Project

Solenoidal Tracker at RHIC (STAR)

Subproject

Time Projection Chamber - Wheel and Supports

Title

Stress and Deflection of the TPC Support Arm

1.0 Introduction

The following note covers the stress and deflection calculations for the TPC Support Arm, TPC513-E-1(LBL 21F0926). This is a revision and extension of an earlier STAR Note, ST 0117. The major differences between these two notes are a reduced TPC weight, a somewhat different TPC end attachment scheme, a change in the beam's cross-section and the elimination of welding.

The TPC rests on four TPC Support Arms. The arms are bolted to the TPC Magnet Inner Ring near the 3 and 9 o'clock locations at either end of the STAR Detector. A vertical pin connection fixed to a bracket on the TPC Wheel transfers the weight of the TPC to each arm. The pin rests on a pair of spherical washers so that the bending of the arm does not transmit a moment to the TPC Wheel. The support arrangement is quasi-kinematic. All pins are adjustable in y (vertical) and are free to move a small amount upward but cannot move downward. One pin is fixed in x and z, one pin is free to move in x but not z and one pin, diagonally opposed to the fixed pin, can move freely in x and z. The position of the pins in the fixed direction is adjustible within a +/- 1/8" range. Refer to LBL 24A8006 Support System Assembly.

2.0 Calculations

2.1 Vertical Force on Support Arm

$$W_{TPC} := 25000\ lb$$

$$N_{arm} := 4$$

$$Fy := \frac{-W_{TPC}}{4}$$

2.2 Material Properties of Support Arm

The support arm is fabricated from 6061-T6 aluminum alloy.

Yield stress $\sigma_y := 36000 \frac{lb}{in^2}$

Elastic Modulus $E := 10 \cdot 10^6 \frac{lb}{in^2}$

Poisson's Ratio $\nu := 0.34$

Modulus of Rigidity $G := \frac{E}{2 \cdot (1 + \nu)}$ $G = 3.731 \cdot 10^6 \ \frac{lb}{in^2}$

2.3 Beam Region of Arm

For calculation purposes the Support Arm will be divided into the beam and mounting flange regions. The "beam region" is the portion of the arm extending from the knee nearest the magnet to the enlarged end at the 6 inch bore for the TPC location adjusters.

2.3.1 Torsional Stress and Deflection

The arm will be treated as a rectilinear beam of constant cross section for simplicity. While the beam tapers from 19.4 to 12.75 inches in height along its length, for a first approximation only the smallest vertical and horizontal dimensions will be used.

Following the treatment in Art. 9.2 of Roark's Formulas for Stress and Strain, 6th Ed.

Table 20, 4. Torsion of a solid rectangular section.

half height $a := \frac{12.75 \text{ in}}{2}$ half width $b := \frac{3.679 \text{ in}}{2}$

$$K := a \cdot b^3 \cdot \left[\frac{16}{3} - 3.36 \cdot \frac{b}{a} \cdot \left(1 - \frac{b^4}{12 \cdot a^4} \right) \right] \qquad K = 173.181 \ in^4$$

where K is a form factor in the equation $\theta := \frac{T \cdot L}{K \cdot G}$

The load is offset by roughly 3.4 inches from the center line of the arm. The length of the arm from the "knee" near the mounting point to the enlarged area at the approach to the 6 inch bore is about 30 inches.

$L := 30 \text{ in}$

Torque is a product of the load offset and load,

$T := 3.4 \text{ in} \cdot Fy$

The twist in the arm, in radians is,

$\theta := \frac{T \cdot L}{K \cdot G}$ $\theta = -9.865 \cdot 10^{-4}$

The maximum shear stress from torsion (at the midpoint of the longer sides) in the thin section of the arm is,

$$\tau := \frac{3 \cdot T}{8 \cdot a \cdot b^2} \cdot \left[1 + 0.6095 \cdot \frac{b}{a} + 0.8865 \cdot \left(\frac{b}{a}\right)^2 - 1.8023 \cdot \left(\frac{b}{a}\right)^3 + 0.91 \cdot \left(\frac{b}{a}\right)^4 \right]$$

$$\tau = -447.982 \cdot \frac{lb}{in^2}$$

2.3.2 Stress and deflection exclusive of torsion

In addition to torsion, the arm is subject to bending and a vertical shear load.

Bending stress

The load (support pin) is 34 inches from the mounting end knee along an axis parallel to the mid-line of the arm.

$$L_b := 34 \text{ in}$$

$$M := Fy \cdot L_b$$

The area moment of inertia is,

$$I := \frac{(2 \cdot b) \cdot (2 \cdot a)^3}{12}$$

The distance to the extreme fiber is the half height, a

$$\sigma_b := \frac{M \cdot a}{I}$$

$$\sigma_b = -2.132 \cdot 10^3 \cdot \frac{lb}{in^2}$$

Shear stress due to vertical load

$$\tau_{vl} := \frac{Fy}{(2 \cdot a) \cdot (2 \cdot b)}$$

$$\tau_{vl} = -133.242 \cdot \frac{lb}{in^2}$$

Deflection

The Support Arm is tappered over a significant portion of its length. To estimate its deflection two limiting cases will be calculated, a uniform beam having dimensions equal to the smallest cross section of the arm and a uniformly tapered beam of constant width.

Case 1. Uniform cross section of 12.75 inches by 3.679 inches.

$$\delta_{Umax} := \frac{-Fy \cdot L_b^3}{3 \cdot E \cdot I} \qquad \delta_{Umax} = 0.013\ in$$

Case 2. Linearly decreasing cross section depth:[1]

Area moments of inertia, fixed end $I_B := \frac{(2 \cdot a) \cdot (19.2\ in)^3}{12}$

, free end $I_A := I$

$$\frac{I_B}{I_A} = 11.835$$

From Roark Art. 7.8 Table 13a Case 1a for area moment of inertia ratio of 8

$$\delta_{Vmax} := 0.171 \cdot \delta_{Umax}$$

$$\delta_{Vmax} = 2.203 \cdot 10^{-3}\ in$$

2.3.3 Combined Stress State

The bending stress is greatest at the top and bottom of the beam and zero at the midpoint whereas the shear stress due to torsion is maximum at the mid-point. The shear stress due to the vertical load is roughly constant over the entire cross section. To evaluate the combined stress state at the surface of the mid-height of the beam a Von Mises equivalent stress can be calculated.

In terms of a local coordinate system with an origin at the center of the beam's cross section and oriented with z directed along the length of the beam, y vertical and x horizontal.

$$\tau_{xz} := \tau_{vl} \qquad \text{and} \qquad \tau_{yz} := \tau$$

In terms of non-principal axis the Mises criterion[2] and eliminating all nonzero terms:

$$Y := \left(3 \cdot \tau_{xz}^{2} + 3 \cdot \tau_{yz}^{2}\right)^{\frac{1}{2}}$$

$$Y = 809.52 \cdot \frac{lb}{in^2}$$

2.4 Mounting Flange Region

2.4.1 Stress in mounting flange.

The stress calculations given in SN0117 show a safety factor of 4.4 based on a mounting flange of 18 inches by 3 inches. The current design calls for a mounting flange of 19.2 inches by 3.4 inches thick. Given that the actual load is 73% of the value assumed in the earlier note, the safety factor for the current design is in excess of 5.

3.0 Conclusions

The stresses in the TPC Support Arm fall well within the Safety Factor of 3 on yield for 6061-T6 aluminum called for in the STAR design guidelines. The deflections are very small and can easily be accomodated by the TPC Support System.