References

[1] *On the energy loss of fast particles by ionization,* L. Landau, J. Phys. **VIII** (1944) 201.

[2] *Calculation of energy deposition in argon gas for NA35 and NA49 TPCs,* H. Bichsel, p. 57, Annual Report, Nuclear Physics Laboratory, April 1993

[3] *Energy loss straggling functions needed in NA49 and STAR,* H. Bichsel, p. 42, Annual Report, Nuclear Physics Laboratory, April 1994

[4] *Energy deposition and particle identification in a TPC,* H. Bichsel, p. 46, Annual Report, Nuclear Physics Laboratory, April 1995

[5] *Energy deposition by low energy electrons in Ar and other gases,* H. Bichsel, p. 49, Annual Report, Nuclear Physics Laboratory, April 1996

[6] *Review of Particle Physics,* Eur. Phys. J. C **15** (2000) 1-878.

[7] *Straggling and Particle Identification in Silicon Detectors,* Hans Bichsel, Nucl. Inst. Meth. **78**, 277-284 (1970).

[8] *Charged particle-matter interactions.* Hans Bichsel, ch. 87 in Atomic, molecular and optical physics handbook, G. Drake, ed., Amer. Inst. Phys. (1996)

[9] *Inelastic electronic collision cross sections for Monte Carlo calculations.* Hans Bichsel, Nucl. Inst. Meth. B **52**, 136 (1990)

[10] *Straggling in thin silicon detectors.* Hans Bichsel, Rev. Mod. Phys. **60** (1988) 663.

[11] *Comparison of calculational methods for straggling in thin absorbers.* Hans Bichsel and Roberta P. Saxon, Phys. Rev. **A 11**, 1286 (1975).

[12] *Relativistic charged particle identification by energy loss* W. W. M. Allison and J. H. Cobb, Ann. Rev. Nucl. Part. Sci. **30** (1980) 253.

[13] *Energy-loss in thin layers of argon.* Hans Bichsel, STAR Note **SN 0418** (18 March 2000).

NPL-PC::[bichsel]CBBBFIG.tex 15:15 21.12.2001

III. Conclusions.

From Figs. 4 and 5 it appears that a single Bichsel function

$$B(\beta\gamma) = \beta^{-2} \cdot (1.2101 - 0.01384\beta\gamma + 0.03401\beta^2\gamma^2 - 0.004868\beta^3\gamma^3) \tag{1}$$

will give

$$\Delta_p(x, \beta\gamma)/x = b(x) \cdot B(\beta\gamma) \tag{2}$$

for $x > 2$ cm and $0.5 \leq \beta\gamma \leq 3.16$ with an error of less than 2%, with $b(x)$ given in Table I.

From Fig. 5 it is clear that the use of $x < 2$ cm will lead to further complexities which can be understood from the description of $f(\Delta)$ in STAR NOTE 418 [13]. The Bethe-Bloch function $dE/dx(\beta\gamma)$ is 12% lower than $B(\beta\gamma)$ at $\beta\gamma = 1$ and 12% higher at $\beta\gamma = 1000$. It must be clearly understood that Bichsel functions for truncated mean values for full tracks cannot be related in a simple fashion to the functions for single segments. This aspect will be discussed in a forthcoming Note.

Table I. The scaling function $b(x)$ of Eq. (2) at $\beta\gamma = 1.585$. The function $B(\beta\gamma)$ was a least-squares-fit to the function for $x = 8$ cm. The values fluctuate by less than 2% with $\beta\gamma$.

$x(cm)$	2	4	8	10	20	40	80
$b(x)$	0.851	0.928	1.000	1.025	1.101	1.166	1.226

A additional difference between the Bethe-Bloch function $dE/dx(\beta\gamma)$ and the "Bichsel function" $B(\Delta_p/x, \beta\gamma)$ is not evident from Fig. 3. It is seen if the functions are scaled to a common value at e.g. the minimum ionization. Scaled functions are shown in Fig. 4. Clearly, the Bethe-Bloch dE/dx function is not an accurate representation of the dependence of Δ_p/x on $\beta\gamma$ and x. A more detailed and instructive representation of the dependance on x and $\beta\gamma$ can be obtained with the function shown in Fig. 5. Note that the function for $x = 1$ cm differs considerably from the others.

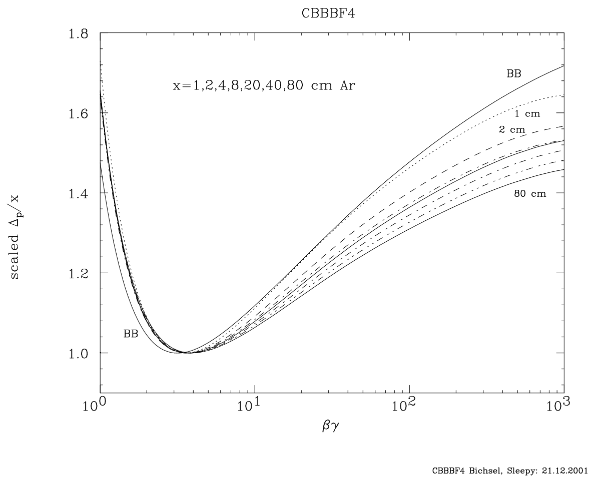

Figure 4: Bichsel functions $g\Delta_p/x$ compared to the Bethe-Bloch function. The coefficient g is a scaling factor chosen such that the functions Δ_p/x coincide at minimum ionization. **Note:** functions for truncated mean values differ from the functions shown here.

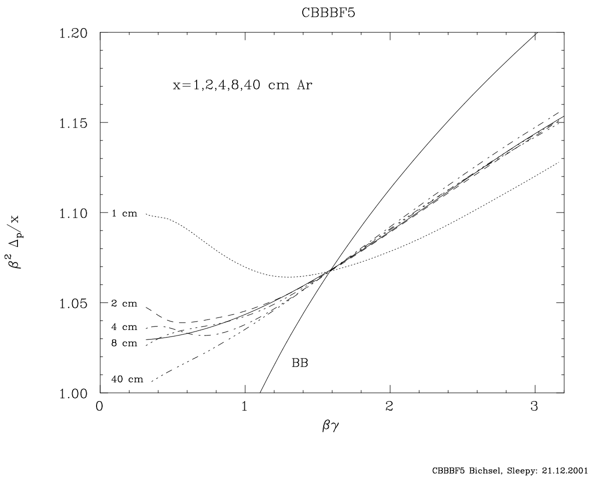

Figure 5: The function $\beta^2\Delta_p/x$ scaled to a common value at $\beta\gamma = 1.513$. Solid line BB: scaled Bethe-Bloch function BB. The function equivalent to Eq. (1) is given by the solid line fitted to the $x = 8$ cm function.

For an accurate analysis in particle identification, we must use the most probable energy loss Δ_p as the parameter associated with $f(\Delta)$. For silicon detectors it was found that Δ_p/x depends on x (Table V in [10]). A similar dependence for Ar is seen in Fig. 2. Thus the ratio of the Bethe-Bloch dE/dx to I_p/x or Δ_p/x depends on x. This is shown in Fig. 3.

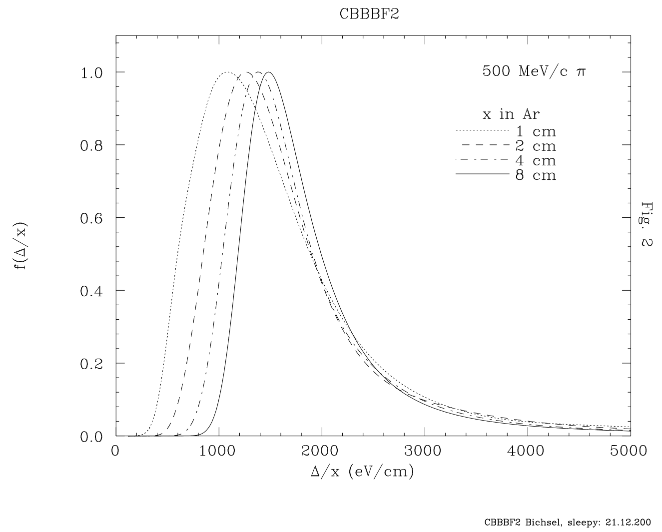

Figure 2: Straggling (energy loss) functions $f(\Delta/x)$ [note that Δ/x is called dE/dx in current literature of particle physics] for $p = 500$ MeV/c pions, for $x = 1, 2, 4, 8$ cm.

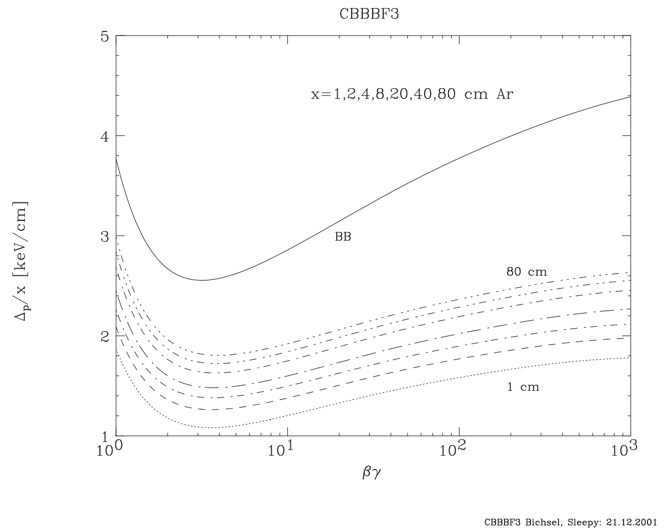

Figure 3: Values of Δ_p/x for segment lengths x as a function of particle momentum (given by $\beta\gamma$). The Bethe-Bloch function, Eq. (23.1) in ref [8], is shown by the solid line BB.

$< \Delta >= x \cdot S(\beta)$=3044 eV where $S = dE/dx$ is the Bethe-Bloch function, Eq. (23.1) in [6]. It is more than twice Δ_p. In other contexts, dE/dx is called the stopping power. For present purposes, $f(\Delta)$ is the same for all particles with charge ± 1 and the same $\beta\gamma$ (e.g. 1.8 MeV/c e, 0.5 GeV/c π, 1.8 GeV/c K or 3.4 GeV/c protons). For large Δ, there are some differences, e.g. for $\beta\gamma = 50$, at $\Delta = 1$ MeV, the electron spectrum is 0.2% higher than the pion spectrum.

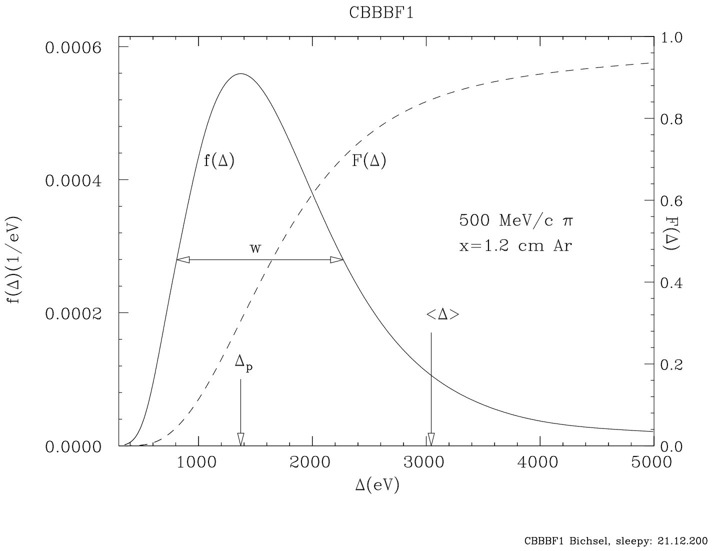

Figure 1: The solid line shows the probability density function $f(\Delta)$ for energy losses Δ of p = 500 MeV/c pions ($\beta\gamma$ = 3.58) crossing a volume V of argon. The dashed line shows the cumulative density function $F(\Delta)$ used for Monte Carlo calculations. For an experimentally measured function, $< \Delta >$ will depend strongly on the largest observed value of Δ.

Abstract.

The Bethe-Bloch equation gives an inaccurate representation of the functional dependance on particle energy of the relevant quantity needed for the evaluation of the ionization spectra from detectors used for particle identification. Instead, "Bichsel functions" are presented here. These functions depend on segment length as well as on truncation method. Only the former dependance is discussed here. An important result : it is inadvisable to use segments less than 2 cm in length.

I. Introduction.

The quantity observed in a TPC or a silicon array is the ionization I in a small volume V along the track of a fast charged particle. The length of the segment of the track inside the volume is x. The energy lost along x by the particle is Δ (using Landau notation, [1]). The ratio $\omega = \Delta/I$ is almost constant. Usually, I is called dE and is assumed to be equivalent to Δ, and an "energy calibration" is made to determine ω. Also, x is usually called dx, and then I/x is called "dE/dx". For the present purpose, various effects are neglected. They are, for example, the escape of δ rays from V [2, 3, 4, 5], the diffusion of electrons during their traversal to the proportional counters, recombination etc.

A function frequently used to describe I is the "straggling function". It is the probability density function (PDF) [6, 7] $f(I, x, \beta)$ where β is the speed of the particle. This note will consider $f(\Delta, x, \beta)$ instead of $f(I, x, \beta)$. A comparison of the two functions can be found e.g. in [5].

II. Calculation of straggling functions

A realistic simulation of the traversal of a particle through a segment of length x in the detector can be done with a Monte Carlo calculation [8, 9]. For each particle an energy loss Δ is obtained, and the straggling function is obtained by binning the Δ into $f(\Delta)$. A faster and more accurate method is the use of convolutions ([10, 11, 12, 4]). The arguments x and β are always implicit in $f(\Delta)$, but are not always given.

As an example, and to define some concepts, a function $f(\Delta)$ calculated with convolutions is given in Fig. 1, for p = 500 MeV/c pions ($\beta\gamma$ = 3.58) traversing a segment of length x = 1.2 cm. Important parameters describing the function are the most probable energy loss $\Delta_p(x, \beta)$ (the position of the maximum of the straggling function, at 1371 eV), and the full-width-at-half maximum (fwhm) $w(x, \beta)$=1463 eV. The mean energy loss is also shown. It is

University of Washington Nuclear Physics Laboratory

Comparison of Bethe-Bloch and Bichsel functions.

Hans Bichsel
Telephone: 206-329-2792, FAX: 206-685-4634
e-mail: bichsel@gluon.npl.washington.edu
Center for Experimental Nuclear Physics and Astrophysics
Box 354290 University of Washington
Seattle, WA 98195-4290

December 21, 2001

Ultrarelativistic Heavy Ions Group

References

[1] *On the energy loss of fast particles by ionization,* L. Landau, J. Phys. **VIII** (1944) 201.

[2] *Calculation of energy deposition in argon gas for NA35 and NA49 TPCs,* H. Bichsel, p. 57, Annual Report, Nuclear Physics Laboratory, April 1993

[3] *Energy loss straggling functions needed in NA49 and STAR,* H. Bichsel, p. 42, Annual Report, Nuclear Physics Laboratory, April 1994

[4] *Energy deposition and particle identification in a TPC,* H. Bichsel, p. 46, Annual Report, Nuclear Physics Laboratory, April 1995

[5] *Energy deposition by low energy electrons in Ar and other gases,* H. Bichsel, p. 49, Annual Report, Nuclear Physics Laboratory, April 1996

[6] *Review of Particle Physics,* Eur. Phys. J. C **15** (2000) 1-878.

[7] *Straggling and Particle Identification in Silicon Detectors,* Hans Bichsel, Nucl. Inst. Meth. **78**, 277-284 (1970).

[8] *Charged particle-matter interactions.* Hans Bichsel, ch. 87 in Atomic, molecular and optical physics handbook, G. Drake, ed., Amer. Inst. Phys. (1996)

[9] *Inelastic electronic collision cross sections for Monte Carlo calculations.* Hans Bichsel, Nucl. Inst. Meth. B **52**, 136 (1990)

[10] *Straggling in thin silicon detectors.* Hans Bichsel, Rev. Mod. Phys. **60** (1988) 663.

[11] *Comparison of calculational methods for straggling in thin absorbers.* Hans Bichsel and Roberta P. Saxon, Phys. Rev. **A 11**, 1286 (1975).

[12] *Relativistic charged particle identification by energy loss* W. W. M. Allison and J. H. Cobb, Ann. Rev. Nucl. Part. Sci. **30** (1980) 253.

[13] *Energy-loss in thin layers of argon.* Hans Bichsel, STAR Note **SN 0418** (18 March 2000).

NPL-PC::[bichsel]CBBBFIG.tex 15:15 21.12.2001

III. Conclusions.

From Figs. 4 and 5 it appears that a single Bichsel function

$$B(\beta\gamma) = \beta^{-2} \cdot (1.2101 - 0.01384\beta\gamma + 0.03401\beta^2\gamma^2 - 0.004868\beta^3\gamma^3) \tag{1}$$

will give

$$\Delta_p(x, \beta\gamma)/x = b(x) \cdot B(\beta\gamma) \tag{2}$$

for $x > 2$ cm and $0.5 \leq \beta\gamma \leq 3.16$ with an error of less than 2%, with $b(x)$ given in Table I.

From Fig. 5 it is clear that the use of $x < 2$ cm will lead to further complexities which can be understood from the description of $f(\Delta)$ in STAR NOTE 418 [13]. The Bethe-Bloch function $dE/dx(\beta\gamma)$ is 12% lower than $B(\beta\gamma)$ at $\beta\gamma = 1$ and 12% higher at $\beta\gamma = 1000$. It must be clearly understood that Bichsel functions for truncated mean values for full tracks cannot be related in a simple fashion to the functions for single segments. This aspect will be discussed in a forthcoming Note.

Table I. The scaling function $b(x)$ of Eq. (2) at $\beta\gamma = 1.585$. The function $B(\beta\gamma)$ was a least-squares-fit to the function for $x = 8$ cm. The values fluctuate by less than 2% with $\beta\gamma$.

$x(cm)$	2	4	8	10	20	40	80
$b(x)$	0.851	0.928	1.000	1.025	1.101	1.166	1.226

A additional difference between the Bethe-Bloch function $dE/dx(\beta\gamma)$ and the "Bichsel function" $B(\Delta_p/x, \beta\gamma)$ is not evident from Fig. 3. It is seen if the functions are scaled to a common value at e.g. the minimum ionization. Scaled functions are shown in Fig. 4. Clearly, the Bethe-Bloch dE/dx function is not an accurate representation of the dependence of Δ_p/x on $\beta\gamma$ and x. A more detailed and instructive representation of the dependance on x and $\beta\gamma$ can be obtained with the function shown in Fig. 5. Note that the function for $x = 1$ cm differs considerably from the others.

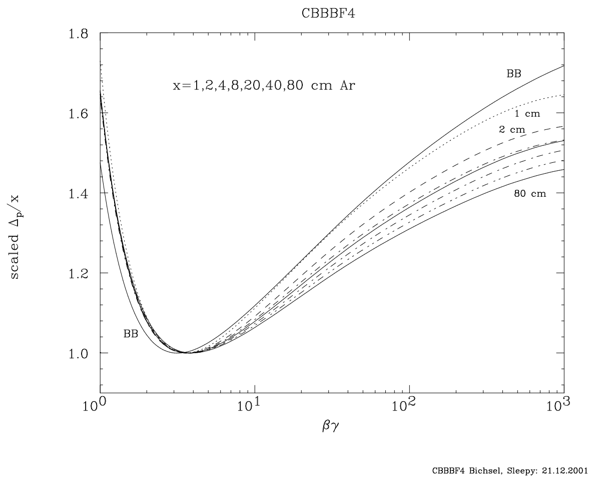

Figure 4: Bichsel functions $g\Delta_p/x$ compared to the Bethe-Bloch function. The coefficient g is a scaling factor chosen such that the functions Δ_p/x coincide at minimum ionization. **Note:** functions for truncated mean values differ from the functions shown here.

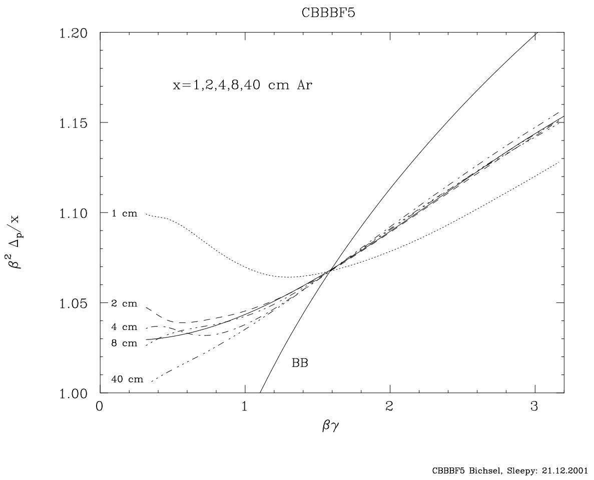

Figure 5: The function $\beta^2\Delta_p/x$ scaled to a common value at $\beta\gamma = 1.513$. Solid line BB: scaled Bethe-Bloch function BB. The function equivalent to Eq. (1) is given by the solid line fitted to the $x = 8$ cm function.

For an accurate analysis in particle identification, we must use the most probable energy loss Δ_p as the parameter associated with $f(\Delta)$. For silicon detectors it was found that Δ_p/x depends on x (Table V in [10]). A similar dependence for Ar is seen in Fig. 2. Thus the ratio of the Bethe-Bloch dE/dx to I_p/x or Δ_p/x depends on x. This is shown in Fig. 3.

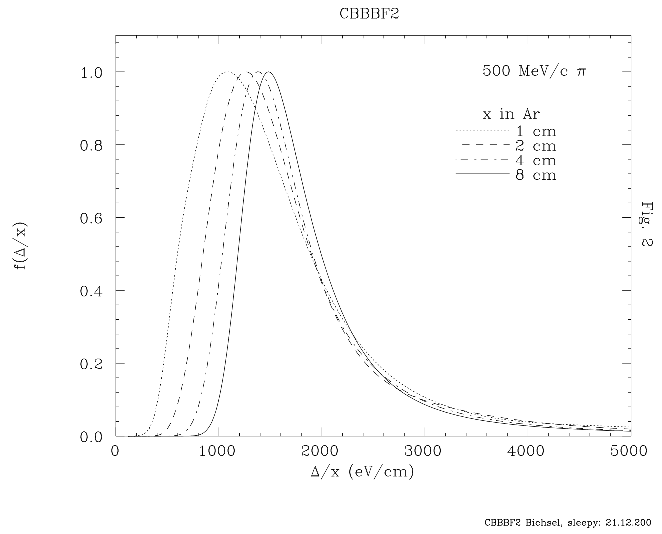

Figure 2: Straggling (energy loss) functions $f(\Delta/x)$ [note that Δ/x is called dE/dx in current literature of particle physics] for $p = 500$ MeV/c pions, for $x = 1, 2, 4, 8$ cm.

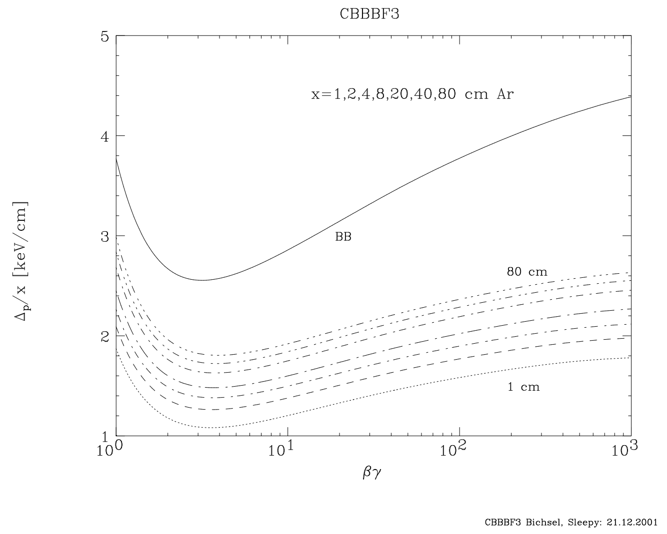

Figure 3: Values of Δ_p/x for segment lengths x as a function of particle momentum (given by $\beta\gamma$). The Bethe-Bloch function, Eq. (23.1) in ref [8], is shown by the solid line BB.

$< \Delta >= x \cdot S(\beta)$=3044 eV where $S = dE/dx$ is the Bethe-Bloch function, Eq. (23.1) in [6]. It is more than twice Δ_p. In other contexts, dE/dx is called the stopping power. For present purposes, $f(\Delta)$ is the same for all particles with charge ± 1 and the same $\beta\gamma$ (e.g. 1.8 MeV/c e, 0.5 GeV/c π, 1.8 GeV/c K or 3.4 GeV/c protons). For large Δ, there are some differences, e.g. for $\beta\gamma = 50$, at $\Delta = 1$ MeV, the electron spectrum is 0.2% higher than the pion spectrum.

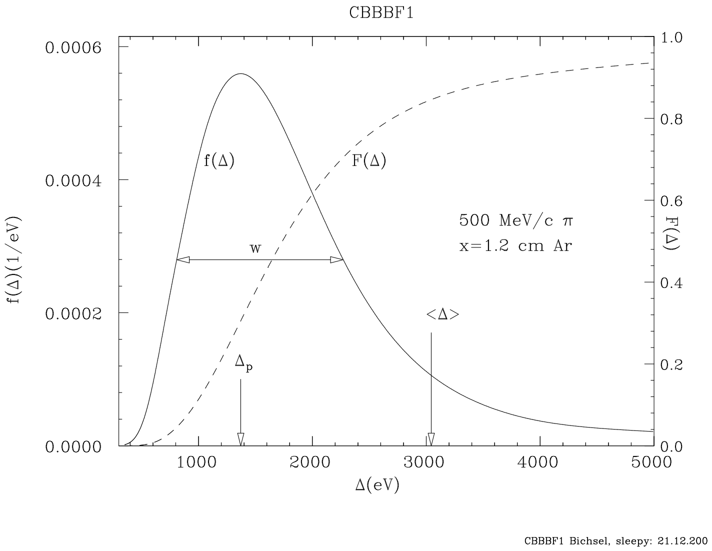

Figure 1: The solid line shows the probability density function $f(\Delta)$ for energy losses Δ of p = 500 MeV/c pions ($\beta\gamma$ = 3.58) crossing a volume V of argon. The dashed line shows the cumulative density function $F(\Delta)$ used for Monte Carlo calculations. For an experimentally measured function, $< \Delta >$ will depend strongly on the largest observed value of Δ.

Abstract.

The Bethe-Bloch equation gives an inaccurate representation of the functional dependance on particle energy of the relevant quantity needed for the evaluation of the ionization spectra from detectors used for particle identification. Instead, "Bichsel functions" are presented here. These functions depend on segment length as well as on truncation method. Only the former dependance is discussed here. An important result : it is inadvisable to use segments less than 2 cm in length.

I. Introduction.

The quantity observed in a TPC or a silicon array is the ionization I in a small volume V along the track of a fast charged particle. The length of the segment of the track inside the volume is x. The energy lost along x by the particle is Δ (using Landau notation, [1]). The ratio $\omega = \Delta/I$ is almost constant. Usually, I is called dE and is assumed to be equivalent to Δ, and an "energy calibration" is made to determine ω. Also, x is usually called dx, and then I/x is called "dE/dx". For the present purpose, various effects are neglected. They are, for example, the escape of δ rays from V [2, 3, 4, 5], the diffusion of electrons during their traversal to the proportional counters, recombination etc.

A function frequently used to describe I is the "straggling function". It is the probability density function (PDF) [6, 7] $f(I, x, \beta)$ where β is the speed of the particle. This note will consider $f(\Delta, x, \beta)$ instead of $f(I, x, \beta)$. A comparison of the two functions can be found e.g. in [5].

II. Calculation of straggling functions

A realistic simulation of the traversal of a particle through a segment of length x in the detector can be done with a Monte Carlo calculation [8, 9]. For each particle an energy loss Δ is obtained, and the straggling function is obtained by binning the Δ into $f(\Delta)$. A faster and more accurate method is the use of convolutions ([10, 11, 12, 4]). The arguments x and β are always implicit in $f(\Delta)$, but are not always given.

As an example, and to define some concepts, a function $f(\Delta)$ calculated with convolutions is given in Fig. 1, for p = 500 MeV/c pions ($\beta\gamma$ = 3.58) traversing a segment of length x = 1.2 cm. Important parameters describing the function are the most probable energy loss $\Delta_p(x, \beta)$ (the position of the maximum of the straggling function, at 1371 eV), and the full-width-at-half maximum (fwhm) $w(x, \beta)$=1463 eV. The mean energy loss is also shown. It is

University of Washington Nuclear Physics Laboratory

Comparison of Bethe-Bloch and Bichsel functions.

Hans Bichsel
Telephone: 206-329-2792, FAX: 206-685-4634
e-mail: bichsel@gluon.npl.washington.edu
Center for Experimental Nuclear Physics and Astrophysics
Box 354290 University of Washington
Seattle, WA 98195-4290

December 21, 2001

Ultrarelativistic Heavy Ions Group