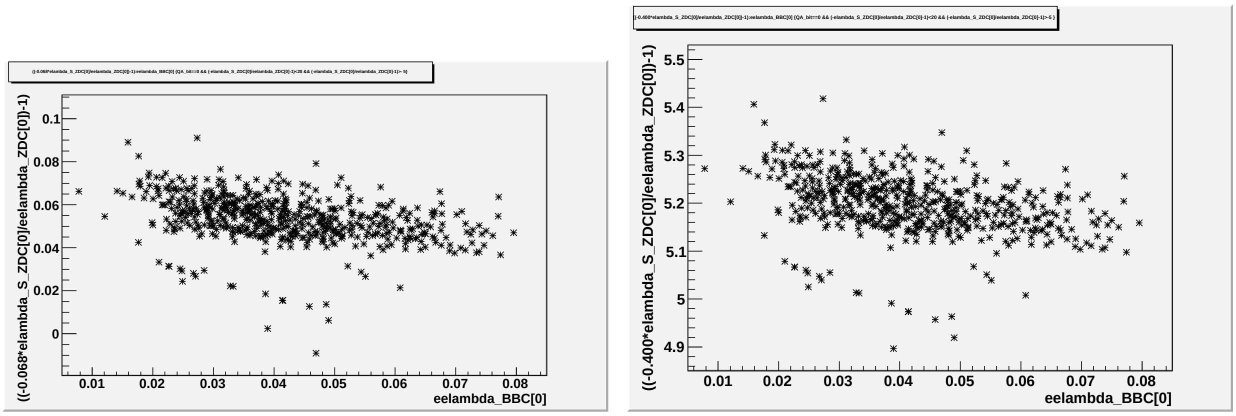
(a) **(b)**

Figure 17: d for the ZDC_N (see text) for assumptions (a) $\epsilon_S = 0.068$ and (b) $\epsilon_S = 0.40$.



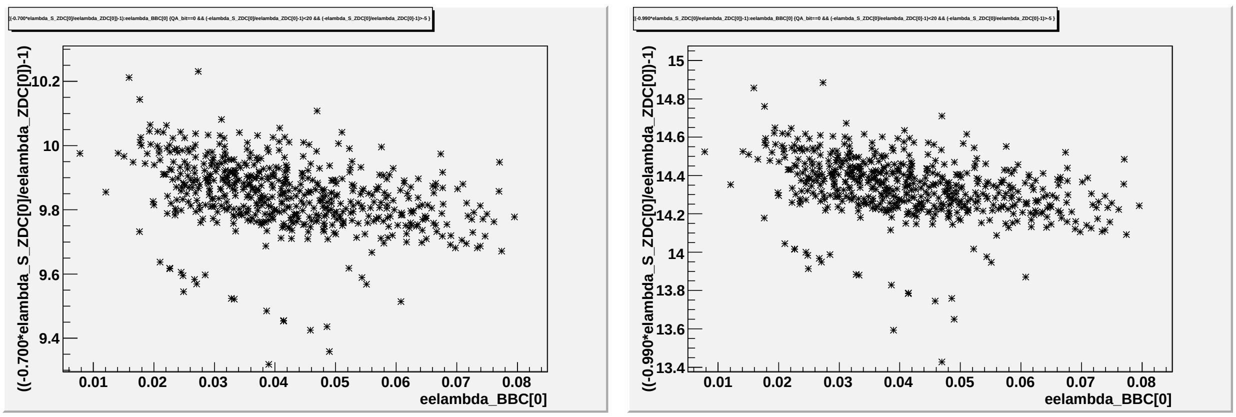
(a) (b)

Figure 18: d for the ZDC_N (see text) for assumptions (a) $\epsilon_S = 0.70$ and (b) $\epsilon_S = 0.99$.



(a) (b)

Figure 19: d for the ZDC_S (see text) for assumptions (a) $\epsilon_N = 0.068$ and (b) $\epsilon_N = 0.40$.



(a) (b)

Figure 20: d for the ZDC_S (see text) for assumptions (a) $\epsilon_N = 0.70$ and (b) $\epsilon_N = 0.99$.