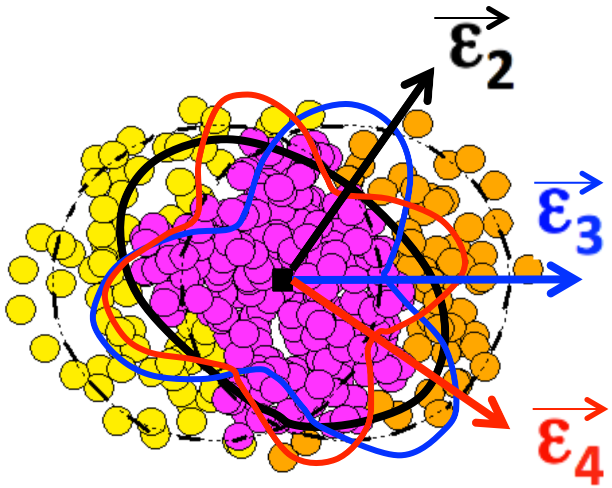
Event-shape fluctuations and flow correlations

Jiangyong Jia

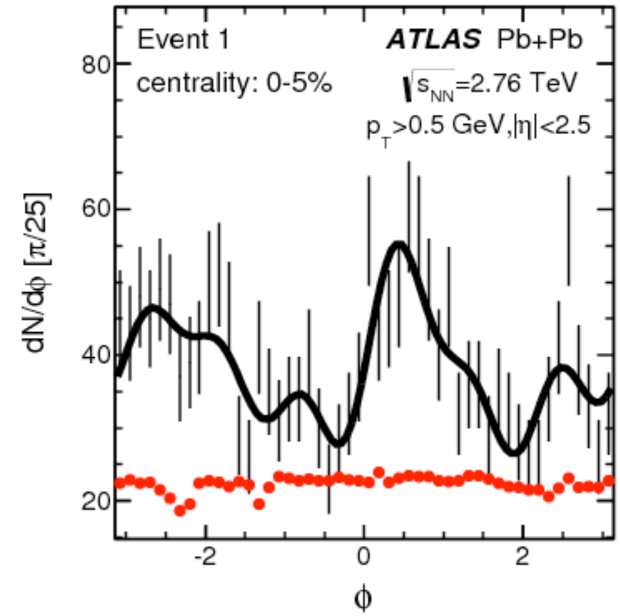
ATHIC 2014 Conference

8/5-8/9, 2014, Osaka

Geometry and harmonic flow



Collective expansion



$$\vec{\epsilon}_n \equiv \epsilon_n e^{in\Phi_n^*} \equiv -\frac{\langle r^n e^{in\phi} \rangle}{\langle r^n \rangle}$$

$$\frac{dN}{d\phi} \propto 1 + 2\sum_n v_n \cos n(\phi - \Phi_n)$$

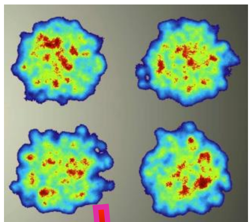
$$\vec{v}_n \equiv v_n e^{in\Phi_n}$$

- Probes: initial geometry and transport properties of QGP
  - How $(\varepsilon_n, \Phi_n^*)$ are transferred to (v_n, Φ_n)?
  - What is the nature of final state (non-linear) dynamics?
  - What is the nature of longitudinal flow dynamics?

Event-by-event observables

Many little bangs

1104.4740, 1209.2323,1203.5095 ,1312.3572



$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{\rm evts}} \frac{dN_{\rm evts}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$

Moments:

$$\langle \cos(n_1\phi_1 + n_2\phi_2 ... + n_m\phi_m) \rangle = \qquad \Sigma n_i = 0$$

$$\langle v_{n_1} v_{n_2} ... v_{n_m} \cos(n_1\Phi_{n_1} + n_2\Phi_{n_2} ... + n_m\Phi_{n_m}) \rangle$$

Examples:

$$\langle \cos(n\phi_1 - n\phi_2 + n\phi_3 - n\phi_4) \rangle = \left\langle v_n^4 \cos(n\Phi_n - n\Phi_n + n\Phi_n - n\Phi_n) \right\rangle = \left\langle v_n^4 \right\rangle$$

$$\langle \cos(n\phi_1 - n\phi_2 + m\phi_3 - m\phi_4) \rangle = \left\langle v_n^2 v_m^2 \cos(n\Phi_n - n\Phi_n + m\Phi_m - m\Phi_m) \right\rangle = \left\langle v_n^2 v_m^2 \right\rangle$$

$$\langle \cos(3\phi_1 + 3\phi_2 - 6\phi_3) \rangle = \langle v_3^2 v_6 \cos 6(\Phi_3 - \Phi_6) \rangle$$

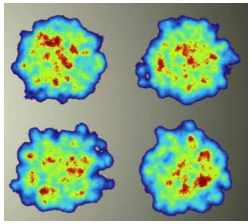
Cumulants obtained by combining with lower order correlators:

$$\langle \cos(n\phi_1 - n\phi_2 + n\phi_3 - n\phi_4) \rangle_c = \left\langle v_n^4 \right\rangle - 2 \left\langle v_n^2 \right\rangle^2 \qquad p(v_n)$$

$$\langle \cos(n\phi_1 - n\phi_2 + m\phi_3 - m\phi_4) \rangle_c = \left\langle v_n^2 v_m^2 \right\rangle - \left\langle v_n^2 \right\rangle \left\langle v_m^2 \right\rangle \qquad p(v_n, v_m)$$

Event-by-event observables

Many little bangs



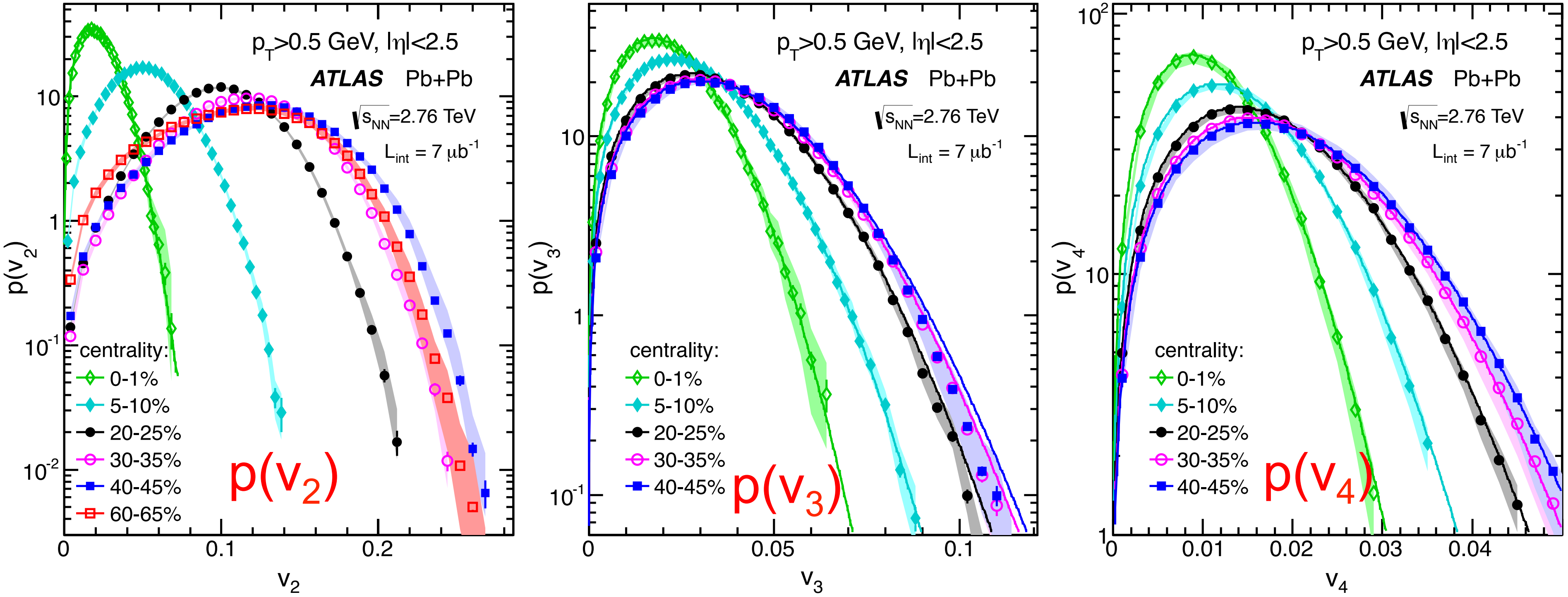
1104.4740, 1209.2323,1203.5095 ,1312.3572

$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{\text{evts}}} \frac{dN_{\text{evts}}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$

	pdf's	cumulants	event-shape method
Flow-amplitudes	$p(v_n)$	$v_n\{2k\},\ k = 1,2,...$	NA
	$p(v_n, v_m)$	$\langle v_n^2 v_m^2 \rangle - \langle v_n^2 \rangle \langle v_m^2 \rangle$	yes
	$p(v_n, v_m, v_l)$	$\langle v_n^2 v_m^2 v_l^2 \rangle + 2\langle v_n^2 \rangle \langle v_m^2 \rangle \langle v_l^2 \rangle -$ $\langle v_n^2 v_m^2 \rangle \langle v_l^2 \rangle - \langle v_m^2 v_l^2 \rangle \langle v_n^2 \rangle - \langle v_l^2 v_n^2 \rangle \langle v_m^2 \rangle$	yes
	...	Obtained recursively as above	yes
EP-correlation	$p(\Phi_n, \Phi_m, ...)$	$\langle v_n^{c_n} v_m^{c_m} ... \cos(c_n n\Phi_n + c_m m\Phi_m + ...)\rangle$ $\sum_k kc_k = 0$	yes
Mixed-correlation	$p(v_l, \Phi_n, \Phi_m, ...)$	$\langle v_l^2 v_n^{c_n} v_m^{c_m} ... \cos(c_n n\Phi_n + c_m m\Phi_m + ...)\rangle -$ $\langle v_l^2 \rangle \langle v_n^{c_n} v_m^{c_m} ... \cos(c_n n\Phi_n + c_m m\Phi_m + ...)\rangle$ $\sum_k kc_k = 0$	yes

Flow fluctuation: $p(v_n)$

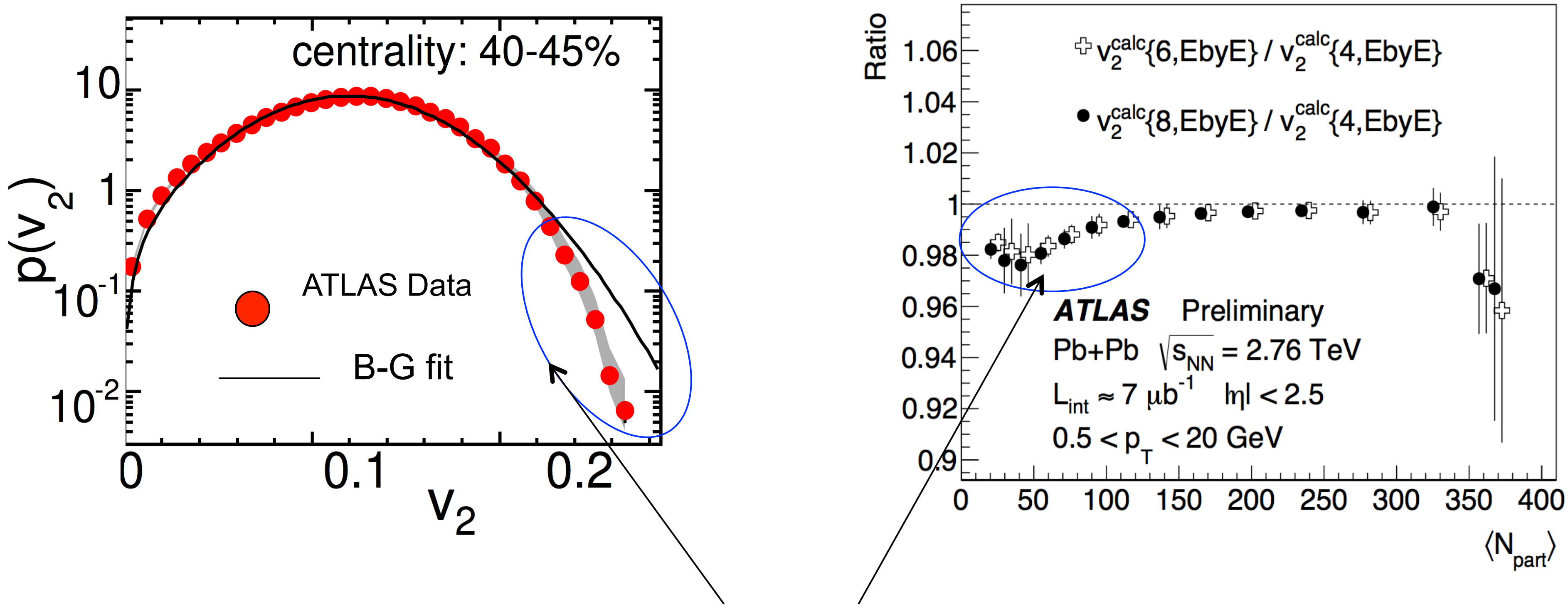
$p(v_2)$, $p(v_3)$ and $p(v_4)$ distributions



$$v_n\{4\}^4 = 2\langle v_n^2\rangle^2 - \langle v_n^4\rangle \neq 0 \text{ for } n = 2,3$$

- The non-zero $v_n\{4,6..\}$ either due to
  - average geometry such as $v_2^{RP}\neq 0$ or
  - non-Gaussianness in the flow fluctuation

Cumulants from $p(v_2)$



Non-Gaussian behavior

- Non-gaussianess is reflected by a 1-2% change beyond 4th order cumulants

Event-plane correlations $p(\Phi_n,\Phi_m\ldots)$

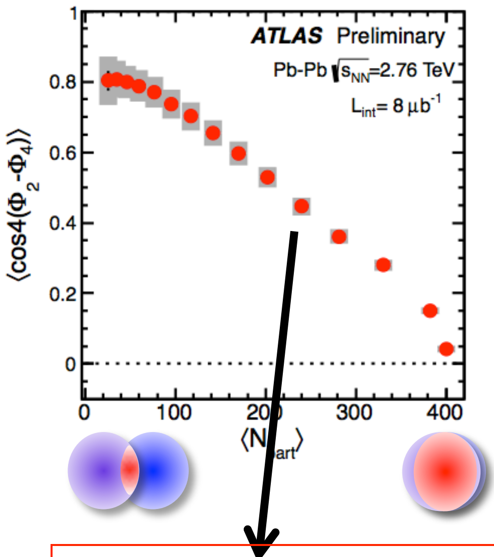
1403.0489

$$\langle\cos 4(\Phi_2 - \Phi_4)\rangle$$
$$\langle\cos 8(\Phi_2 - \Phi_4)\rangle$$
$$\langle\cos 12(\Phi_2 - \Phi_4)\rangle$$
$$\langle\cos 6(\Phi_2 - \Phi_3)\rangle$$
$$\langle\cos 6(\Phi_2 - \Phi_6)\rangle$$
$$\langle\cos 6(\Phi_3 - \Phi_6)\rangle$$
$$\langle\cos 12(\Phi_3 - \Phi_4)\rangle$$
$$\langle\cos 10(\Phi_2 - \Phi_5)\rangle$$

$$\langle\cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5)\rangle$$
$$\langle\cos(-8\Phi_2 + 3\Phi_3 + 5\Phi_5)\rangle$$
$$\langle\cos(2\Phi_2 + 4\Phi_4 - 6\Phi_6)\rangle$$
$$\langle\cos(-10\Phi_2 + 4\Phi_4 + 6\Phi_6)\rangle$$
$$\langle\cos(2\Phi_2 - 6\Phi_3 + 4\Phi_4)\rangle$$
$$\langle\cos(-10\Phi_2 + 6\Phi_3 + 4\Phi_4)\rangle$$

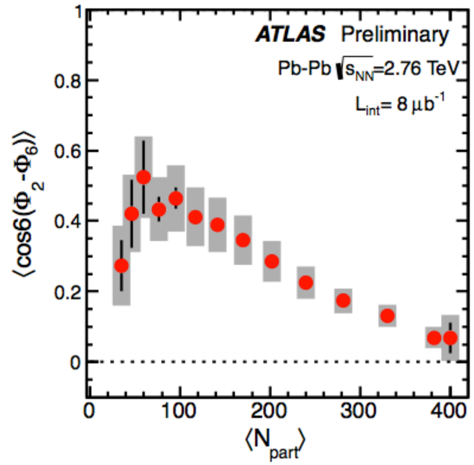
Event-plane correlation results

$\langle \cos 4(\Phi_2 - \Phi_4) \rangle$

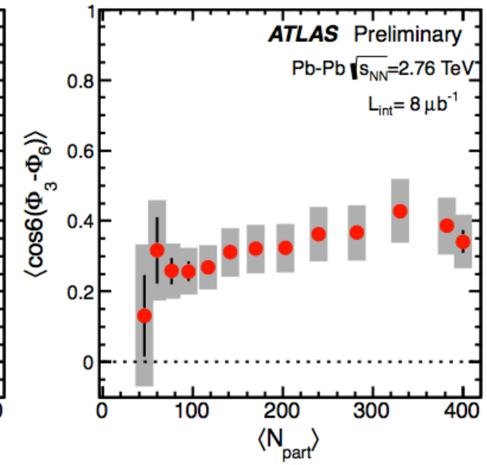


$$v_4 e^{i4\Phi_4} \propto \varepsilon_4 e^{i4\Phi_4^*} + c v_2^2 e^{i4\Phi_2} + \dots$$

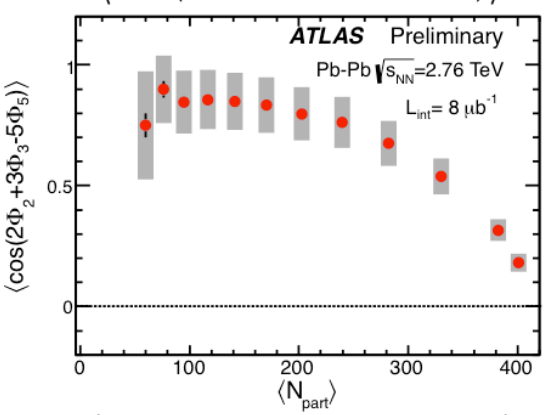
$\langle \cos 6(\Phi_2 - \Phi_6) \rangle$



$\langle \cos 6(\Phi_3 - \Phi_6) \rangle$



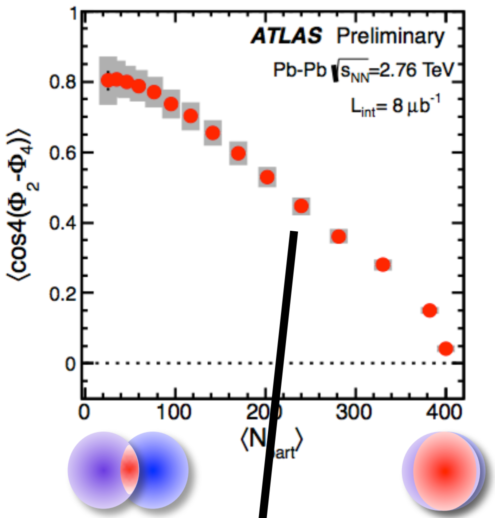
$\langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle$



Teaney & Yan

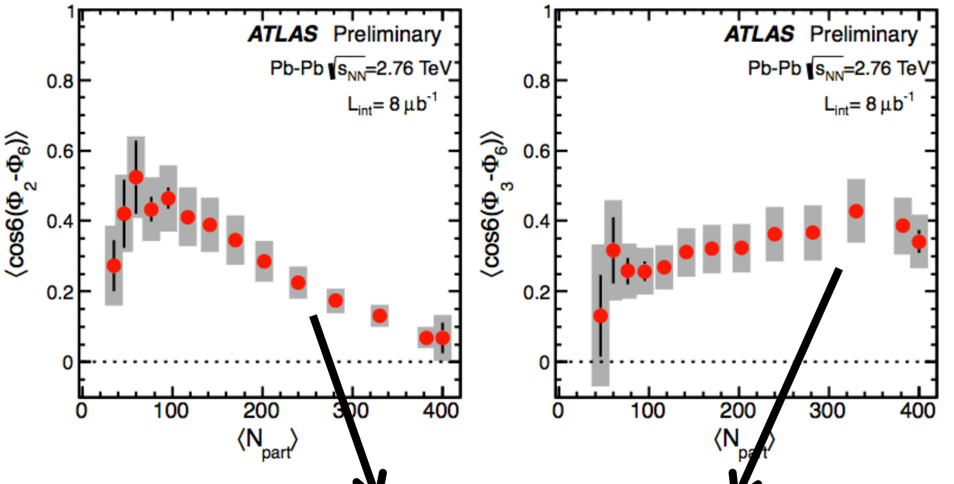
Event plane correlation results

$\langle \cos 4(\Phi_2 - \Phi_4) \rangle$



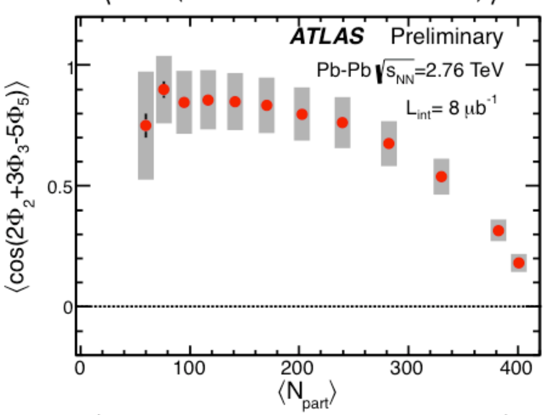
$$v_4 e^{i4\Phi_4} \propto \varepsilon_4 e^{i4\Phi_4^*} + c v_2^2 e^{i4\Phi_2} + \ldots$$

$\langle \cos 6(\Phi_2 - \Phi_6) \rangle$ $\langle \cos 6(\Phi_3 - \Phi_6) \rangle$



$$v_6 e^{i6\Phi_6} \propto \varepsilon_6 e^{i6\Phi_6^*} + c_1 v_2^3 e^{i6\Phi_2} + c_2 v_3^2 e^{i6\Phi_3} + \ldots$$

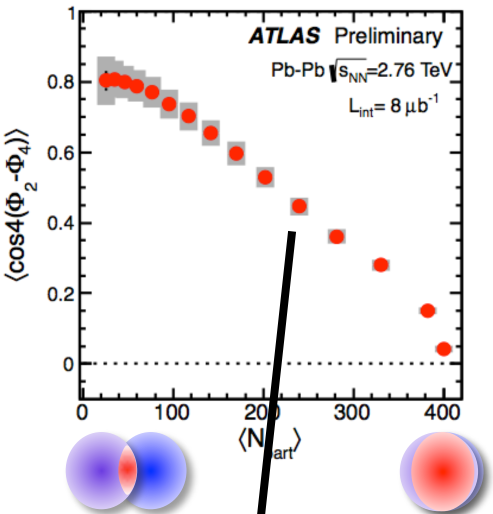
$\langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle$



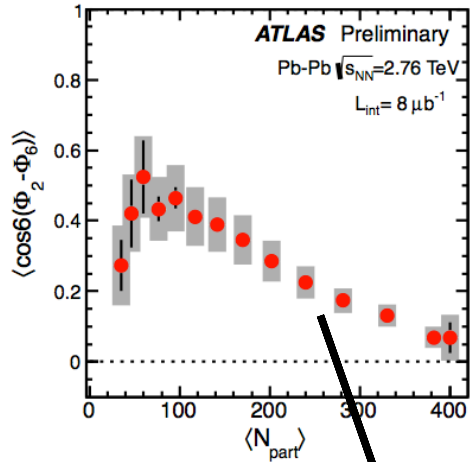
Teaney & Yan

Event plane correlation results

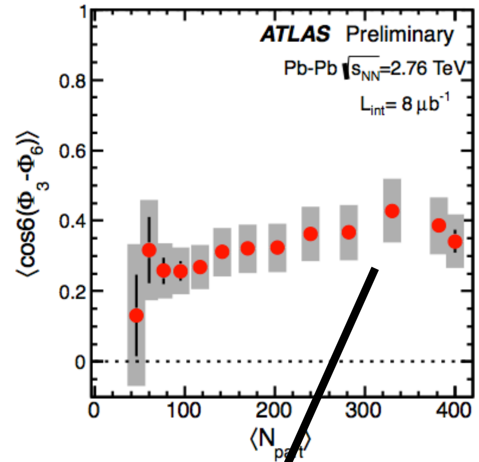
$$\left\langle \cos 4(\Phi_2 - \Phi_4) \right\rangle$$



$$\left\langle \cos 6(\Phi_2 - \Phi_6) \right\rangle$$



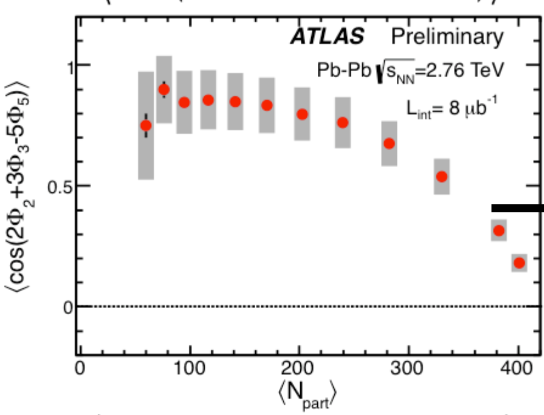
$$\left\langle \cos 6(\Phi_3 - \Phi_6) \right\rangle$$



$$v_4 e^{i4\Phi_4} \propto \varepsilon_4 e^{i4\Phi_4^*} + c v_2^2 e^{i4\Phi_2} + \ldots$$

$$v_6 e^{i6\Phi_6} \propto \varepsilon_6 e^{i6\Phi_6^*} + c_1 v_2^3 e^{i6\Phi_2} + c_2 v_3^2 e^{i6\Phi_3} + \ldots$$

$$\left\langle \cos\left(2\Phi_2 + 3\Phi_3 - 5\Phi_5\right) \right\rangle$$



$$v_5 e^{i5\Phi_5} \propto \varepsilon_5 e^{i5\Phi_5^*} + c v_2 v_3 e^{i(2\Phi_2 + 3\Phi_3)} + \ldots$$

Teaney & Yan

How (ε_n,Φ_n^*) are transferred to (v_n, Φ_n)?

- Flow response is linear for v_2 and v_3: $v_n \propto \varepsilon_n$ and $\Phi_n \approx \Phi_n^*$ i.e.

$$v_2 e^{-i2\Phi_2} \propto \epsilon_2 e^{-i2\Phi_2^*}, \quad v_3 e^{-i3\Phi_3} \propto \epsilon_3 e^{-i3\Phi_3^*}$$

- Higher-order flow arises from EP correlations., e.g. :

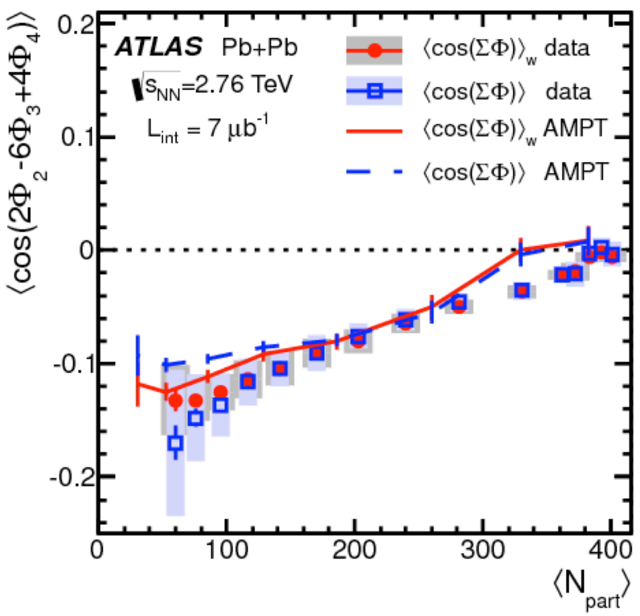
$$v_4 e^{i4\Phi_4} \propto \varepsilon_4 e^{i4\Phi_4^*} + c v_2^2 e^{i4\Phi_2} + \ldots$$

Ollitrault, Luzum, Teaney, Li, Heinz,Chun….

$$v_5 e^{i5\Phi_5} \propto \varepsilon_5 e^{i5\Phi_5^*} + c v_2 v_3 e^{i(2\Phi_2+3\Phi_3)} + \ldots$$

$$v_6 e^{i6\Phi_6} \propto \varepsilon_6 e^{i6\Phi_6^*} + c_1 v_2^3 e^{i6\Phi_2} + c_2 v_3^2 e^{i6\Phi_3} + c_3 v_2 \varepsilon_4 e^{i\left(2\Phi_2+4\Phi_4^*\right)} \ldots$$

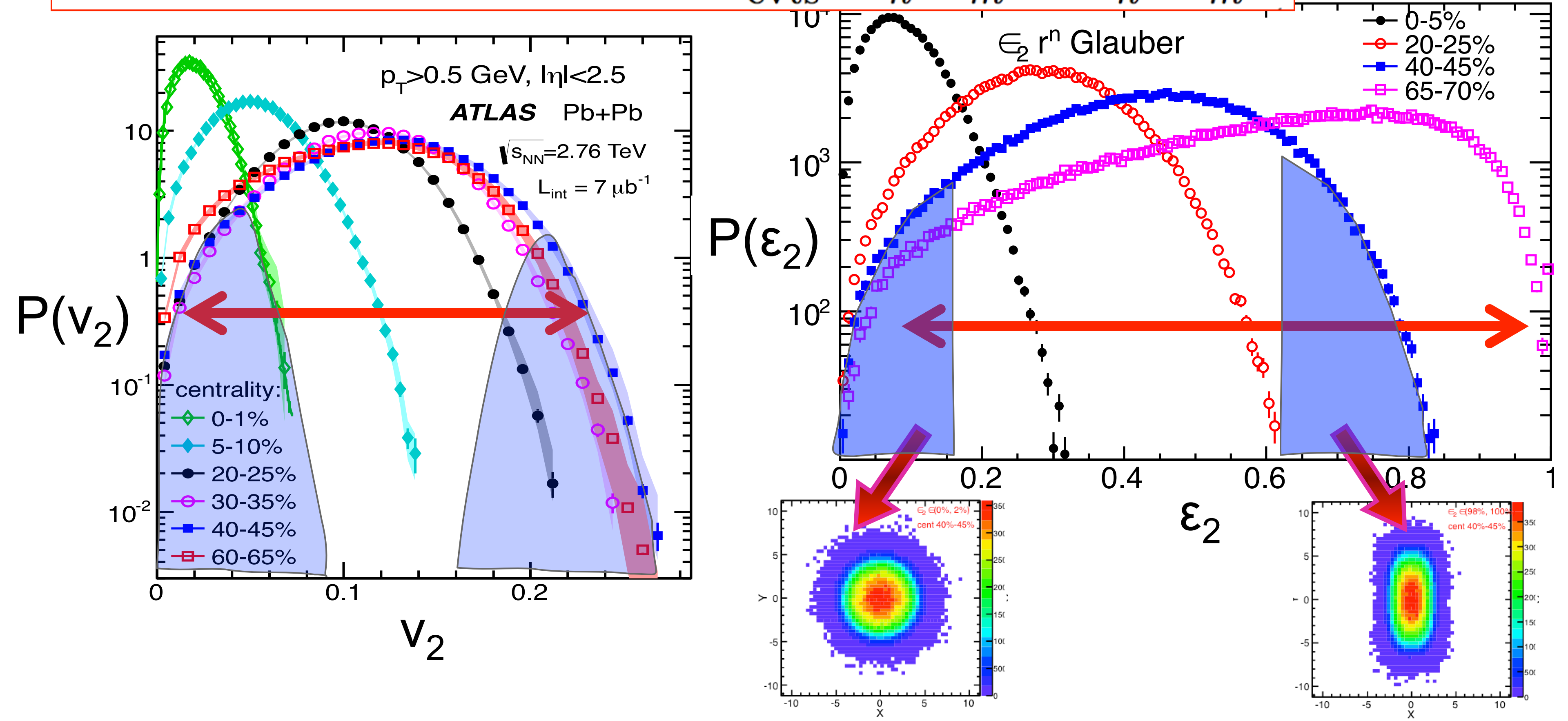
- Some correlators lack intuitive explanation e.g. $<\cos(2\Phi_2-6\Phi_3+4\Phi_4>$ correlation
  - Although described by EbyE hydro and AMPT



Event-shape selection technique

Can we do better?

$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{\text{evts}}} \frac{dN_{\text{evts}}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$



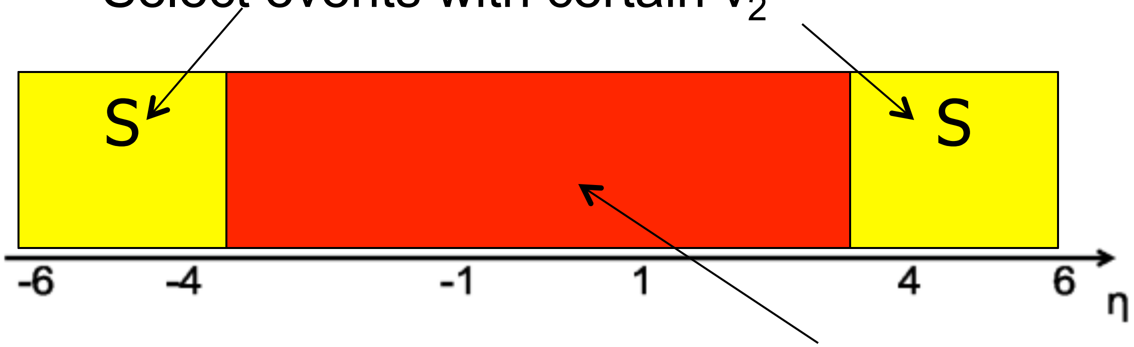
- Study the variation of v_n at fixed centrality but varying event-geometry: "event-shape-selected v_n measurements

Event-shape selection technique

Schukraft, Timmins, and Voloshin, arXiv:1208.4563

Huo, Mohapatra, JJ arxiv:1311.7091

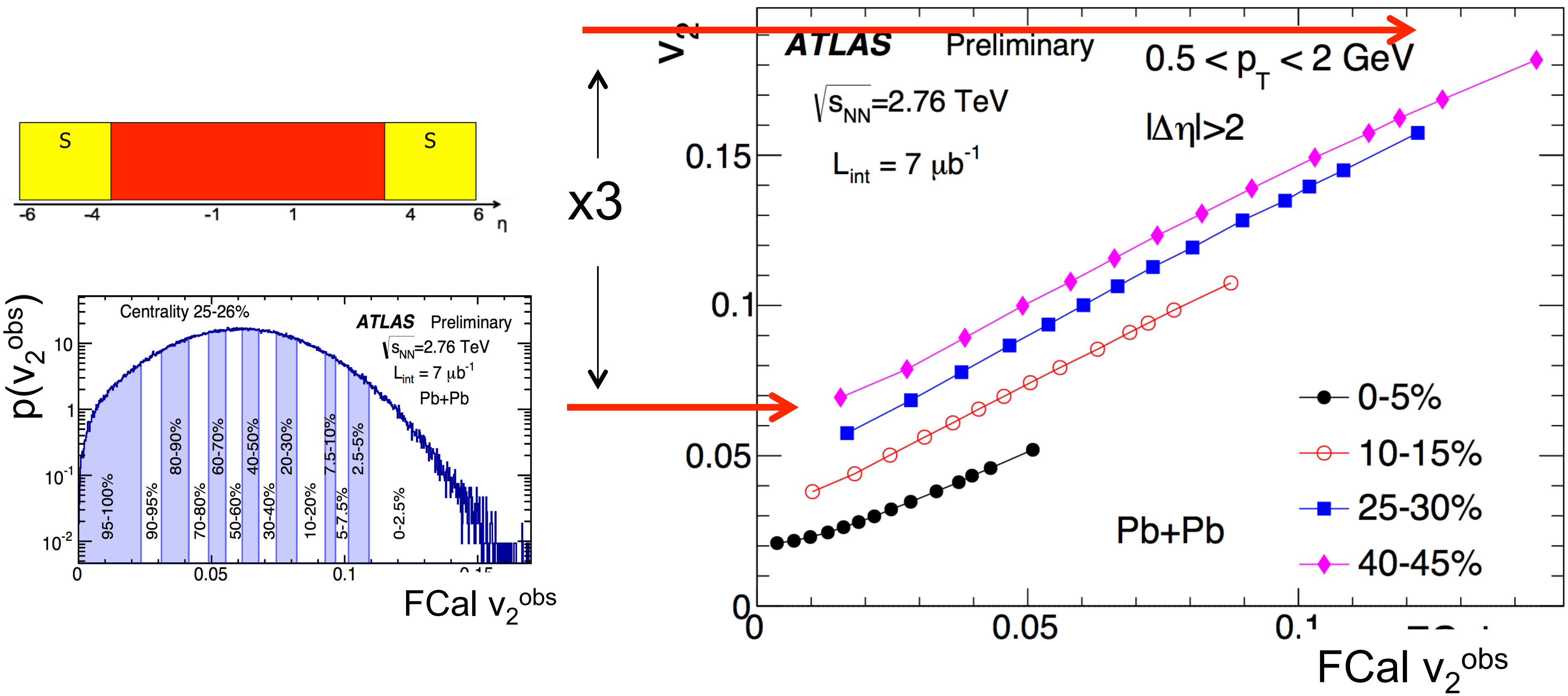
Select events with certain v_2^{obs}



$p(v_n)$, $p(v_n,v_m)$or $p(\Phi_n,\Phi_{m,\ldots})$

$$\vec{v}_n^{obs} = \frac{1}{\Sigma w}(\Sigma w \cos n\phi_n, \Sigma w \sin n\phi_n), w = p_T \;, \qquad v_n^{obs} = \left|\vec{v}_n^{obs}\right|$$

More info by selecting on event-shape

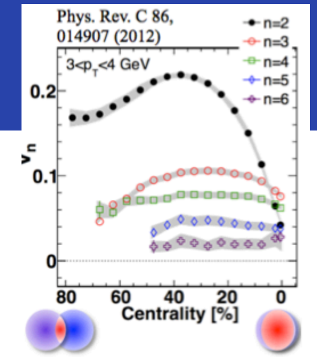


Fix centrality then select events with certain v_2^{obs} in large rapidity:

➔ **measure v_n via two-particle correlations in |η|< 2.5**

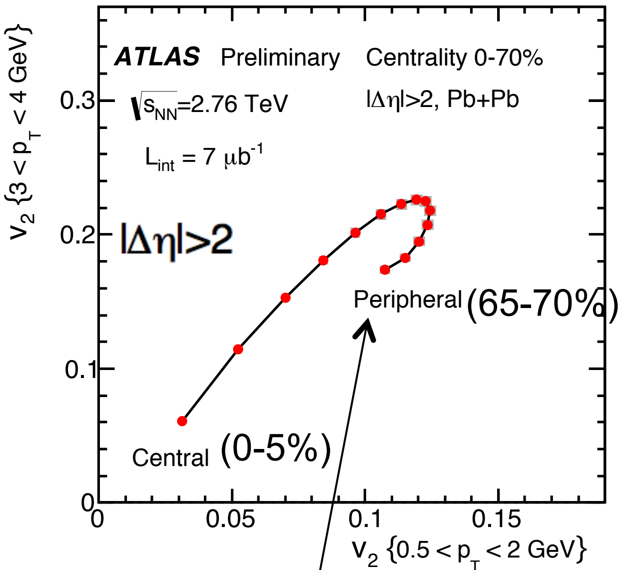
Vary ellipticity by a factor of 3!

v_n-v_2 correlations: centrality dependence

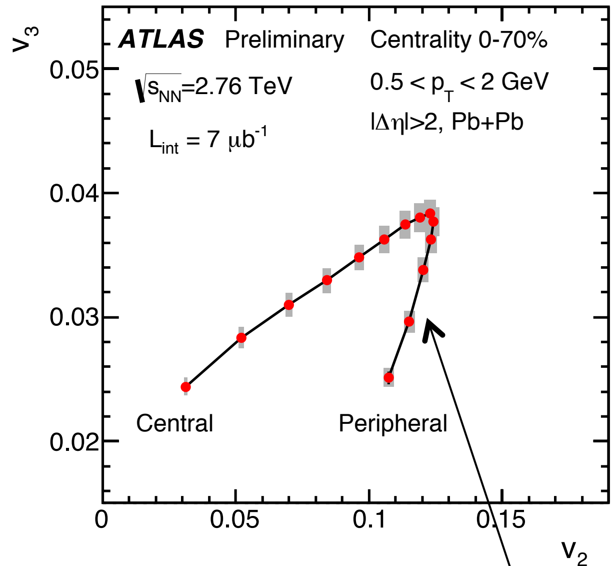


- First correlation without event v_2-selection, 5% steps

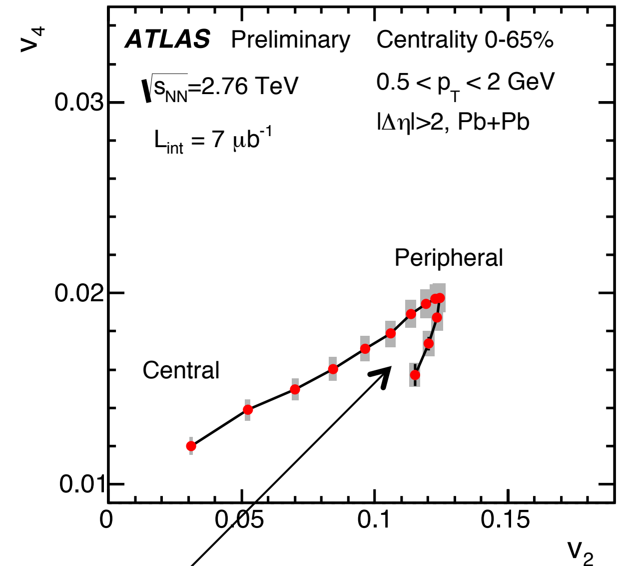
v_2 (p_T^a,b) vs v_2 (p_T^b,b)



v_3 (b) vs v_2 (b)



v_4 (b) vs v_2 (b)



"Boomerang" reflects stronger viscous damping at higher p_T and peripheral

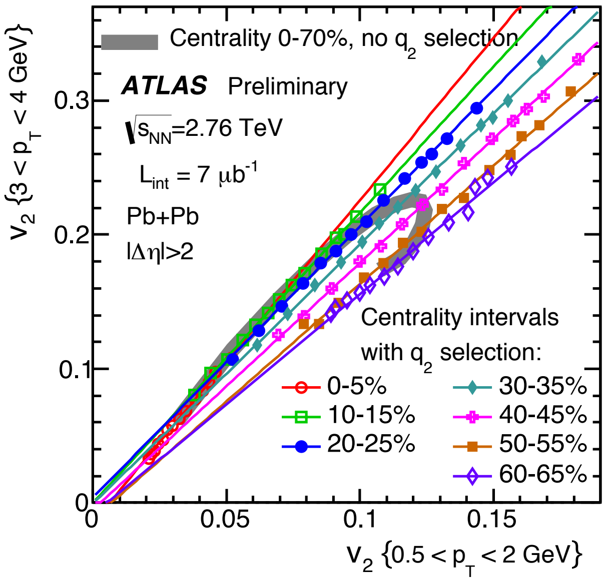
"Boomerang" reflects reflects different centrality dependence, which is also sensitive to the viscosity effect.

v_n-v_2 correlations: within fixed centrality

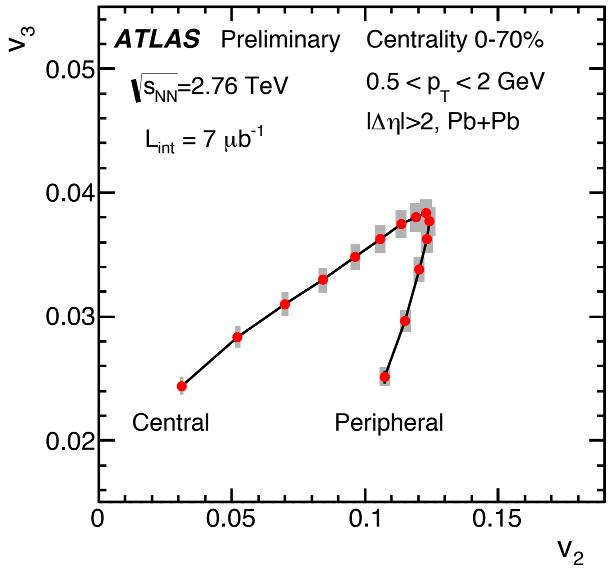
- Fix system size and vary the ellipticity!

Probe $p(v_n,v_2)$

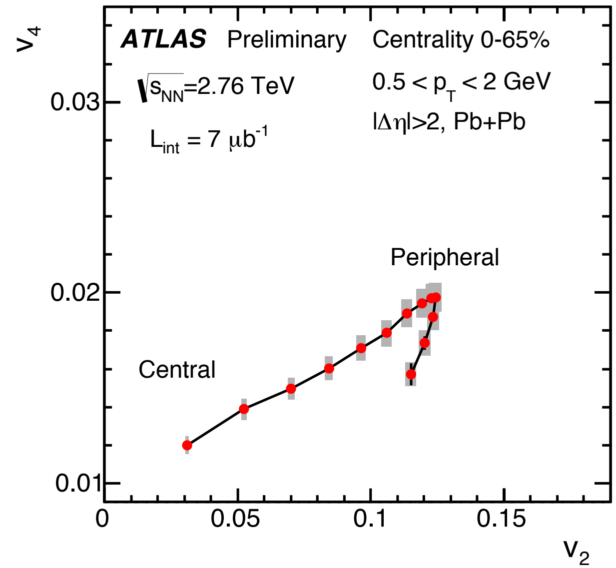
v_2 (higher p_T)



v_3



v_4

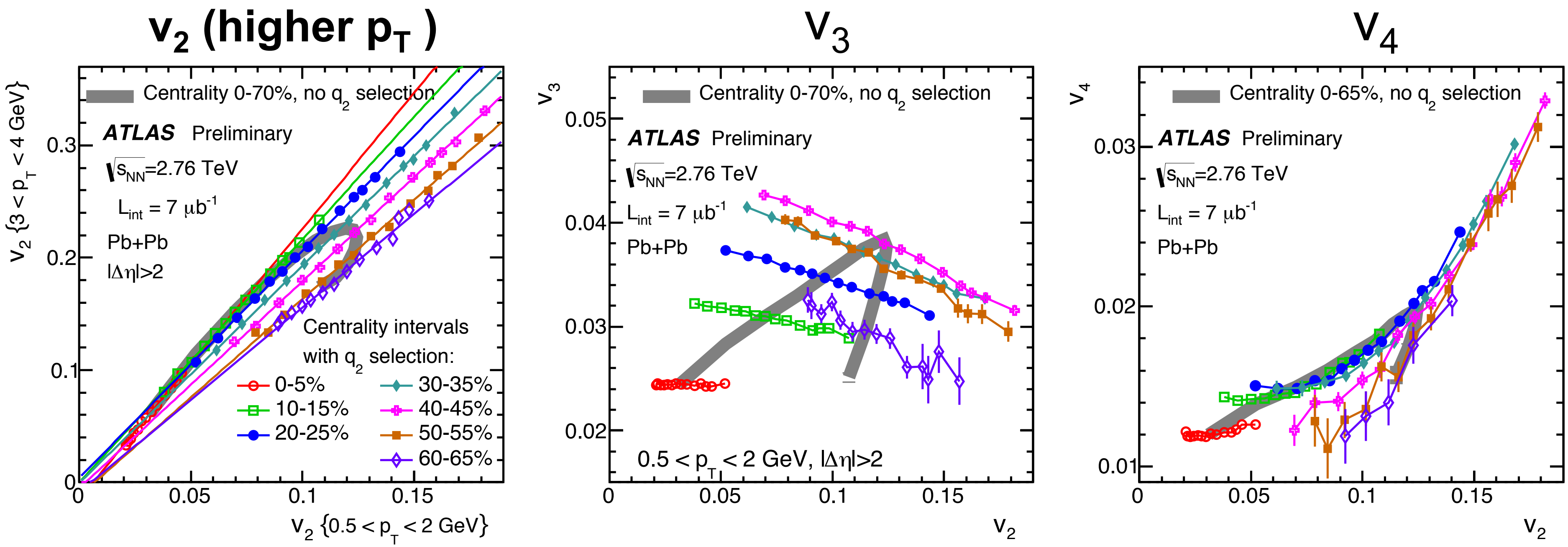


Linear correlation for forward v_2-selected bin→viscous damping controlled by system size, not shape

v_n-v_2 correlations: within fixed centrality

- Fix system size and vary the ellipticity!

Probe $p(v_n,v_2)$



Linear correlation for forward v_2-selected bin→viscous damping controlled by system size, not shape

Clear anti-correlation,

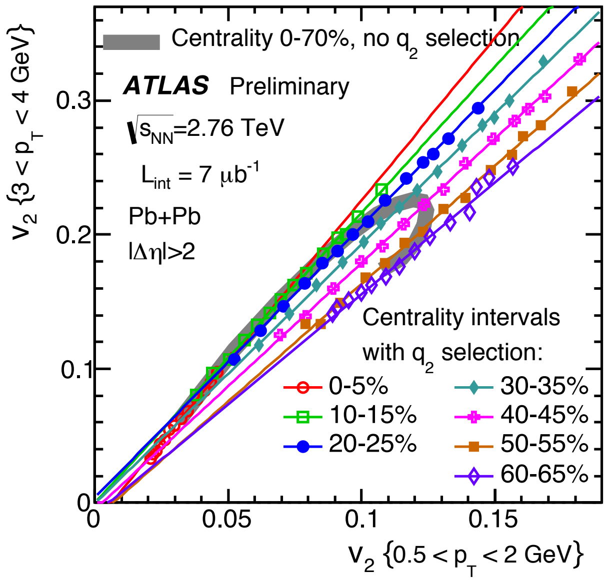
quadratic rise from non-linear coupling to v_2^2

v_n-v_2 correlations: within fixed centrality

- Fix system size and vary the ellipticity!
- Overlay ε_3-ε_2 and ε_4-ε_2 correlations, rescaled

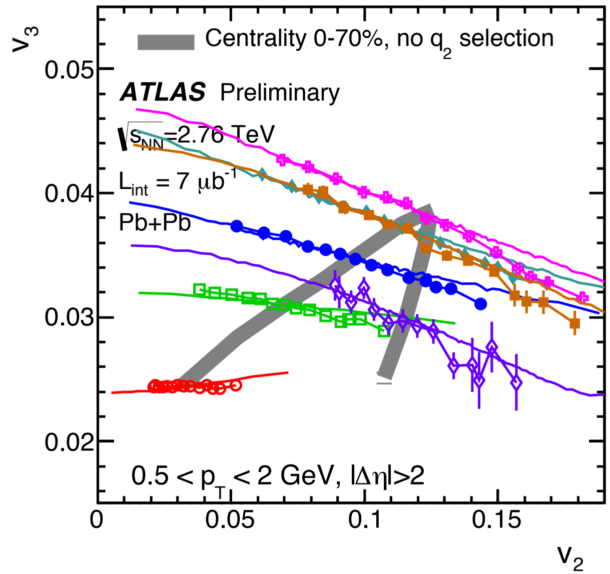
Probe $p(v_n,v_2)$

v_2 (higher p_T)



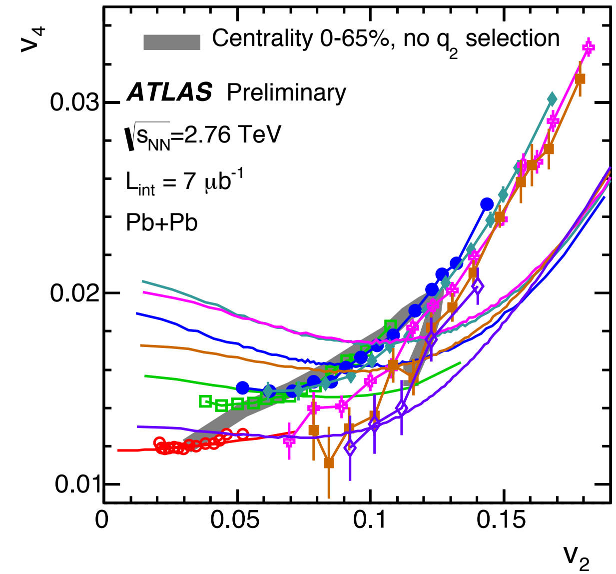
Linear correlation for forward v_2-selected bin→viscous damping controlled by system size, not shape

v_3



Clear anti-correlation, mostly initial geometry effect!!

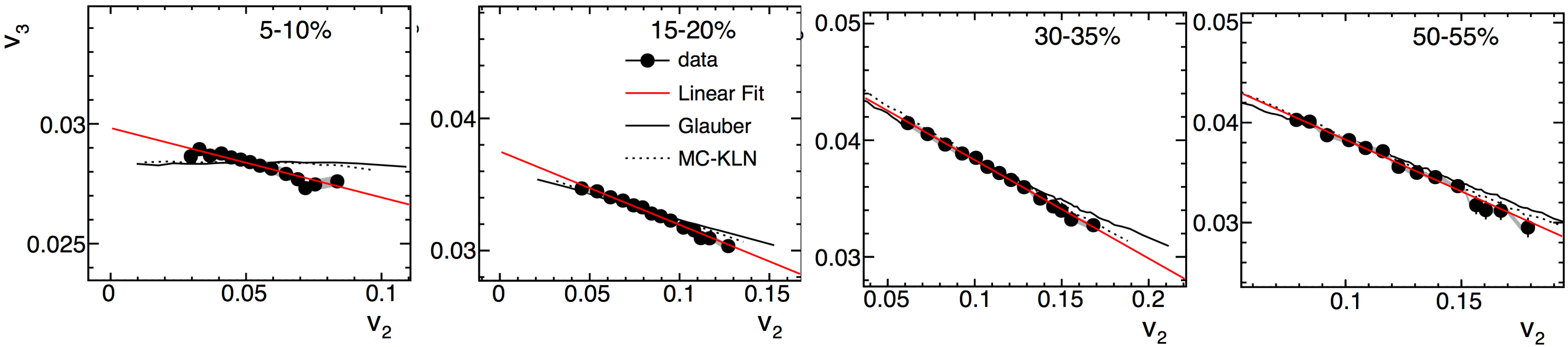
v_4



quadratic rise from non-linear coupling to v_2^2 initial geometry only does not work!!

Initial geometry describe v_3-v_2 but fails v_4-v_2 correlation

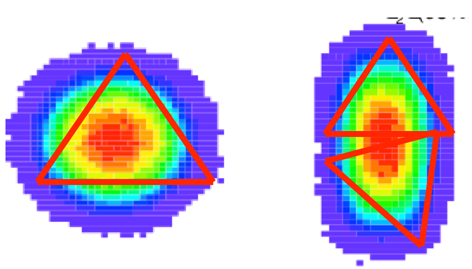
Anti-correlation between v3 and v2

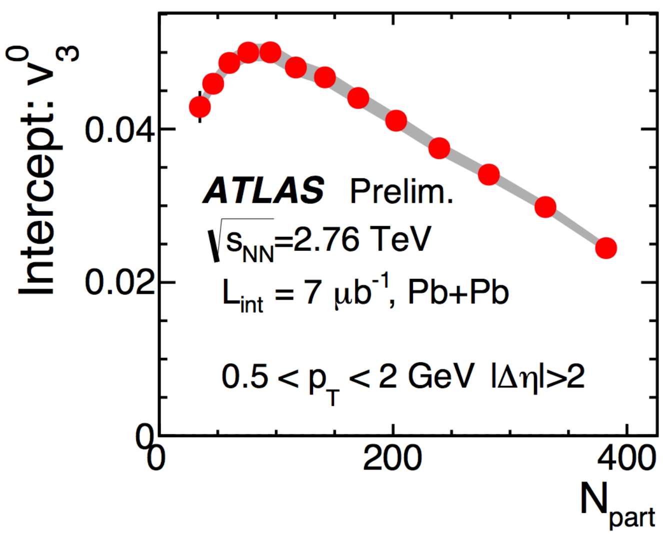


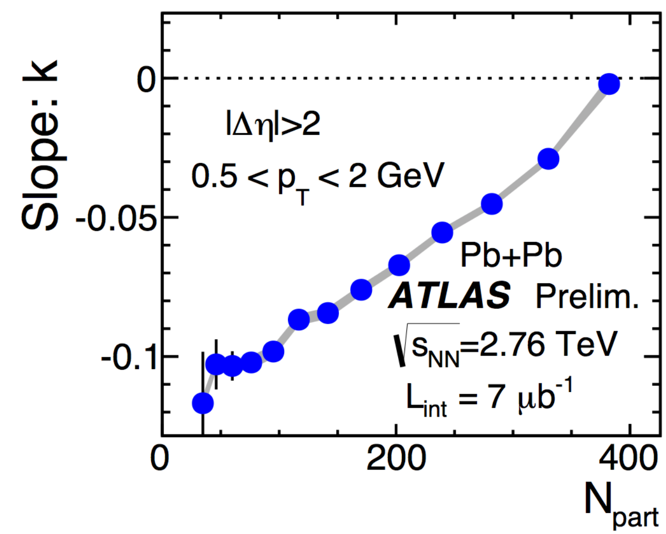
Can be used to fine tune initial geometry models!

- Quantified by a linear fit and extract the intercept and slope

$$v_3 = kv_2 + v_3^0$$



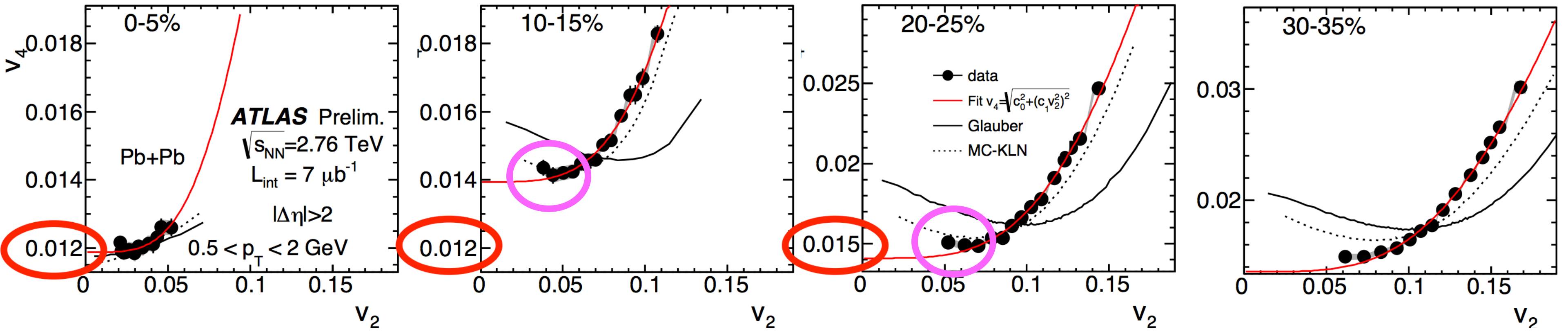




Events with zero ε_2 has larger average ε_3 → larger v_3.

linear (ε_4) and non-linear (v_2^2) component of v_4

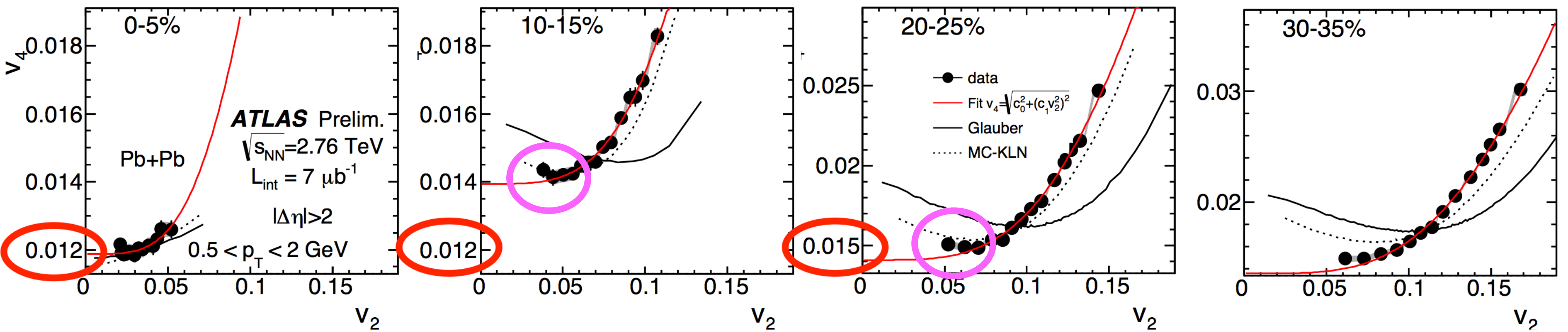
- v_4-v_2 correlation for fixed centrality bin $v_4 e^{i4\Phi_4} = c_0 e^{i\Phi_4^*} + c_1\left(v_2 e^{i2\Phi_2}\right)^2 \Rightarrow$ Fit by $v_4 = \sqrt{c_0^2 + c_1^2 v_2^4}$



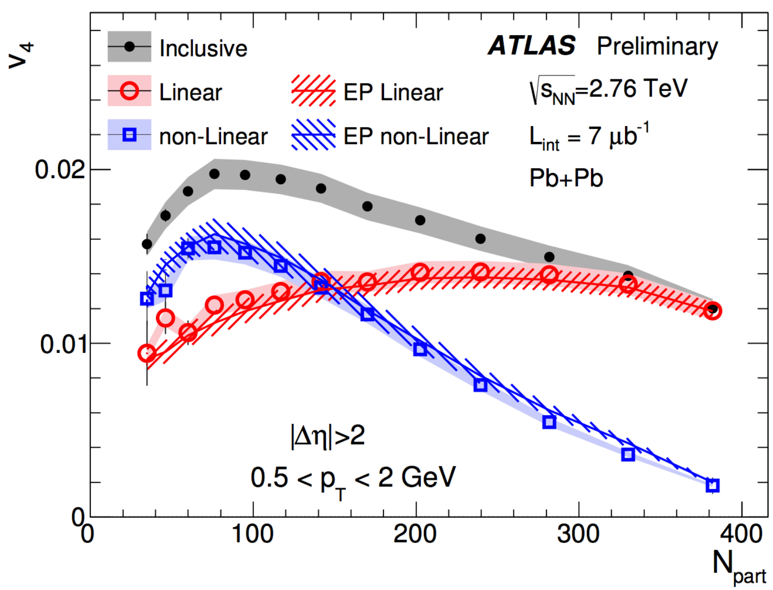
- Fit $v_4 = \sqrt{c_0^2 + c_1^2 v_2^4}$ to separate linear (ε_4) and non-linear (v_2^2) component

linear (ε_4) and non-linear (v_2^2) component of v_4

- v_4-v_2 correlation for fixed centrality bin $v_4 e^{i4\Phi_4} = c_0 e^{i\Phi_4^*} + c_1\left(v_2 e^{i2\Phi_2}\right)^2 \Rightarrow$ Fit by $v_4 = \sqrt{c_0^2 + c_1^2 v_2^4}$



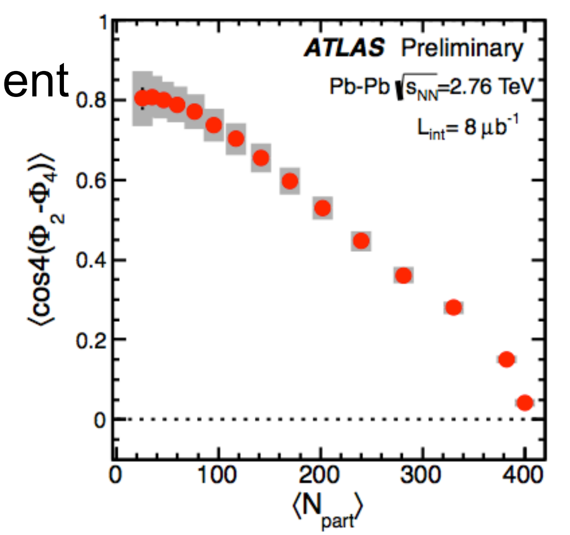
- Fit $v_4 = \sqrt{c_0^2 + c_1^2 v_2^4}$ to separate linear (ε_4) and non-linear (v_2^2) component



predict L and NL component from EP correlations:

$$v_4^{NL} = v_4 \langle \cos 4(\Phi_2 - \Phi_4) \rangle$$

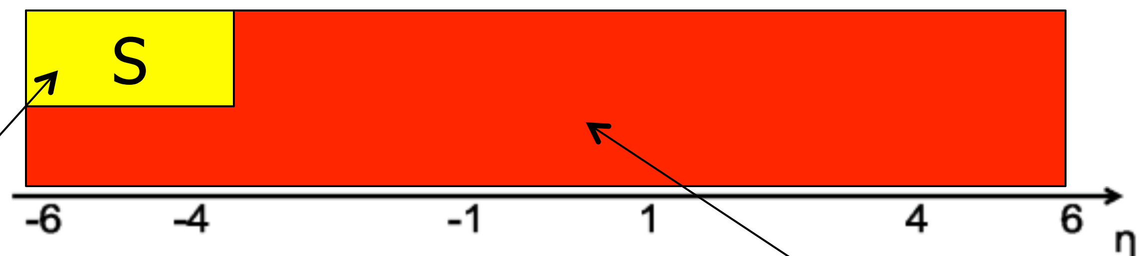
$$v_4^{L} = \sqrt{v_4^2 - (v_4^{NL})^2}$$



What about select on one side?

Schukraft, Timmins, and Voloshin, arXiv:1208.4563

Huo, Mohapatra, JJ arxiv:1311.7091

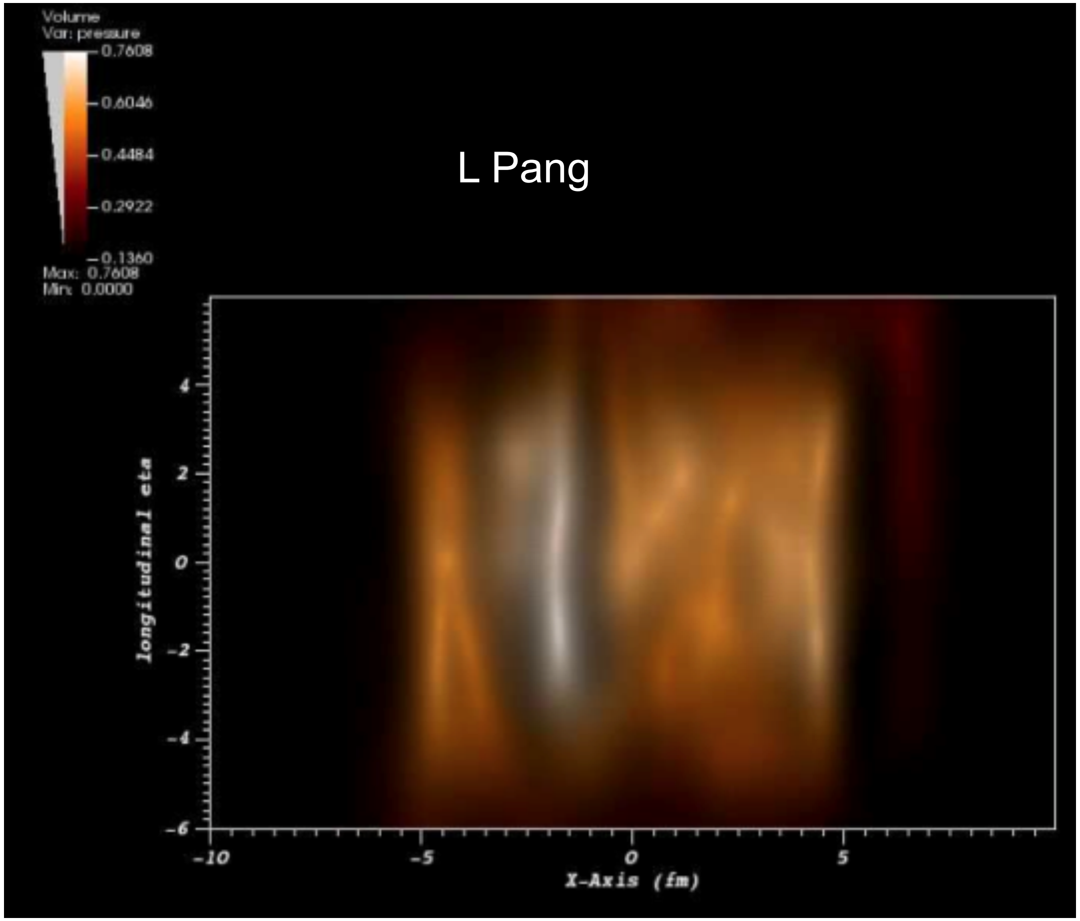


Select events with certain v_2^{obs}

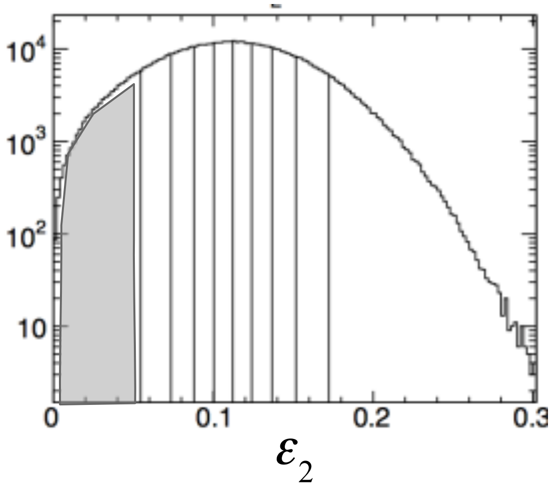
$p(v_n)$, $p(v_n,v_m)$or $p(\Phi_n,\Phi_{m,\ldots})$

AMPT model

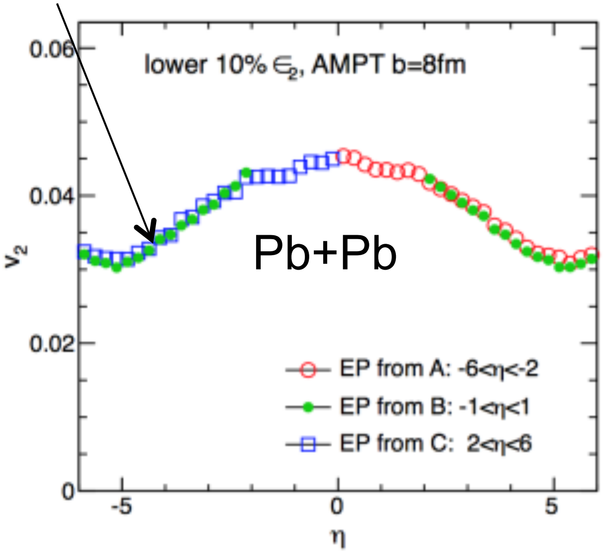
- AMPT model: Glauber+HIJING+transport
  - Has fluctuating geometry and collective flow
  - Longitudinal fluctuations and initial flow



$v_2(\eta)$: select on ε_2



Flow suppressed



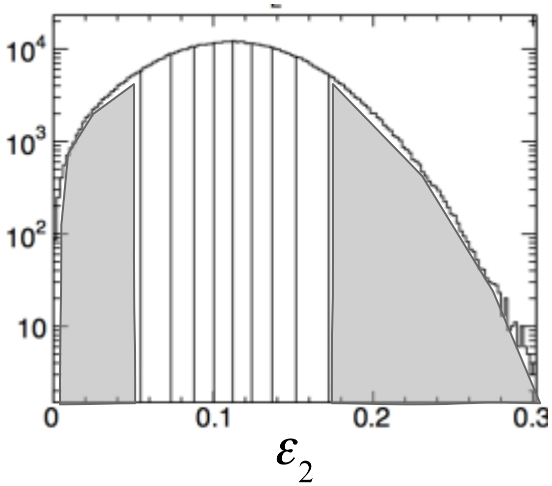
$v_2(\eta)|_{\eta>0}$ when EP in $-6<\eta<-2$

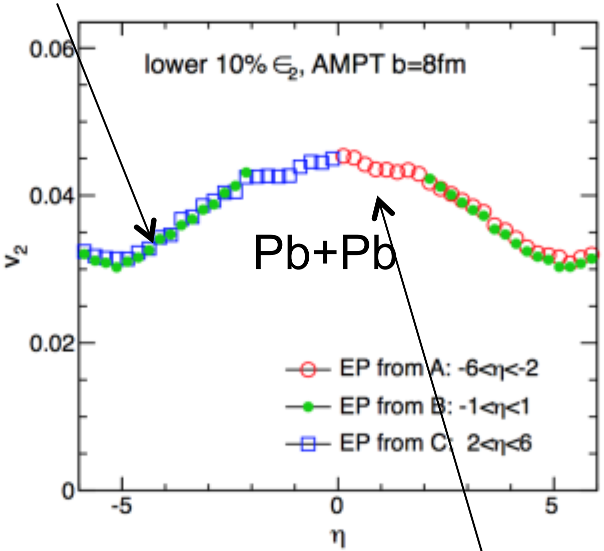
$v_2(\eta)|_{\eta<0}$ when EP in $2<\eta<6$

$v_2(\eta)|_{|\eta|>2}$ when EP in $|\eta|<1$

$v_2(\eta)$: select on ε_2

Flow suppressed



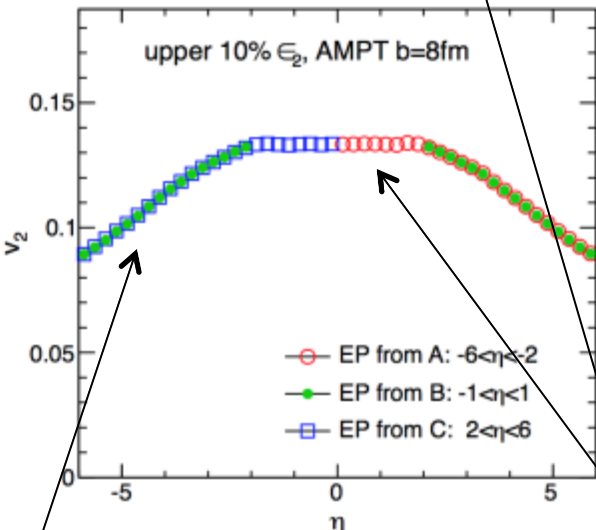


Pb+Pb

$v_2(\eta)|_{\eta>0}$ when EP in $-6<\eta<-2$

$v_2(\eta)|_{\eta<0}$ when EP in $2<\eta<6$

$v_2(\eta)|_{|\eta|>2}$ when EP in $|\eta|<1$

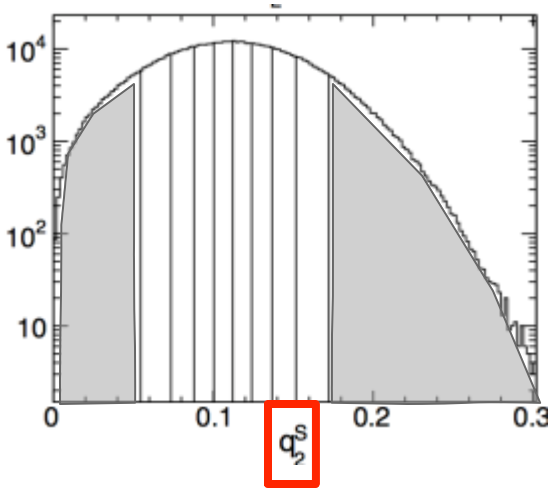


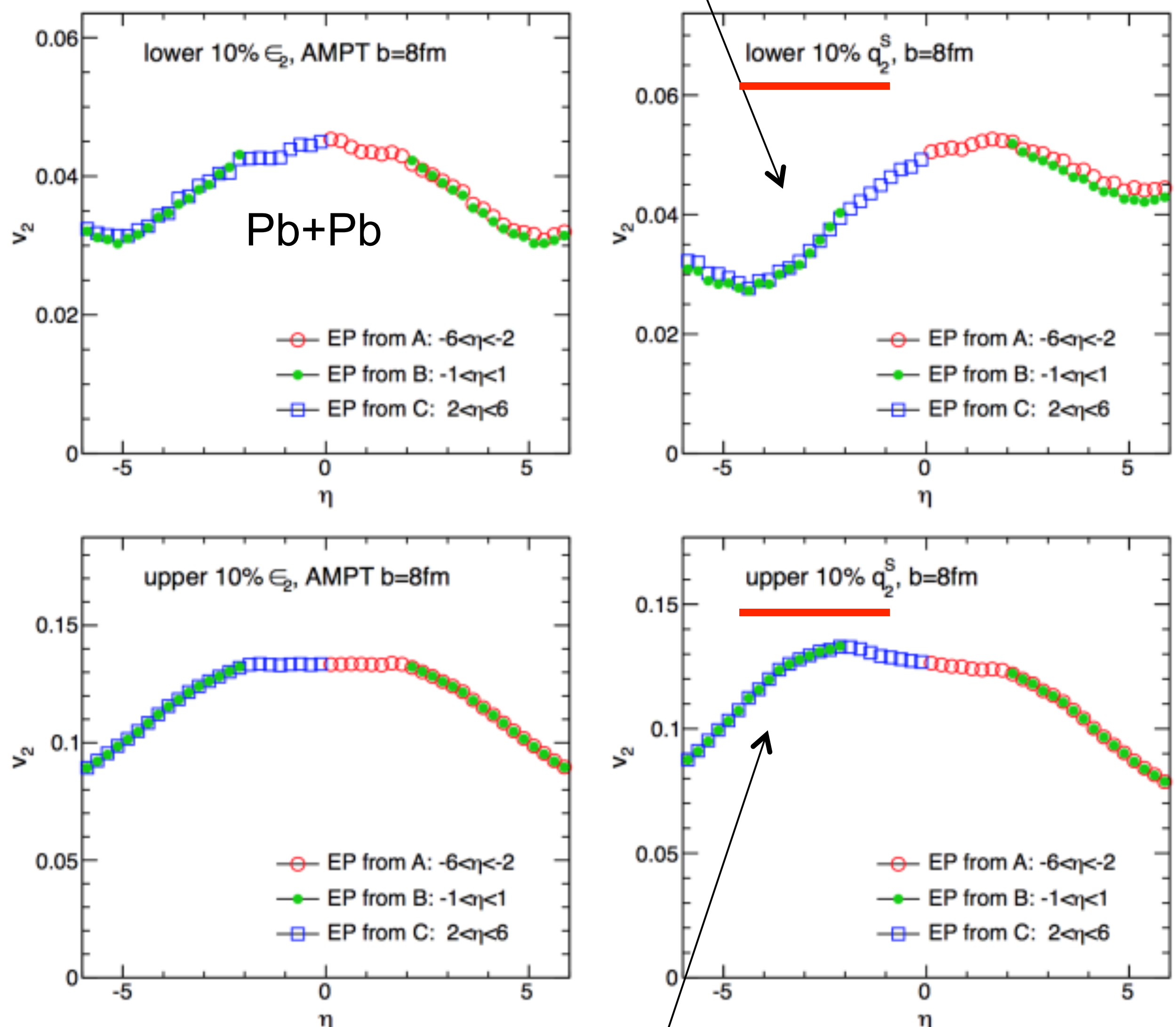
Flow enhanced

Symmetric distribution expected

$v_2(\eta)$: compare with selection on q_2

Suppression of flow in the selection window





$v_2(\eta)|_{\eta>0}$ when EP in $-6<\eta<-2$

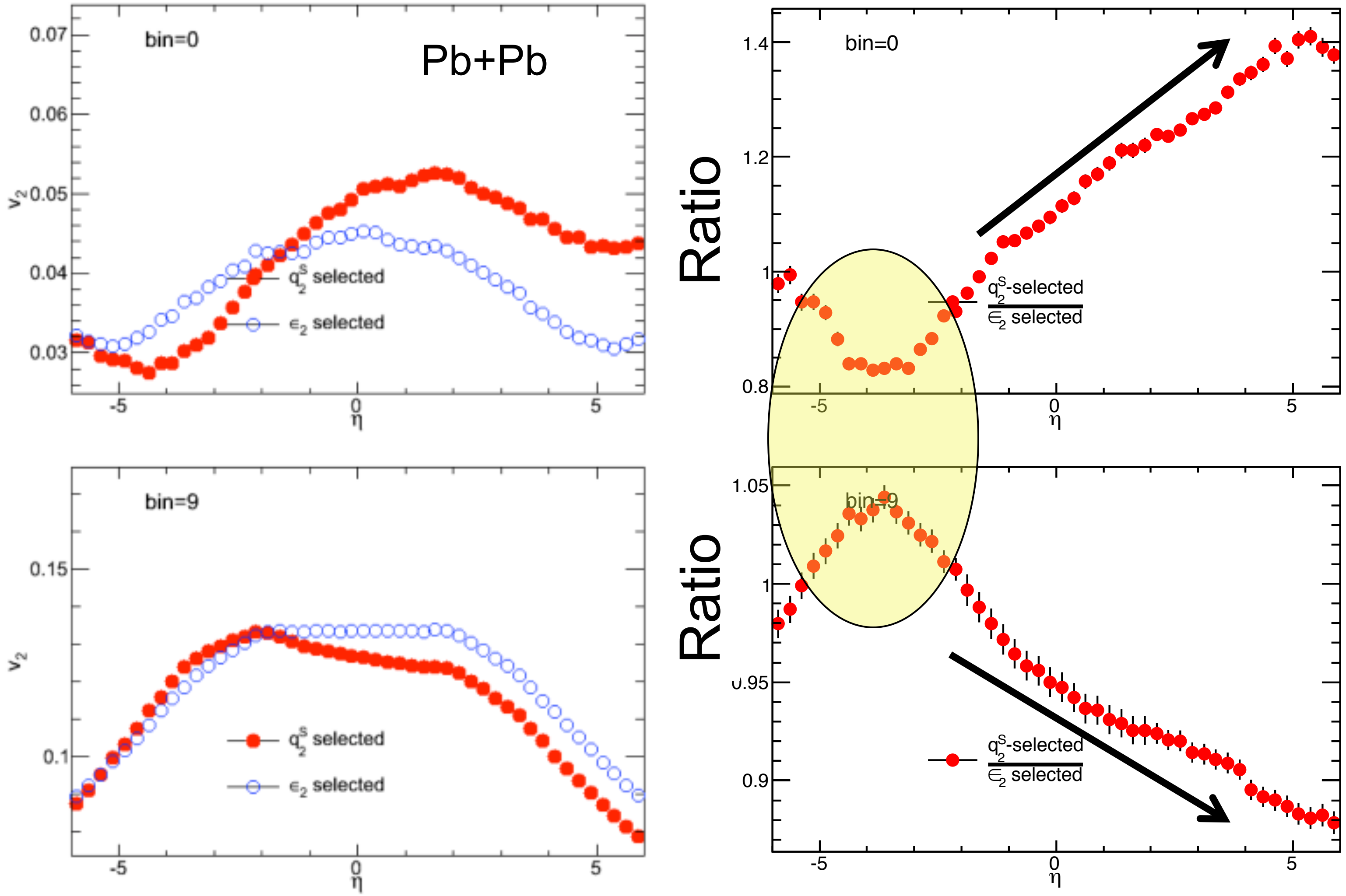
$v_2(\eta)|_{\eta<0}$ when EP in $2<\eta<6$

$v_2(\eta)|_{|\eta|>2}$ when EP in $|\eta|<1$

enhancement of flow in the selection window

What is the origin of $v_2(\eta)$ asymmetry?

- Suppression/enhancement of flow in the selected window
- Decreasing response to flow selection outside the selection window

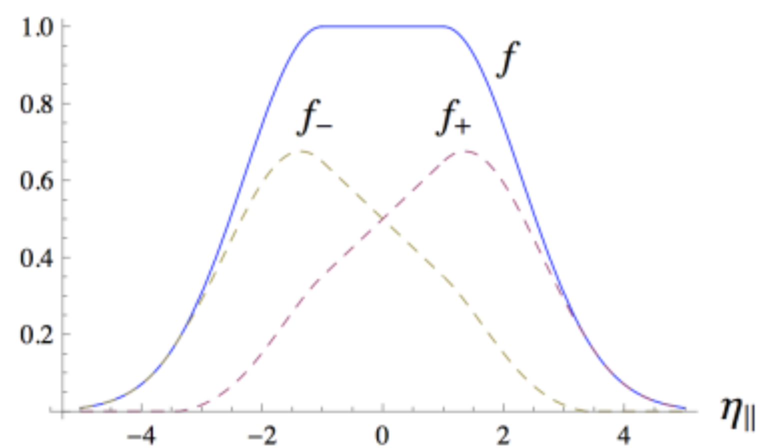


Longitudinal particle production

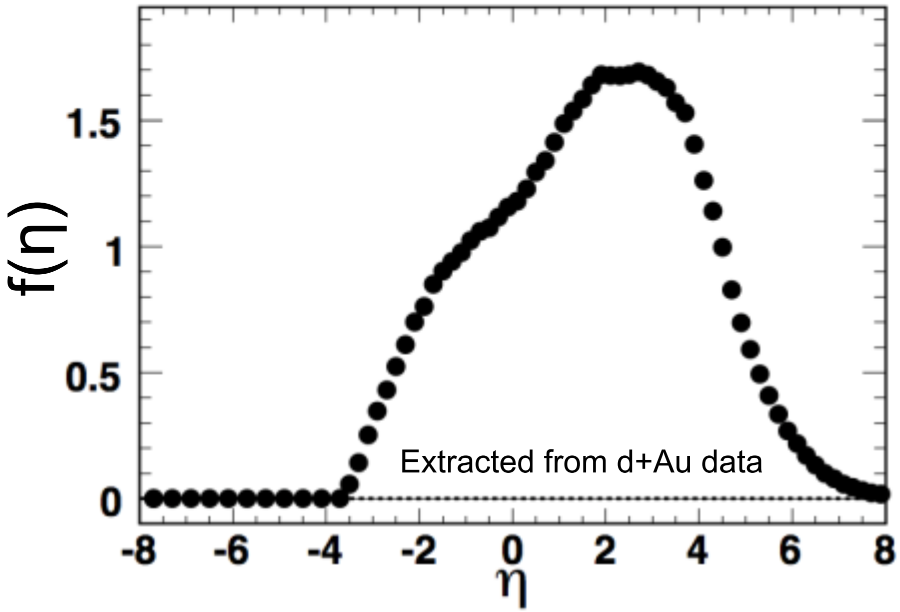
wounded nucleon model Bialas, Bzdak, Zalewski, Wozniak…. STAR/PHOBOS

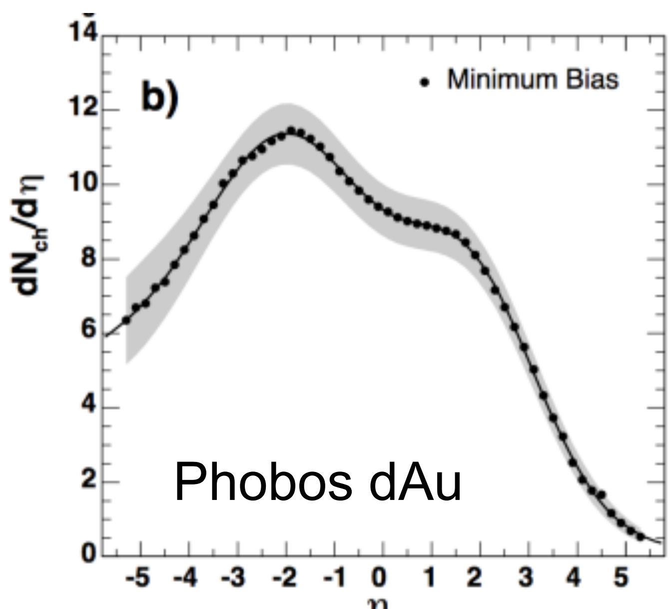
$$dN/d\eta \propto f^{F}(\eta)N^{F}_{part} + f^{B}(\eta)N^{B}_{part}$$

- Assumes that after the collision of two nuclei, the secondary particles are produced by independent fragmentation of wounded nucleons

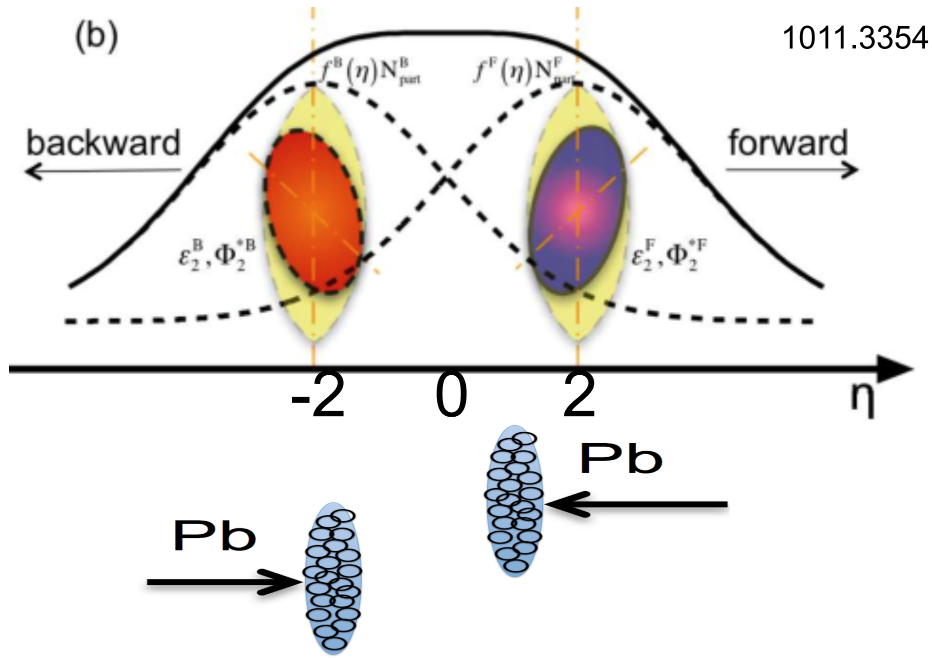


Emission function of one wounded nucleon

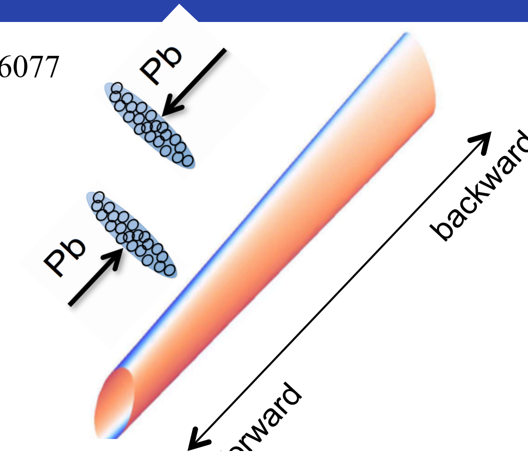


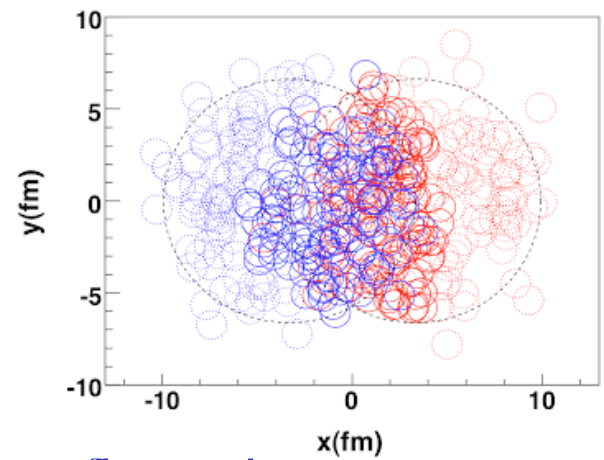


Flow longitudinal dynamics



1011.3354, 1403.6077



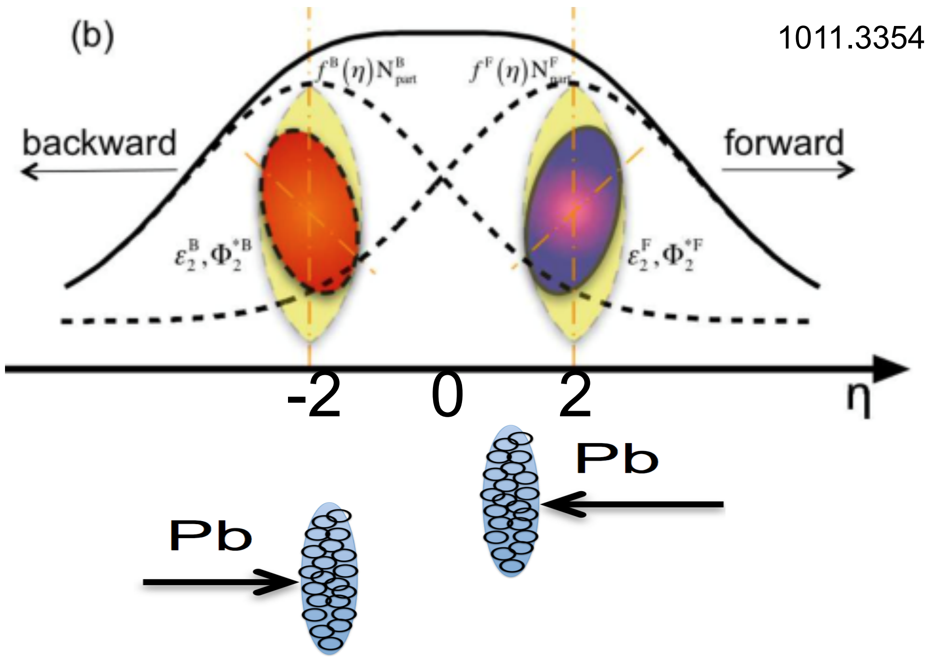


- Shape of participants in two nuclei not the same due to fluctuation

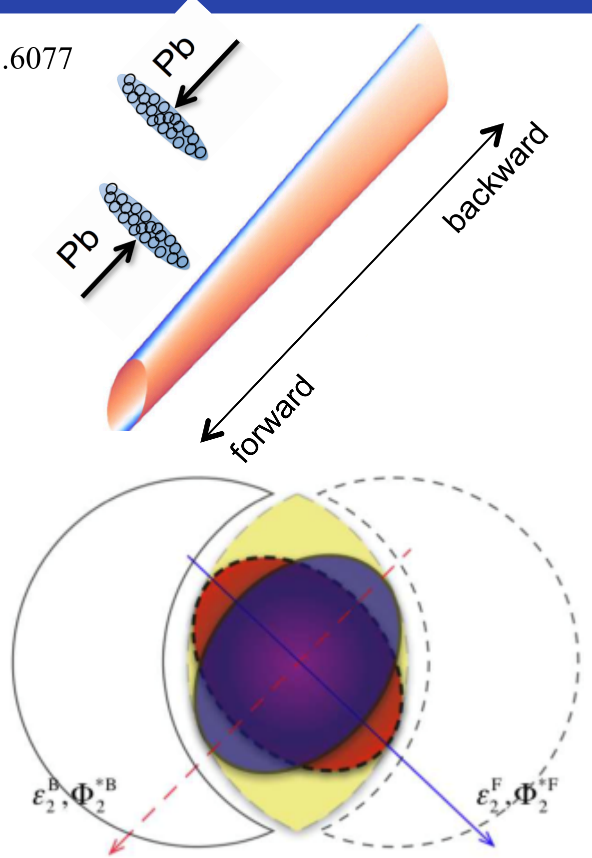
$$\varepsilon_m^{\mathrm{F}},\Phi_m^{*\mathrm{F}} \quad \varepsilon_m^{\mathrm{B}},\Phi_m^{*\mathrm{B}} \quad \varepsilon_m,\Phi_m^{*} \qquad N_{\mathrm{part}}^{\mathrm{F}},N_{\mathrm{part}}^{\mathrm{B}},N_{\mathrm{part}} \qquad \varepsilon_n^{\mathrm{F}},\Phi_n^{*\mathrm{F}} \neq \varepsilon_n^{\mathrm{B}},\Phi_n^{*\mathrm{B}}$$

- Particles are produced by independent fragmentation of wounded nucleons, emission function f (η) not symmetric in η→ Wounded nucleon model

Flow longitudinal dynamics



1011.3354, 1403.6077



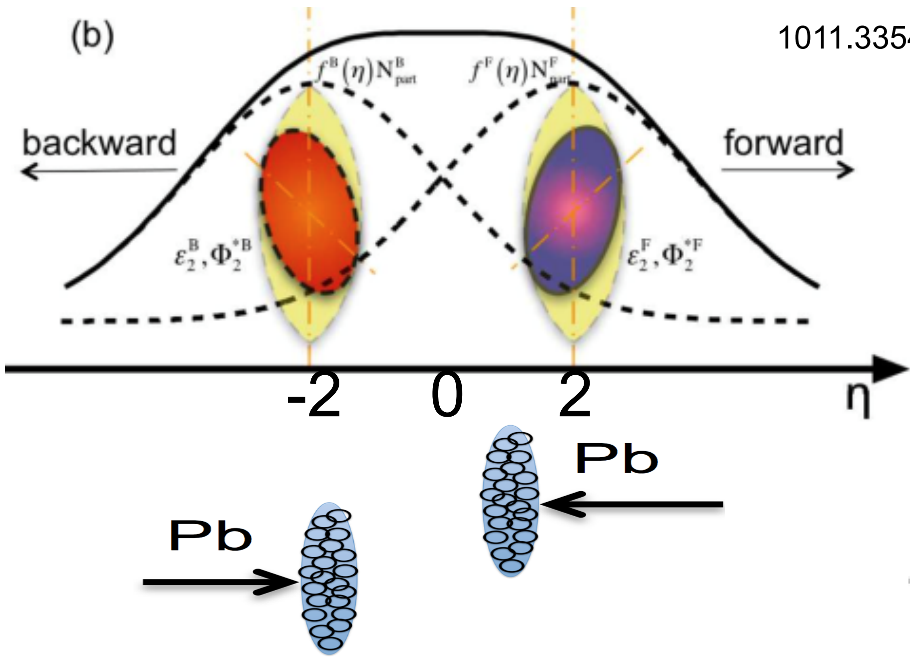
- Shape of participants in two nuclei not the same due to fluctuation

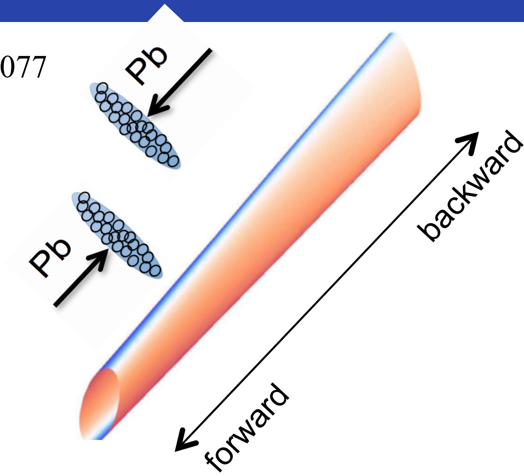
$$\varepsilon^{F}_{m},\Phi^{*F}_{m} \quad \varepsilon^{B}_{m},\Phi^{*B}_{m} \quad \varepsilon_{m},\Phi^{*}_{m} \qquad N^{F}_{part},N^{B}_{part},N_{part} \qquad \varepsilon^{F}_{n},\Phi^{*F}_{n} \neq \varepsilon^{B}_{n},\Phi^{*B}_{n}$$

- Particles are produced by independent fragmentation of wounded nucleons, emission function f (η) not symmetric in η→ Wounded nucleon model

Flow longitudinal dynamics

1011.3354, 1403.6077





$$\varepsilon_n^{\mathrm{F}}, \Phi_n^{*\mathrm{F}} \neq \varepsilon_n^{\mathrm{B}}, \Phi_n^{*\mathrm{B}}$$

- Eccentricity vector interpolates between $\vec{\epsilon}_n^{\mathrm{F}}$ and $\vec{\epsilon}_n^{\mathrm{B}}$

$$\vec{\epsilon}_n^{\mathrm{tot}}(\eta) \approx \alpha(\eta)\vec{\epsilon}_n^{\mathrm{F}} + (1-\alpha(\eta))\vec{\epsilon}_n^{\mathrm{B}} \equiv \epsilon_n^{\mathrm{tot}}(\eta) e^{in\Phi_n^{*\mathrm{tot}}(\eta)}$$

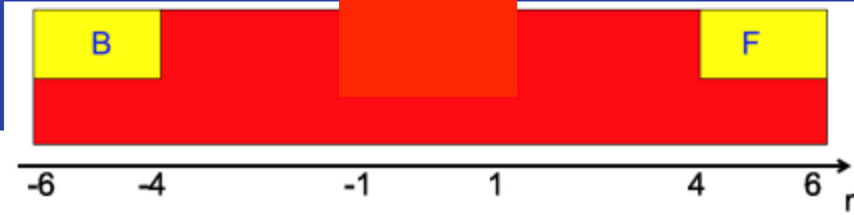
α(η) determined by f(η)

Asymmetry: $\varepsilon_n^{\mathrm{F}} \neq \varepsilon_n^{\mathrm{B}}$

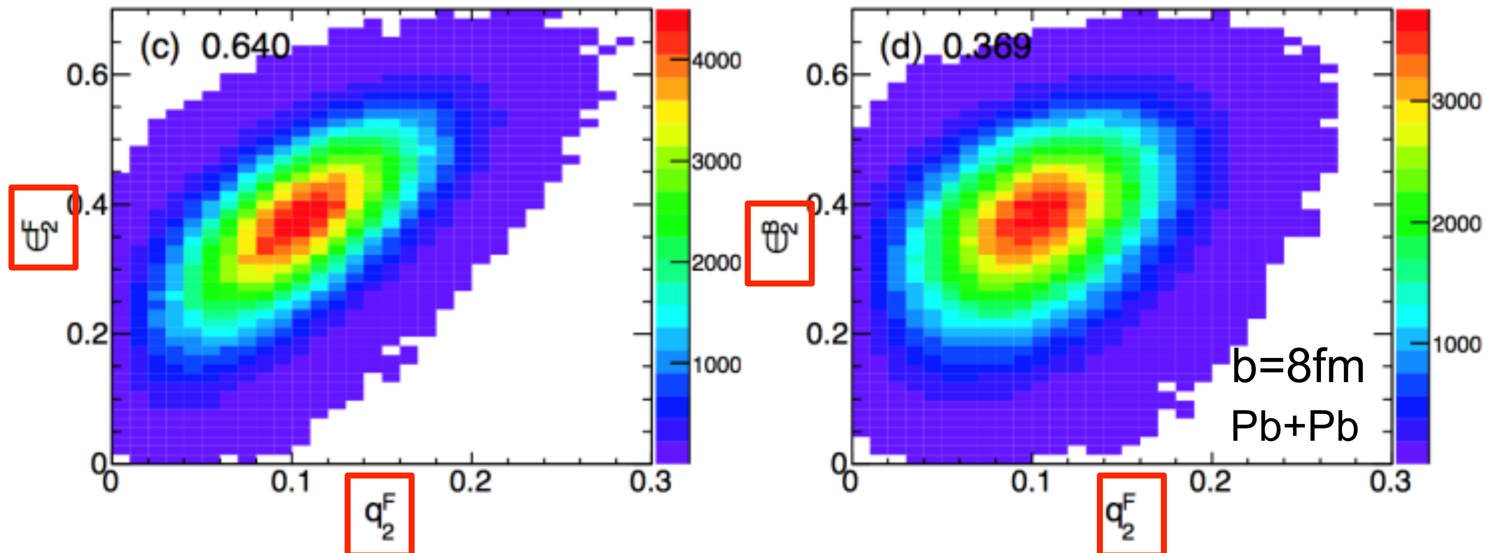
Twist: $\Phi_n^{*\mathrm{F}} \neq \Phi_n^{*\mathrm{B}}$

- Hence $\vec{v}_n(\eta) \approx c_n(\eta)\left[\alpha(\eta)\vec{\epsilon}_n^{\mathrm{F}} + (1-\alpha(\eta))\vec{\epsilon}_n^{\mathrm{B}}\right]$ for n=2,3
  - Picture verified in AMPT simulations, magnitude estimated 1403.6077

What AMPT tell us?

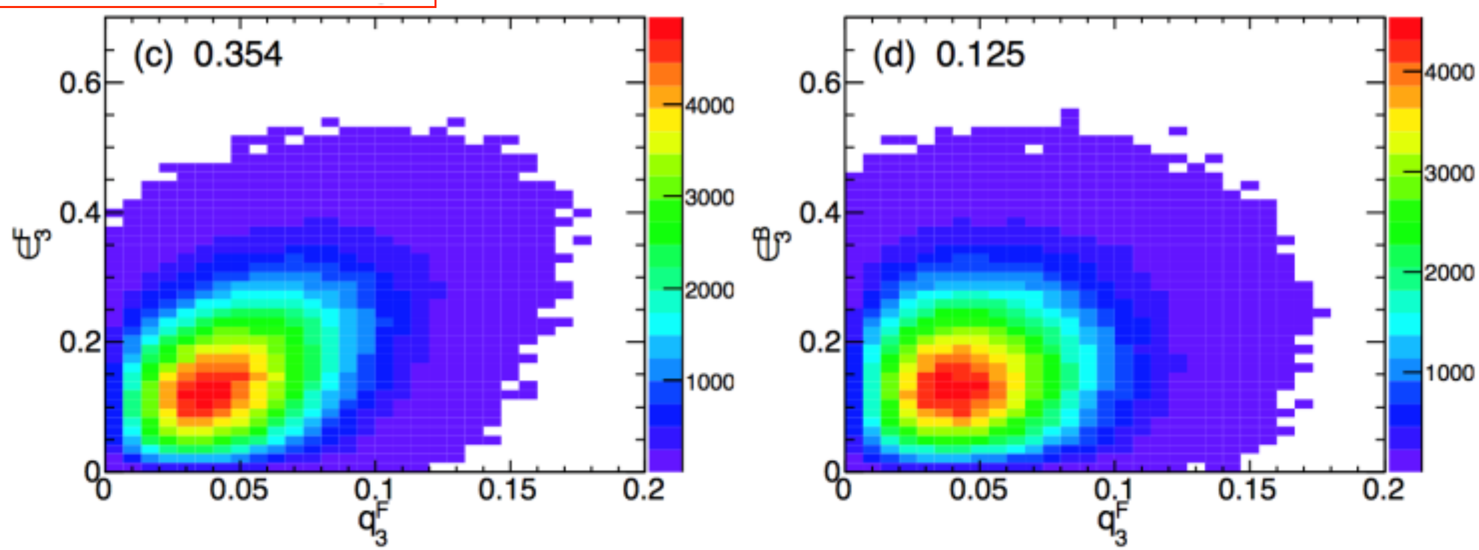


ε_2^F more correlated with q_2^F than q_2^B



ε_3^F more correlated with q_3^F than q_3^B

FB asymmetry survives



What AMPT tell us?

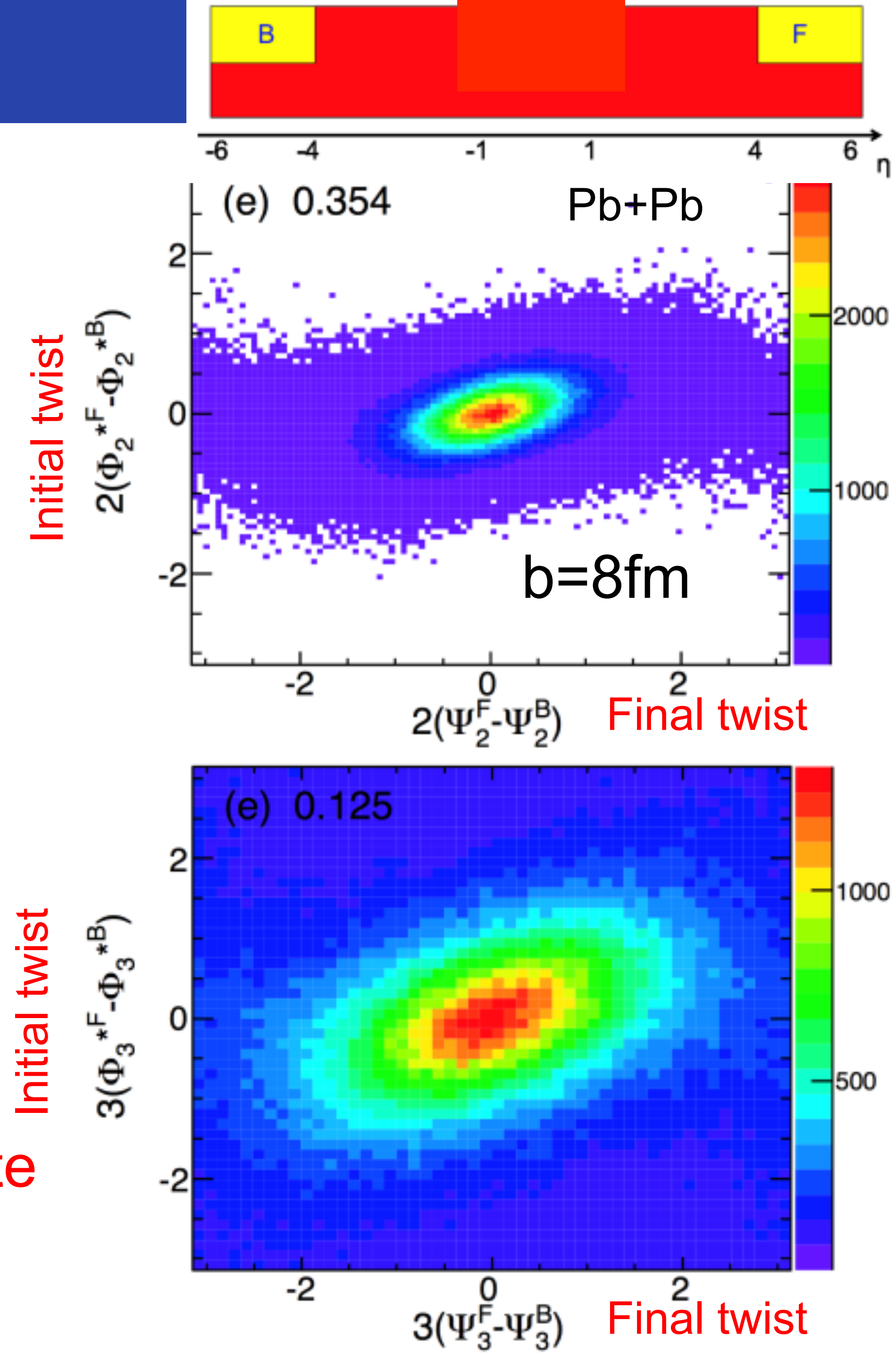
- Twist in initial geometry appears as twist in the final state flow
  - Participant plane angles:

$$\Phi_n^{*F} \quad \Phi_n^{*B}$$

  - Final state event-plane angles

$$\Psi_n^{F} \quad \Psi_n^{B}$$

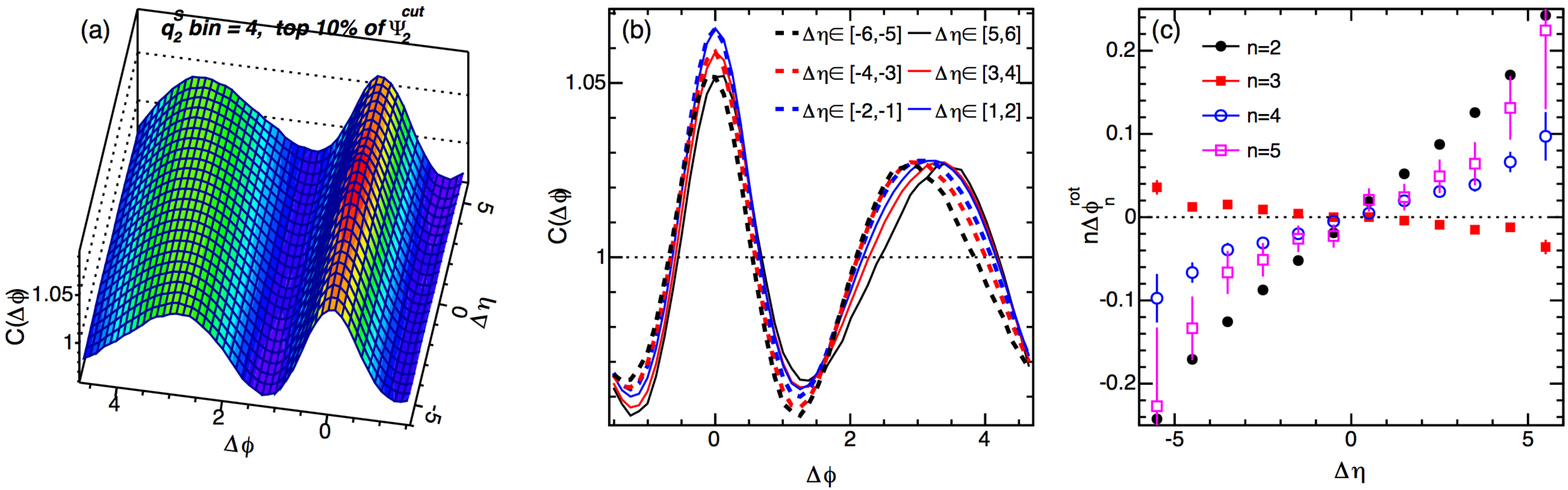
Initial twist survives to final state



Twist seen in simple 2PC analysis

- NO event-plane determination! Just select twist in large η and check correlation at center-rapidity.

$$C(\Delta\phi,\Delta\eta) \propto 1 + 2\sum v_n^{\mathrm{a}} v_n^{\mathrm{b}} \cos\left(n\Delta\phi - n\Delta\Phi_n^{\mathrm{rot}}\right)$$



- Though twist is enforced on q_2, twist also seen for higher order v_n
- Non-linear mixing to the higher order harmonics!! .

Summary-I

- Event-shape fluctuations contains a lot of information

$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{\text{evts}}} \frac{dN_{\text{evts}}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$

- Three complementary methods:

Strong fluctuation within fixed centrality!

	pdf's	cumulants	event-shape method
Flow-amplitudes	$p(v_n)$	$v_n\{2k\},\ k = 1,2,...$	NA
	$p(v_n, v_m)$	$\langle v_n^2 v_m^2 \rangle - \langle v_n^2 \rangle \langle v_m^2 \rangle$	yes
	$p(v_n, v_m, v_l)$	$\langle v_n^2 v_m^2 v_l^2 \rangle + 2\langle v_n^2 \rangle \langle v_m^2 \rangle \langle v_l^2 \rangle -$ $\langle v_n^2 v_m^2 \rangle \langle v_l^2 \rangle - \langle v_m^2 v_l^2 \rangle \langle v_n^2 \rangle - \langle v_l^2 v_n^2 \rangle \langle v_m^2 \rangle$	yes
	...	Obtained recursively as above	yes
EP-correlation	$p(\Phi_n, \Phi_m, ...)$	$\langle v_n^{c_n} v_m^{c_m} ... \cos(c_n n\Phi_n + c_m m\Phi_m + ...)\rangle$ $\sum_k kc_k = 0$	yes
Mixed-correlation	$p(v_l, \Phi_n, \Phi_m, ...)$	$\langle v_l^2 v_n^{c_n} v_m^{c_m} ... \cos(c_n n\Phi_n + c_m m\Phi_m + ...)\rangle -$ $\langle v_l^2 \rangle \langle v_n^{c_n} v_m^{c_m} ... \cos(c_n n\Phi_n + c_m m\Phi_m + ...)\rangle$ $\sum_k kc_k = 0$	yes

Summary-II

- Rich patterns forward/backward EbyE flow fluctuations:

$$\vec{v}_n(\eta) \approx c_n(\eta)\left[\alpha(\eta)\vec{\epsilon}_n^{\,\mathrm{F}} + (1-\alpha(\eta))\vec{\epsilon}_n^{\,\mathrm{B}}\right]$$

Event-shape selection and event twist techniques

- New avenue to study initial state fluctuations, particle production and collective expansion dynamics.