Stony Brook University | The State University of New York

IS2014

Correlations between harmonics

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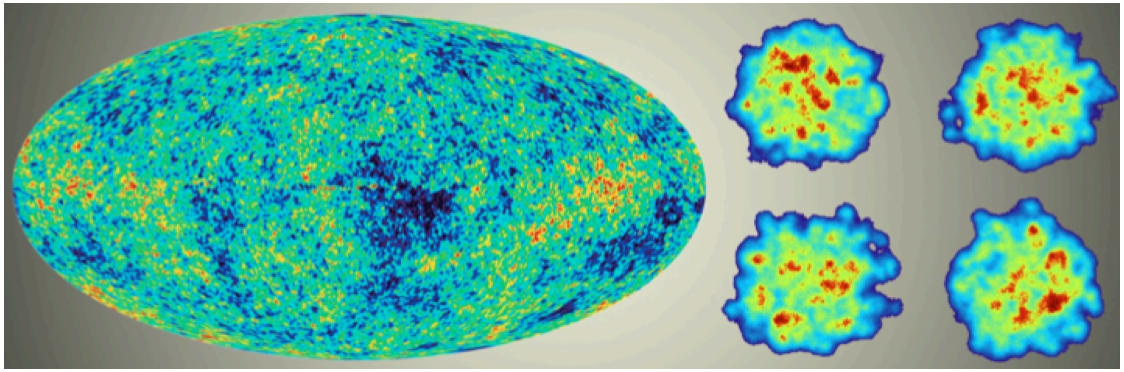
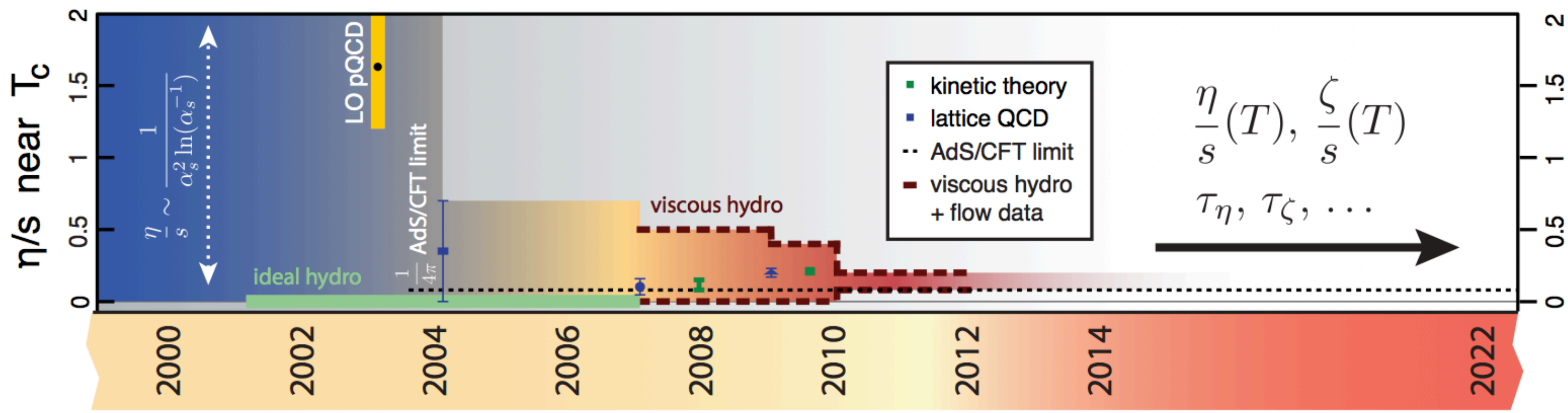
The 2nd International Conference on the
Initial Stages in High-Energy Nuclear Collisions

Recent accomplishments

Improved the space-time picture of bulk matter evolution

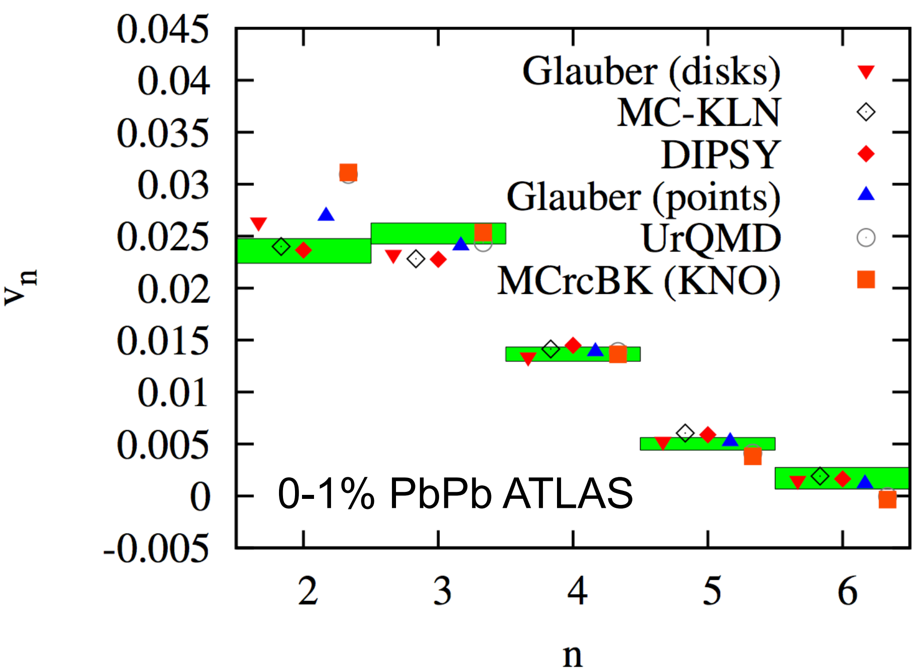


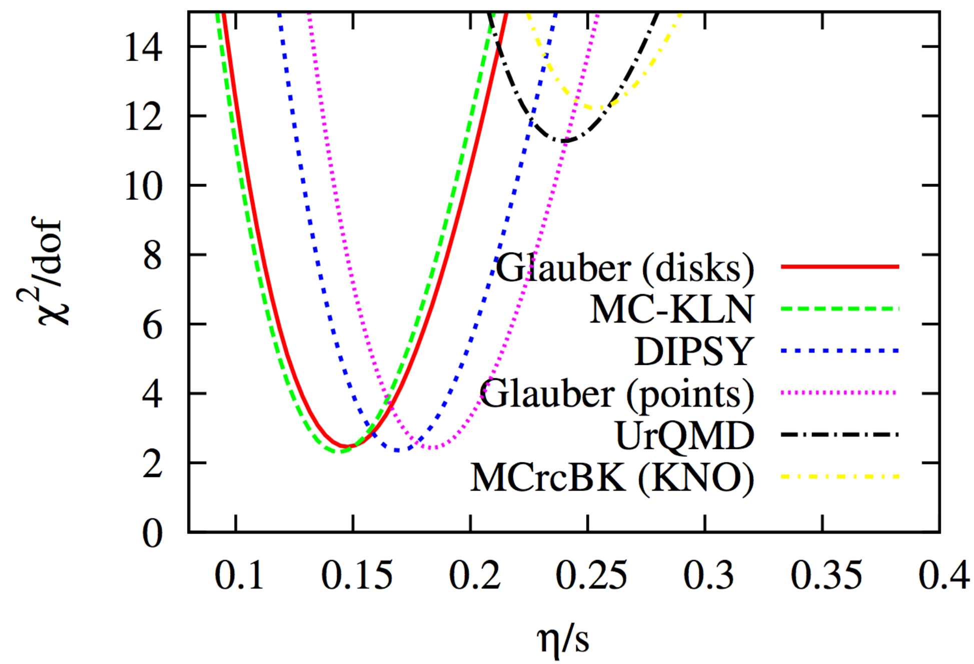
Improved extraction of the medium properties



Challenge: simultaneous control of two unknowns

Simultaneous control of initial geometry and η/s





For traditional observables, i.e.$\langle v_n \rangle$,
experimental error no-longer the limiting factors

Need new observables with new intuitive insights!

Future experimental measurements

- Pb+Pb/Au+Au top energy: as complete as possible
  - Event-by-event flow observables→ initial fluctuation & flow correlation
  - Event shape engineer & engen-mode ana.→ geometry model & hydro. response
  - Longitudinal fluctuations→ particle production & early time dynamics
  - Medium response→ relaxation mechanism for local energy deposition
  - PID v_n → hadronization and hadronic transport

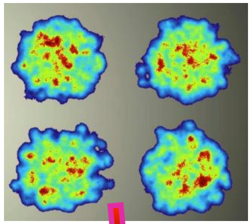
- System size scan
  - e.g. Cu+Au or U+U → hydro response to extreme geometries.
  - p+A vs peripheral A+A→ onset of collectivity? onset of hydrodynamics?

- Energy scan of Au+Au collisions at RHIC
  - Temperature dependence of medium properties, partonic vs hadronic.
    - However lower stats & p_T reach

Event-by-event observables

Many little bangs

1104.4740, 1209.2323,1203.5095 ,1312.3572



$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{\text{evts}}} \frac{dN_{\text{evts}}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$

Moments:

$$\langle \cos(n_1\phi_1 + n_2\phi_2 ... + n_m\phi_m) \rangle = \langle v_{n_1} v_{n_2} ... v_{n_m} \cos(n_1\Phi_{n_1} + n_2\Phi_{n_2} ... + n_m\Phi_{n_m}) \rangle \qquad \Sigma n_i = 0$$

e.g.

$$\langle \cos(n\phi_1 - n\phi_2 + n\phi_3 - n\phi_4) \rangle = \left\langle v_n^4 \cos(n\Phi_n - n\Phi_n + n\Phi_n - n\Phi_n) \right\rangle = \left\langle v_n^4 \right\rangle$$

$$\langle \cos(n\phi_1 - n\phi_2 + m\phi_3 - m\phi_4) \rangle = \left\langle v_n^2 v_m^2 \cos(n\Phi_n - n\Phi_n + m\Phi_m - m\Phi_m) \right\rangle = \left\langle v_n^2 v_m^2 \right\rangle$$

Cumulants obtained by combining with lower order correlators:

$c_n\{4\}$ = $\left\langle v_n^4 \right\rangle - 2 \left\langle v_n^2 \right\rangle^2$ $\qquad p(v_n)$

$c_{n,m}\{2,2\}$ = $\left\langle v_n^2 v_m^2 \right\rangle - \left\langle v_n^2 \right\rangle \left\langle v_m^2 \right\rangle$ $\qquad p(v_n,v_m)$

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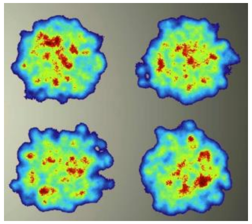

$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{\text{evts}}} \frac{dN_{\text{evts}}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$

	pdf's	cumulants
Flow-amplitudes	$p(v_n)$	$v_n\{2k\},\ k = 1,2,...$
	$p(v_n, v_m)$	$\langle v_n^2 v_m^2\rangle - \langle v_n^2\rangle\langle v_m^2\rangle$
	$p(v_n, v_m, v_l)$	$\langle v_n^2 v_m^2 v_l^2\rangle + 2\langle v_n^2\rangle\langle v^2$ … $\langle v_n^2 v_m^2\rangle\langle v_l^2\rangle - \langle v_m^2 v_l^2$ … $\langle v_m^2\rangle$
	...	Obtained … above
EP-correlation	$p(\Phi_n, \Phi_m, ...)$	$\langle v_n^{c_n} v^c$ … $_n + c_m m\Phi_m + ...))\rangle$ … $k c_k = 0$
Mixed-correlation	$p(v_l, \Phi_n, \Phi_m, ...)$	$\langle v_l^2 v_n^{c_n}$ … $\cos(c_n n\Phi_n + c_m m\Phi_m + ...))\rangle -$ $\langle v_l^2\rangle\langle v_n^{c_n} v_m^{c_m} ... \cos(c_n n\Phi_n + c_m m\Phi_m + ...))\rangle$ $\sum_k k c_k = 0$

a subset moments of PDF
only even modes

Event-by-event observables

Many little bangs

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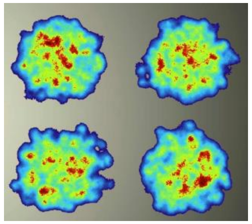


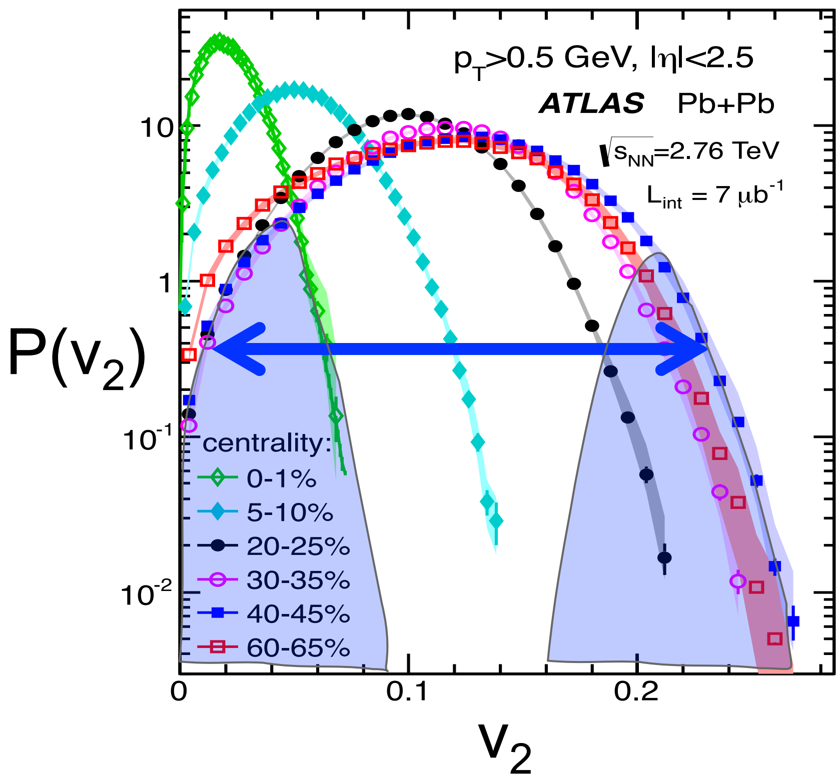
$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{\text{evts}}} \frac{dN_{\text{evts}}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$

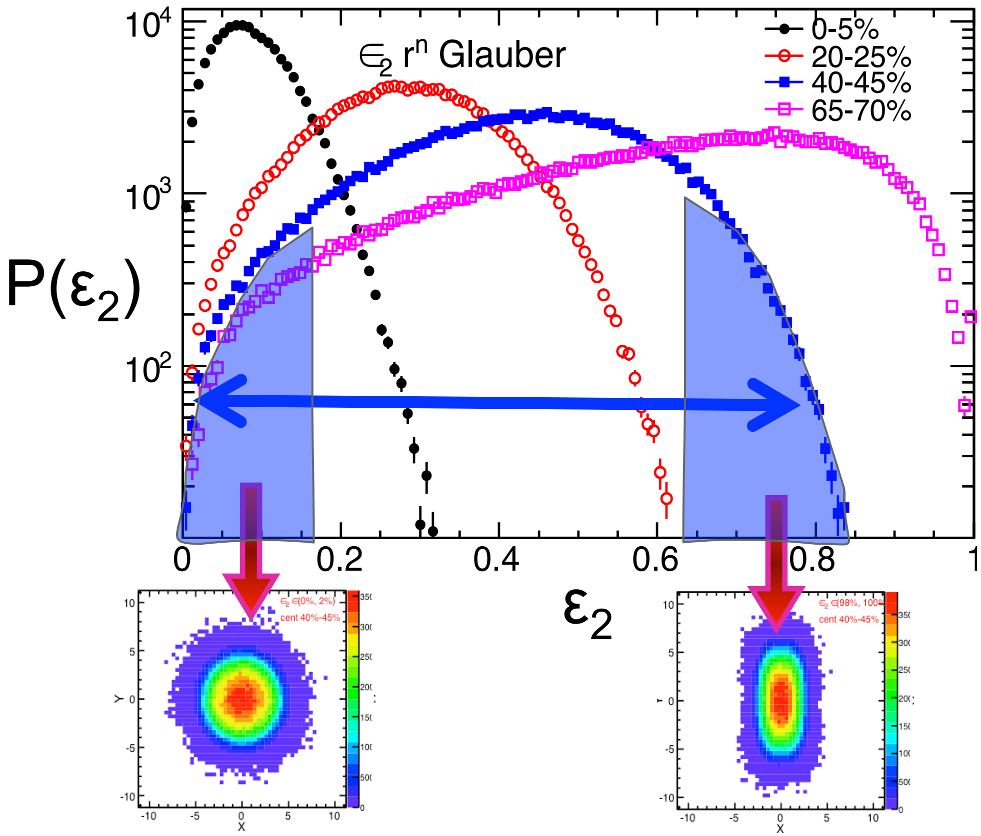
	pdf's	cumulants	event-shape method
Flow-amplitudes	$p(v_n)$	$v_n\{2k\}$, $k = 1,2,...$	NA
	$p(v_n, v_m)$	$\langle v_n^2 v_m^2 \rangle - \langle v_n^2 \rangle \langle v_m^2 \rangle$	yes
	$p(v_n, v_m, v_l)$	$\langle v_n^2 v_m^2 v_l^2 \rangle + 2\langle v_n^2 \rangle \langle v^2$ … $\langle v_n^2 v_m^2 \rangle \langle v_l^2 \rangle - \langle v_m^2 v_l^2$ … $\langle v_m^2 \rangle$	yes
	...	Obtained … above	yes
EP-correlation	$p(\Phi_n, \Phi_m, ...)$	$\langle v_n^{c_n} v^{c}$ … $_n + c_m m\Phi_m + ...)\rangle$ … $\sum_k kc_k = 0$	yes
Mixed-correlation	$p(v_l, \Phi_n, \Phi_m, ...)$	$\langle v_l^2 v_n^{c_n}$ … $\cos(c_n n\Phi_n + c_m m\Phi_m + ...)\rangle -$ $\langle v_l^2 \rangle \langle v_n^{c_n} v_m^{c_m} ... \cos(c_n n\Phi_n + c_m m\Phi_m + ...)\rangle$ $\sum_k kc_k = 0$	yes

a subset moments of PDF
only even modes

Event shape engineering technique

$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{evts}} \frac{dN_{evts}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$





- Measure v_n or EP correlation for different v_2 at fixed centrality

$p(v_n, v_2)$ $p(\Phi_n, \Phi_m, v_2)$

Schukraft, Timmins, and Voloshin, arXiv:1208.4563

Huo, Mohapatra, JJ arxiv:1311.7091

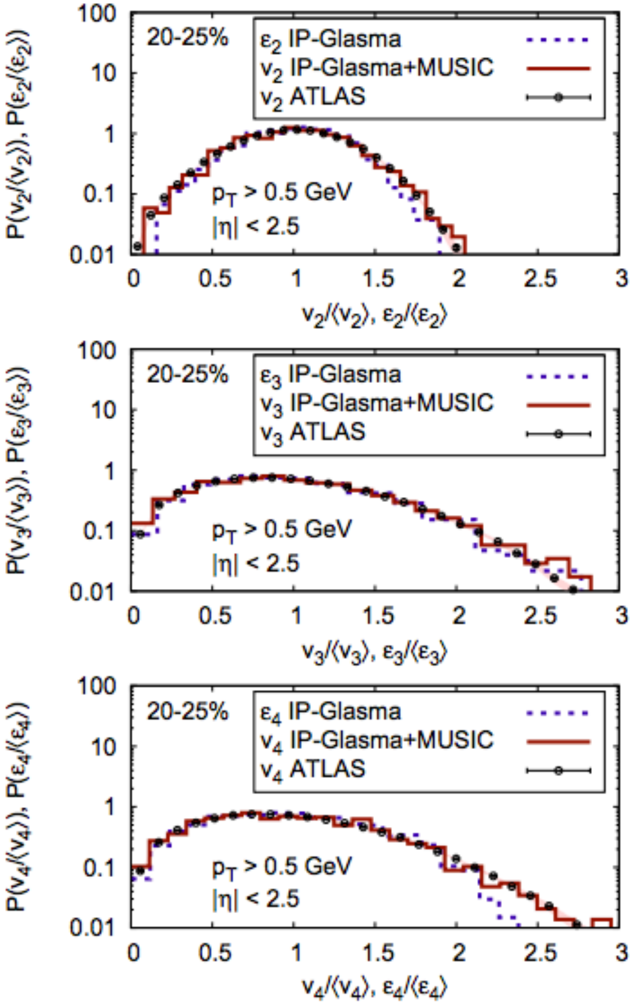
Applications

	pdf's	cumulants	event-shape method
Flow-amplitudes	$p(v_n)$	$v_n\{2k\}$, $k = 1{,}2{,}\ldots$	NA
	$p(v_n, v_m)$	$\langle v_n^2 v_m^2\rangle - \langle v_n^2\rangle\langle v_m^2\rangle$	yes
	$p(v_n, v_m, v_l)$	$\langle v_n^2 v_m^2 v_l^2\rangle + 2\langle v_n^2\rangle\langle v_m^2\rangle\langle v_l^2\rangle -$ $\langle v_n^2 v_m^2\rangle\langle v_l^2\rangle - \langle v_m^2 v_l^2\rangle\langle v_n^2\rangle - \langle v_l^2 v_n^2\rangle\langle v_m^2\rangle$	yes
	...	Obtained recursively as above	yes
EP-correlation	$p(\Phi_n, \Phi_m, \ldots)$	$\langle v_n^{c_n} v_m^{c_m} \ldots \cos(c_n n\Phi_n + c_m m\Phi_m + \ldots)\rangle$ $\sum_k k c_k = 0$	yes
Mixed-correlation	$p(v_l, \Phi_n, \Phi_m, \ldots)$	$\langle v_l^2 v_n^{c_n} v_m^{c_m} \ldots \cos(c_n n\Phi_n + c_m m\Phi_m + \ldots)\rangle -$ $\langle v_l^2\rangle\langle v_n^{c_n} v_m^{c_m} \ldots \cos(c_n n\Phi_n + c_m m\Phi_m + \ldots)\rangle$ $\sum_k k c_k = 0$	yes

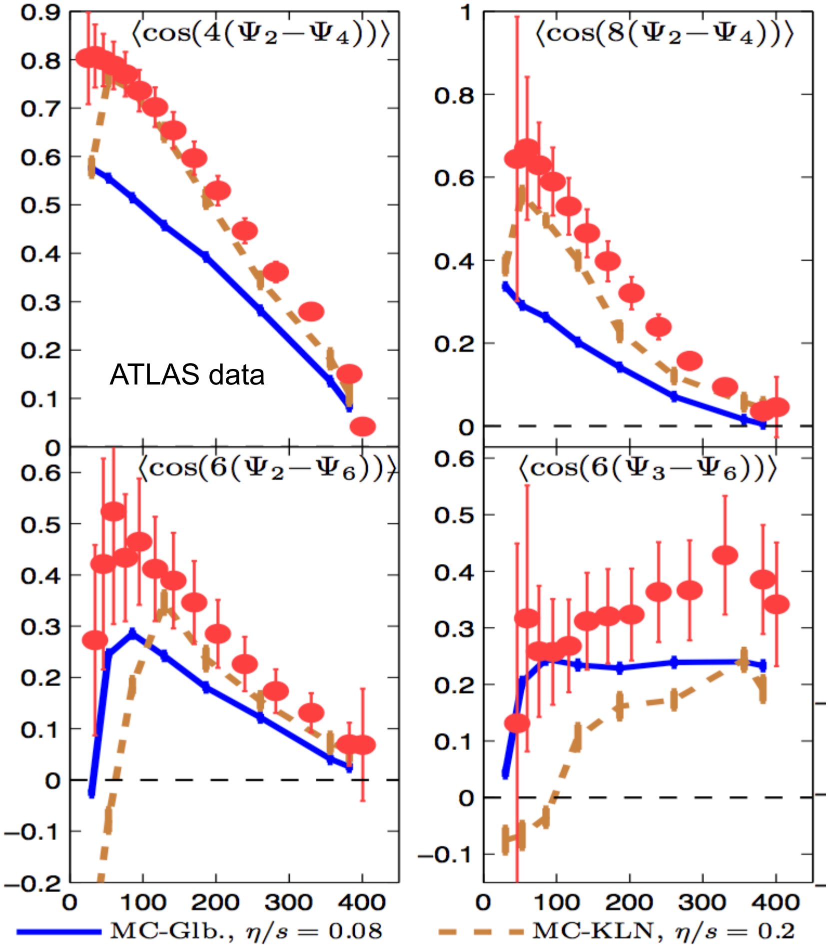
Current measurements of E-by-E distributions

$p(v_2)$, $p(v_3)$, $p(v_4)$

$p(\Phi_n,\Phi_m)$ and $p(\Phi_n,\Phi_m,\Phi_L)$

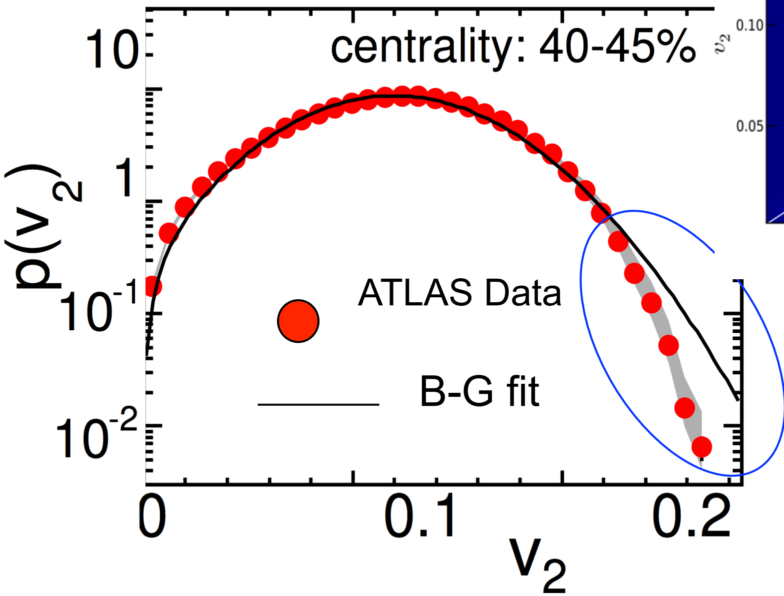


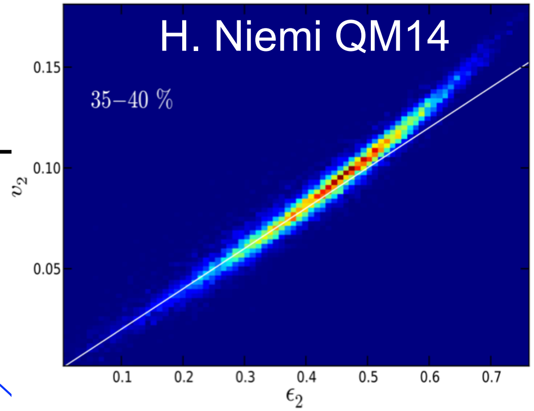
Pb+Pb

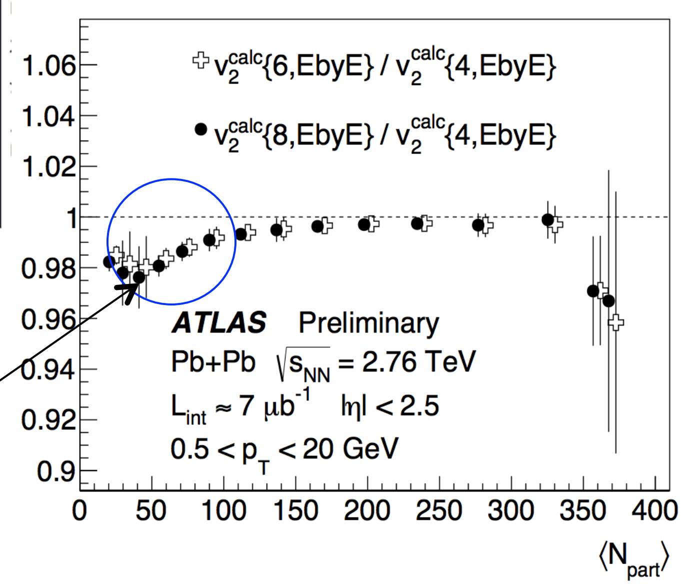


Measured distributions quantitatively described by hydro.

Non-Gaussianity in the p(v_2) distribution







$\langle v_2 \rangle = k \langle \varepsilon_2 \rangle$

Non-Gaussian behavior

- Reflected by a 1-2% change beyond 4th order cumulants
  - Note: p(v_2) contains more information than first few cumulants.

Non-BG in p(ε_2) or non-linearity of response for large ε_2

EP correlation: how $(\varepsilon_n, \Phi_n^*)$ are transferred to (v_n, Φ_n)?

- Flow response is linear for v_2 and v_3: $v_n \propto \varepsilon_n$ and $\Phi_n \approx \Phi_n^*$ i.e.

$$v_2 e^{-i2\Phi_2} \propto \epsilon_2 e^{-i2\Phi_2^*}, \quad v_3 e^{-i3\Phi_3} \propto \epsilon_3 e^{-i3\Phi_3^*}$$

- Higher-order can arise from EP correlations (mode mixing), e.g. :

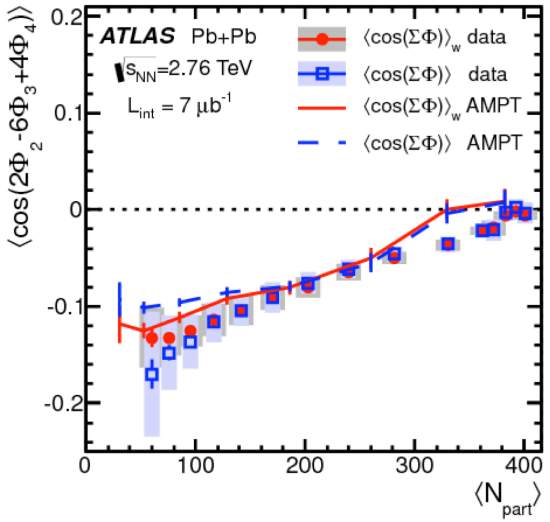
$$v_4 e^{i4\Phi_4} \propto \varepsilon_4 e^{i4\Phi_4^*} + c v_2^2 e^{i4\Phi_2} + \ldots$$

$$v_5 e^{i5\Phi_5} \propto \varepsilon_5 e^{i5\Phi_5^*} + c v_2 v_3 e^{i(2\Phi_2+3\Phi_3)} + \ldots$$

$$v_6 e^{i6\Phi_6} \propto \varepsilon_6 e^{i6\Phi_6^*} + c_1 v_2^3 e^{i6\Phi_2} + c_2 v_3^2 e^{i6\Phi_3} + c_3 v_2 \varepsilon_4 e^{i(2\Phi_2+4\Phi_4^*)} \ldots$$

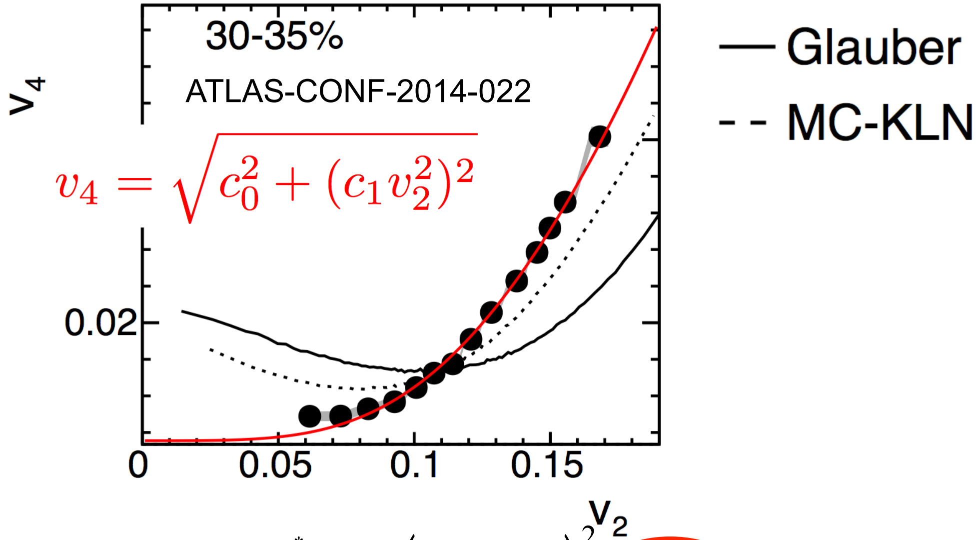
Ollitrault, Luzum, Teaney, Li, Heinz,Chun….
(14 correlators from ATLAS)

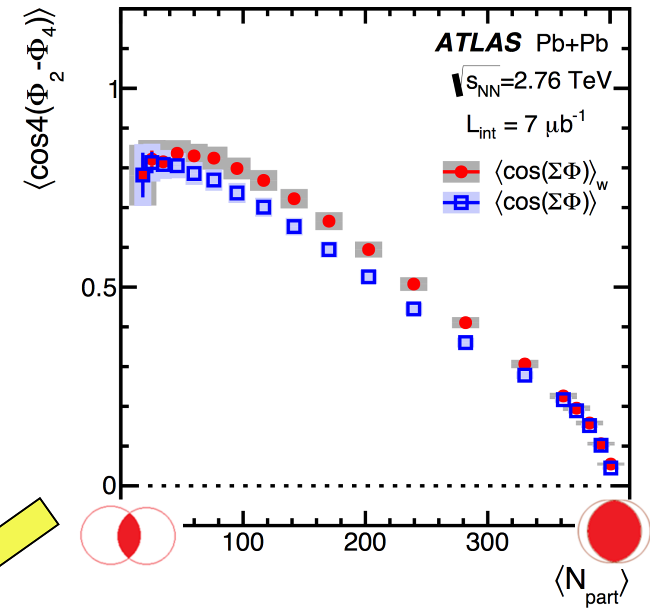
- Some correlators lack intuitive explanation
  e.g. <cos(2Φ_2-6Φ_3+4Φ_4> correlation
  - Although described by EbyE hydro and AMPT



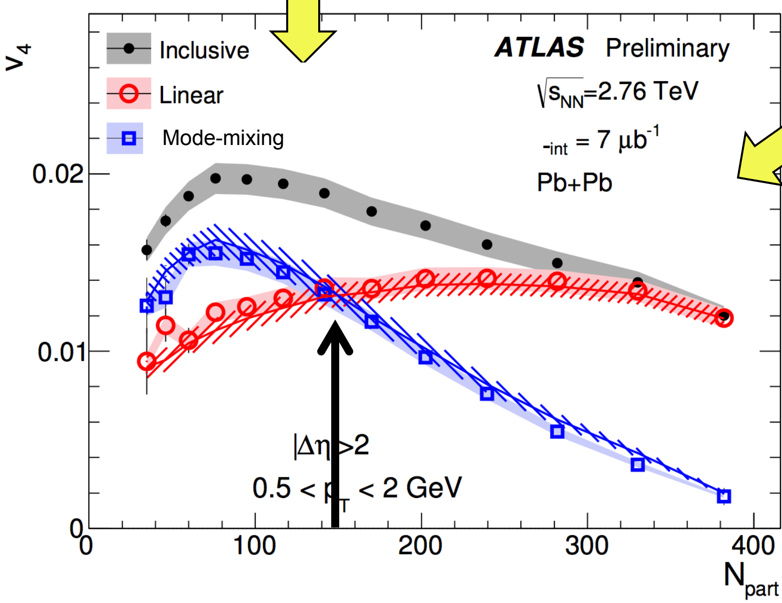
Naturally expect correlation between v_n and v_2

v_4-v_2 correlation from event-shape engineering





$$v_4 e^{i4\Phi_4} = c_0 e^{i4\Phi_4^*} + c_1 \left(v_2 e^{i2\Phi_2}\right)^2 + \ldots$$



$$v_4^{NL} = v_4 \langle\cos 4(\Phi_2 - \Phi_4)\rangle$$

$$v_4^{L} = \sqrt{v_4^2 - (v_4^{NL})^2}$$

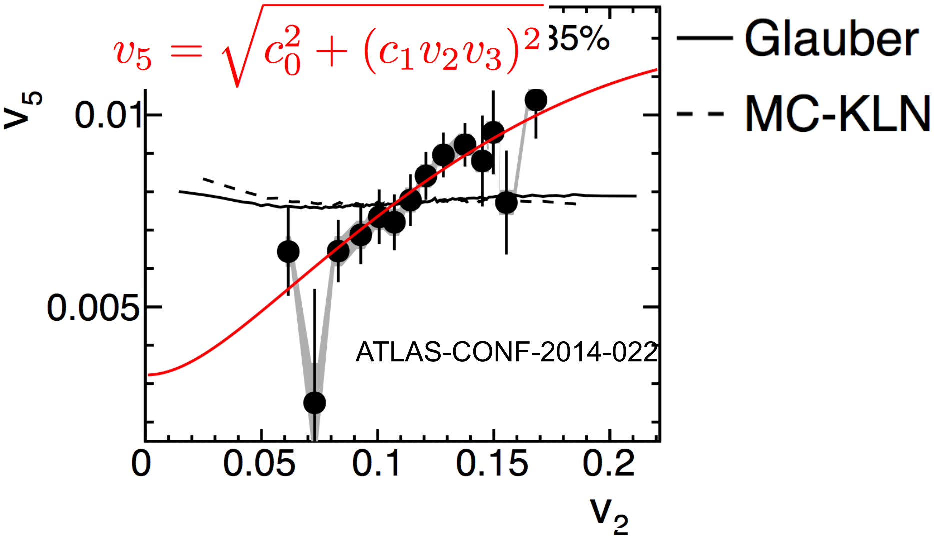
Compare well with expectation from EP correlations.

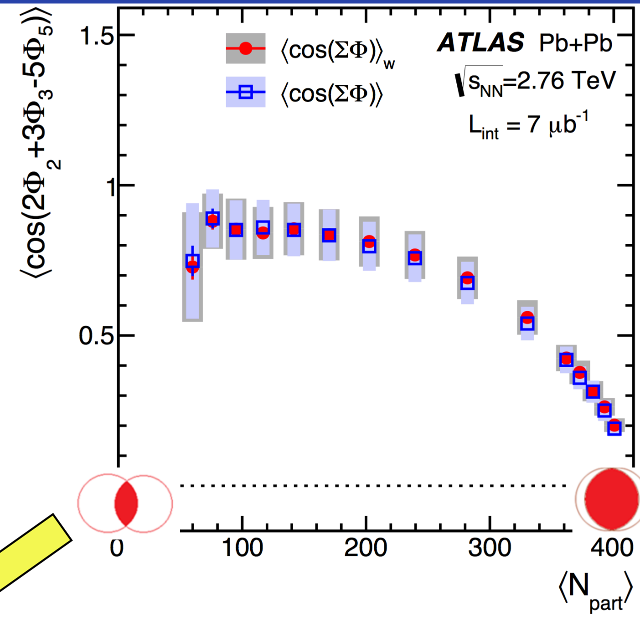
higher-order mode-mixing negligible

Separate geometry and mode-mixing components!!

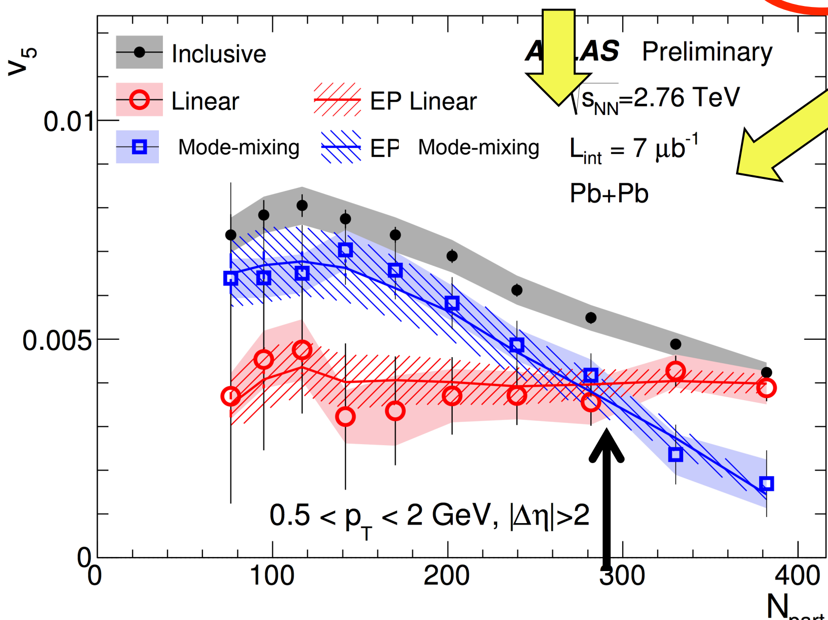
v_5-v_2 correlation from event-shape engineering

$$v_5 = \sqrt{c_0^2 + (c_1 v_2 v_3)^2}$$





$$v_5 e^{i5\Phi_5} \propto c_0 e^{i5\Phi_5^*} + c_1 v_2 v_3 e^{i(2\Phi_2+3\Phi_3)} + \ldots$$



$$v_5^{NL} = v_5 \langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle$$

$$v_5^{L} = \sqrt{v_5^2 - (v_5^{NL})^2}$$

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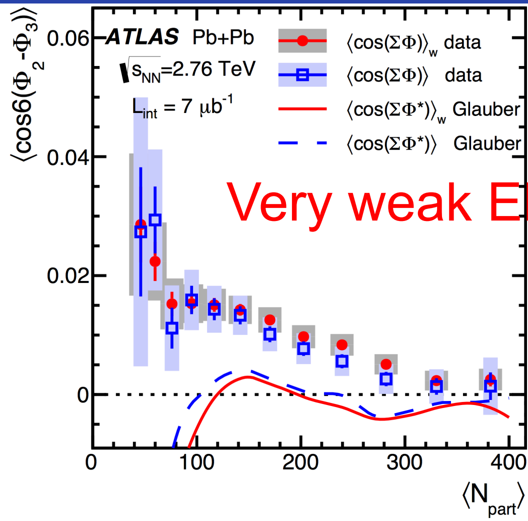
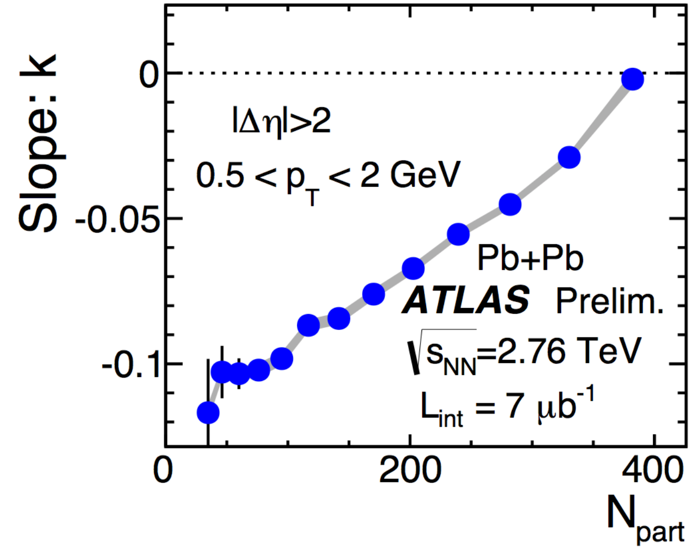
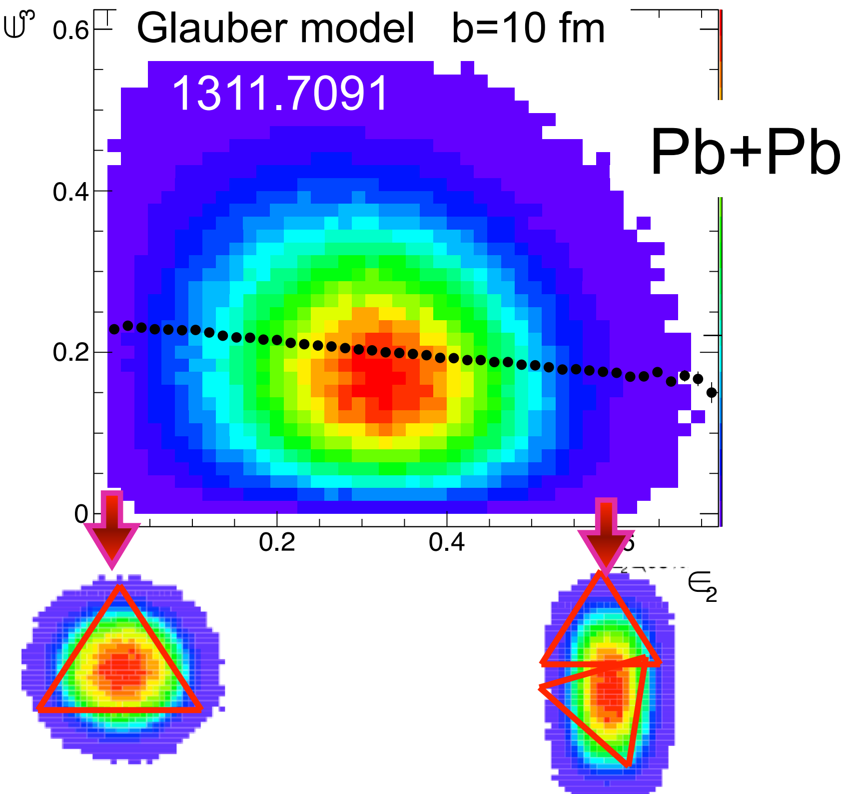
v_3-v_2 correlation from event-shape engineering

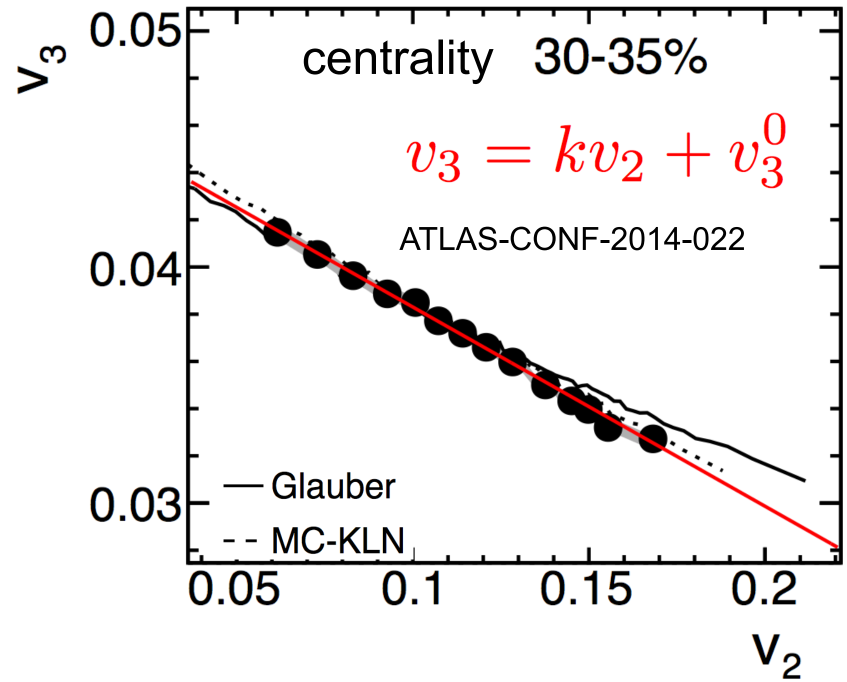


Very weak EP correlations





Direct constraint on initial geometry



Comparison of p+Pb with Pb+Pb

- Basar & Teaney's conformal scaling:

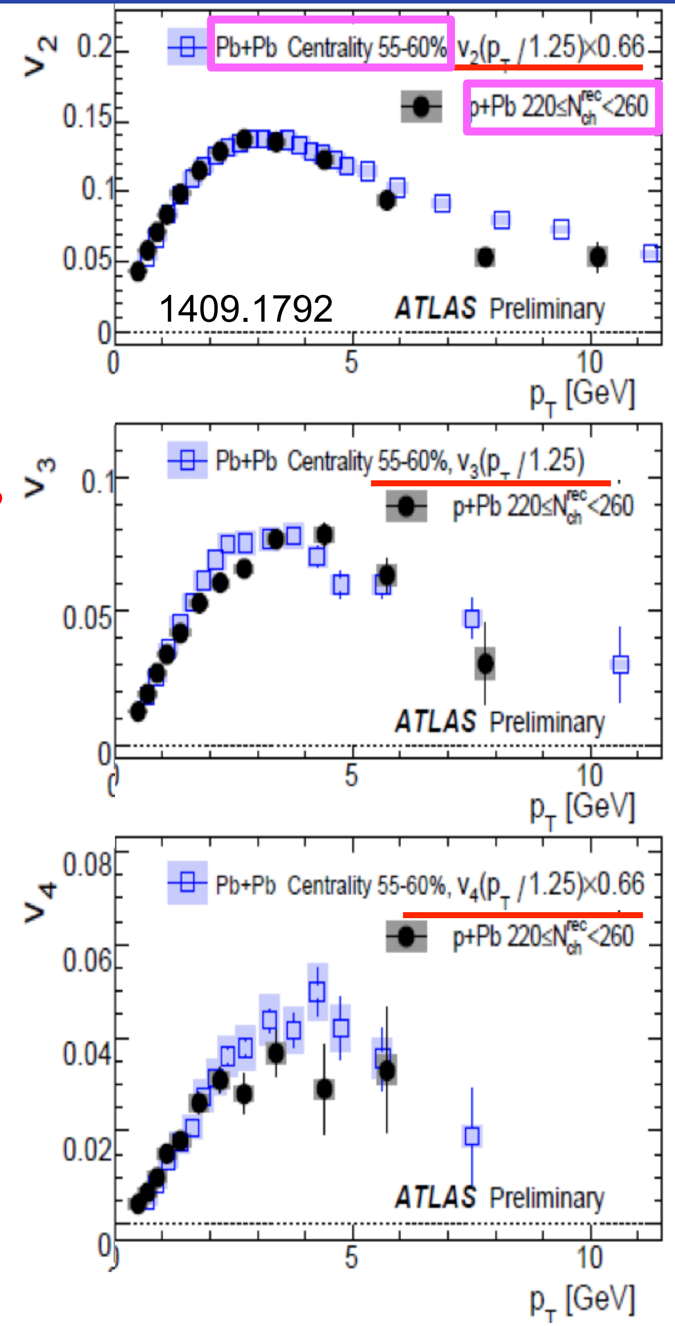
  $V_n(p_T)_{pA} \sim \boldsymbol{a} v_n(p_T/k)_{AA}$ at same multiplicity

  i.e. collective response controlled multiplicity

  - $\langle p_T \rangle_{pPb} = k \langle p_T \rangle_{PbPb}$, k=1.25
  - $a \approx 0.66$ for v_2 → average elliptic geometry $V_2 = a v_2$
  - $a \approx 1.0$ for v_3 → dominated by fluctuations. $V_3 \approx v_3$
  - $a \approx 0.66$ for v_4 → mode-mixing $\propto v_2^2$. but why also 0.66?

linear Mode-mixing linear Mode-mixing

Pb+Pb: $v_4^2 = (v_4^L)^2 + (v_4^{NL})^2$ p+Pb: $V_4^2 = (V_4^L)^2 + (V_4^{NL})^2$



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linear Mode-mixing linear Mode-mixing

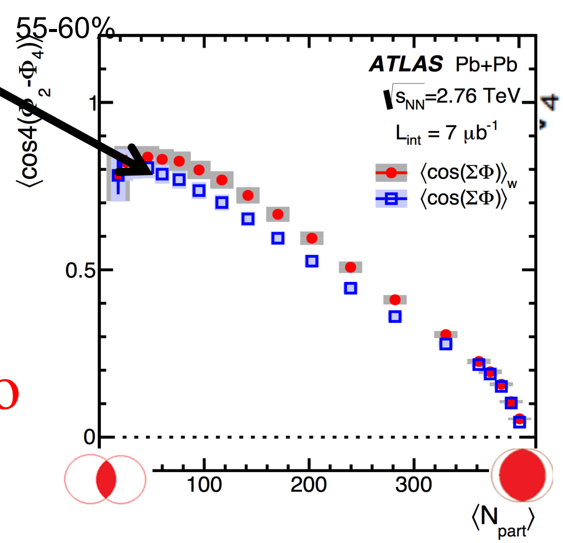
Pb+Pb: $v_4^2 = \left(v_4^L\right)^2 + \left(c v_2^2\right)^2$ $\quad$ p+Pb: $V_4^2 = \left(V_4^L\right)^2 + \left(c V_2^2\right)^2$

Since: $c v_2^2 = b v_4 \; b = 0.837, v_4^L = \sqrt{1-b^2}\, v_4 \quad V_4^L \approx v_4^L \quad V_2 = a v_2 \; a = 0.66$

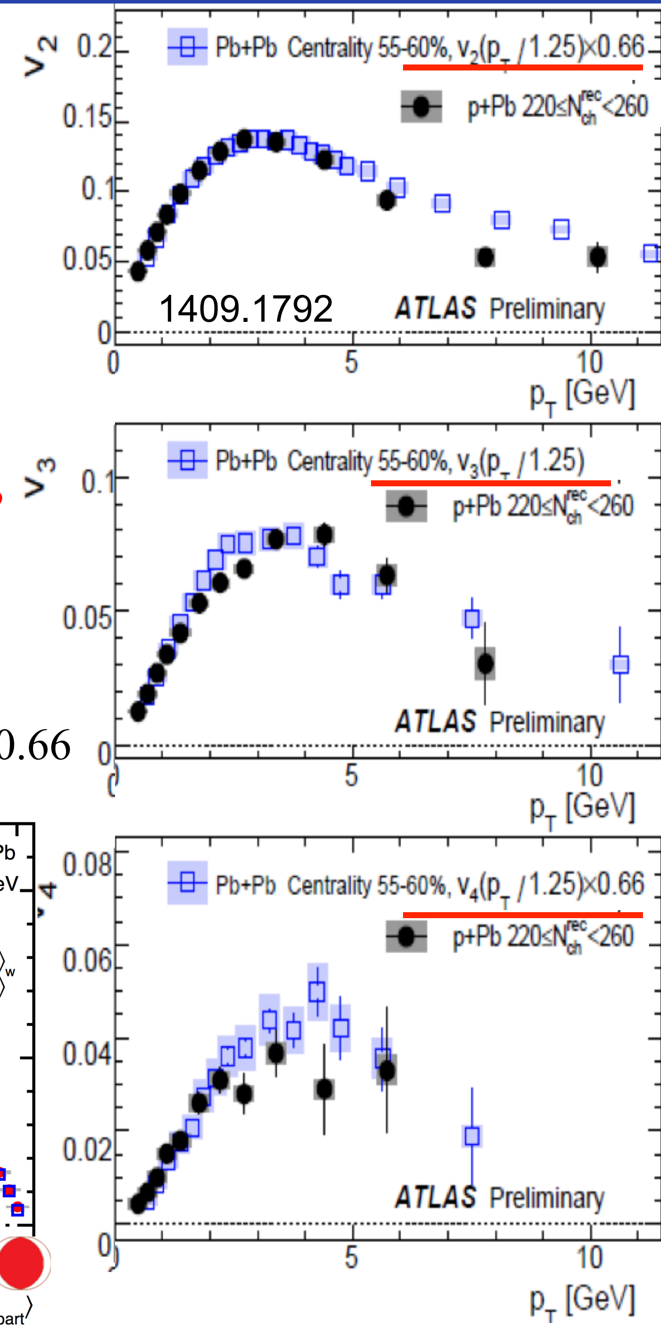
$$\Rightarrow \; c V_2^2 \approx a^2 c v_2^2 = a^2 b v_4$$

$$\Rightarrow \; V_4^2 = \left[1 - b^2 + a^4 \times b^2\right] v_4^2$$

$$\Rightarrow \; V_4 \approx 0.66 v_4$$



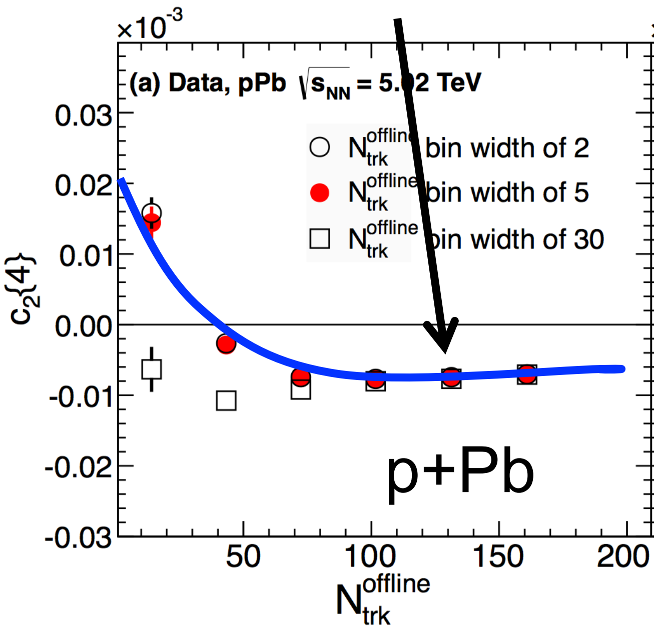
Why extrapolation of hydro prediction works so well?



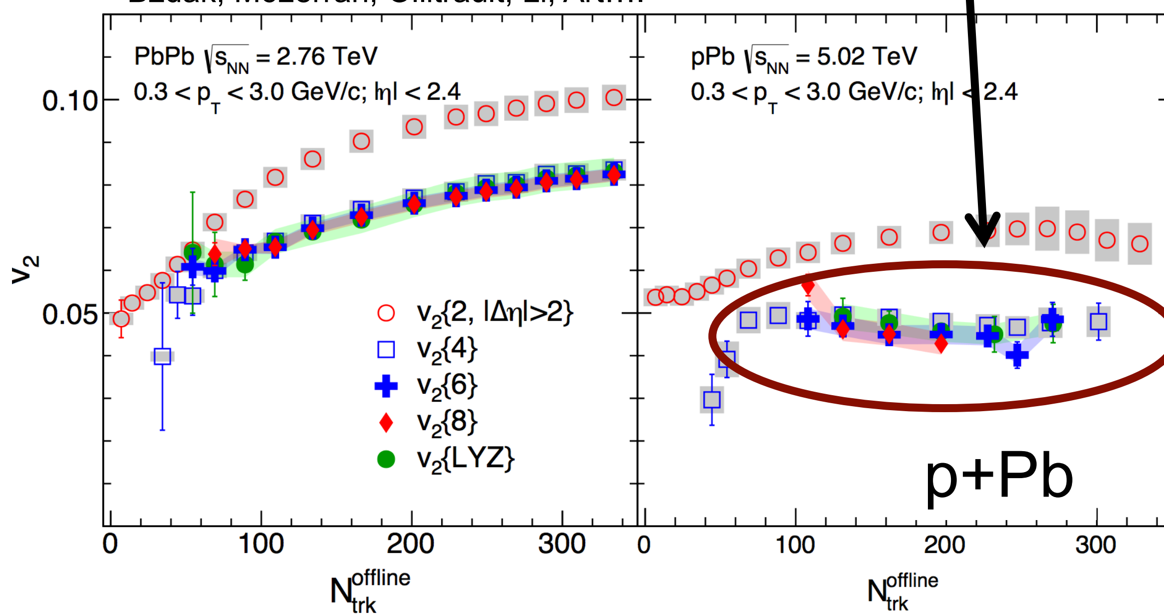
A detour: The meaning of cumulants?

negative $c_2\{4\}$ or real $v_2\{4\}$
Onset of collectivity?

What $v_2\{2k\} \approx$ same means?
~Gaussian?, ~power?



Bzdak, McLerran, Ollitrault, Li, Art….



$$a_n\{2\} \equiv c_n\{2\} = \langle v_n^2 \rangle$$

$$-a_n\{4\} \equiv c_n\{4\} = \langle v_n^4 \rangle - 2\langle v_n^2 \rangle^2$$

$$4a_n\{6\} \equiv c_n\{6\} = \langle v_n^6 \rangle - 9\langle v_n^4 \rangle\langle v_n^2 \rangle + 12\langle v_n^2 \rangle^3$$

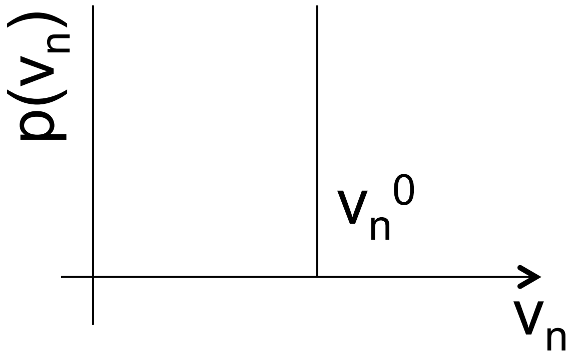
$$-33a_n\{8\} \equiv c_n\{8\} = \langle v_n^8 \rangle - 16\langle v_n^6 \rangle\langle v_n^2 \rangle - 18\langle v_n^4 \rangle^2 + 144\langle v_n^4 \rangle\langle v_n^2 \rangle^2 - 144\langle v_n^2 \rangle^4$$

$$v_n\{2k\} \equiv \mathrm{sgn}(a_n\{2k\}) \sqrt[2k]{|a_n\{2k\}|}$$

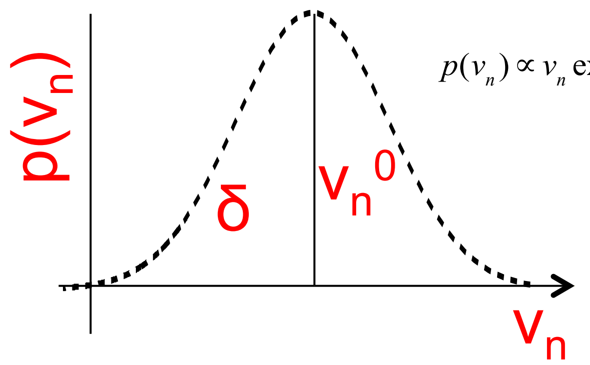
Cumulants $\overset{?}{=}$ $p(v_n)$

Cumulants for azimuthal correlations

No fluctuation



Bessel-Gaussian fluctuation



$$p(v_n) \propto v_n \exp\left(\frac{-(v_n^2 + (v_n^0)^2)}{2\delta_n^2} \right) I_0\left(\frac{v_n v_n^0}{\delta_n^2}\right)$$

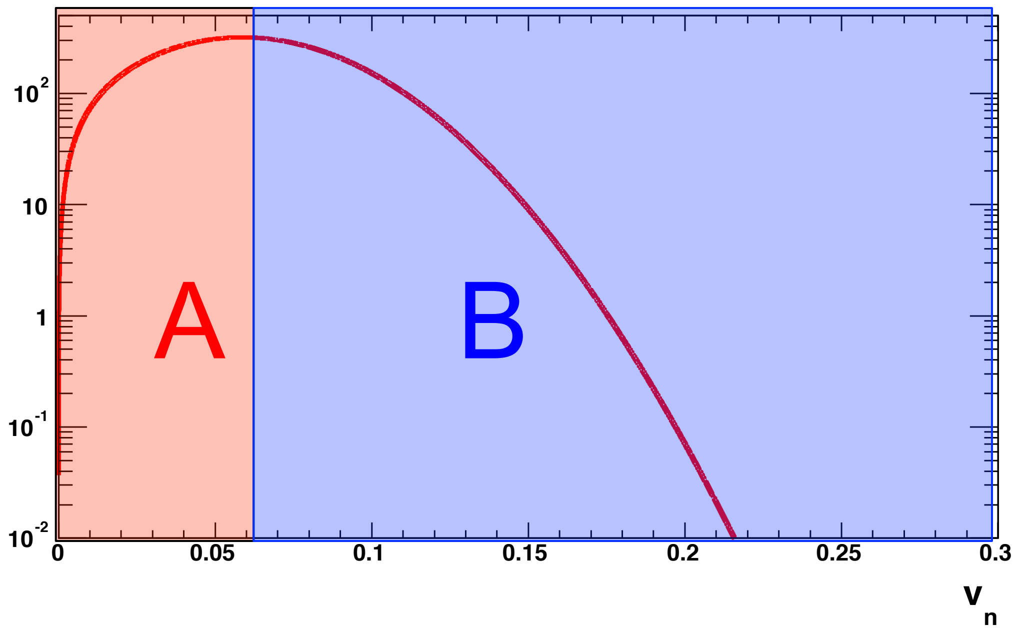
Same answer!:

$$c_n\{2\} = \left(v_n^0\right)^2 + 2\delta^2 \quad c_n\{4\} = -\left(v_n^0\right)^4$$

$$c_n\{6\} = 4\left(v_n^0\right)^6 \quad c_n\{8\} = -33\left(v_n^0\right)^8$$

$$v_n\{4\} = v_n\{6\} = v_n\{8\} = .. = v_n^0$$

Can cumulants capture all details of $p(v_n)$?



$$p(v_n) \propto v_n \exp\left(\frac{-(v_n^2 + (v_n^0)^2}{2\delta_n^2}\right) I_0(\frac{v_n v_n^0}{\delta_n^2})$$

$v_n^0 = \delta\sqrt{2}$ as example

- Divide B-G distri. to 2 equal parts, and calculate cumulants separately.

	$v_n\{2\}$	$v_n\{4\}$	$v_n\{6\}$	$v_n\{8\}$
all	1.414	1	1	1
A	0.851	0.759	0.746	0.744
B	1.809	1.690	1.701	1.701

In units of v_n^0

- v{2} and v{4}→width and mean of p(v), higher-order diff. very small

Higher-order cumulants not sensitive to $p(v_n)$ beyond mean and width?

Cumulants for narrow distribution

- Simple central moment expansion

$$\theta_j = \int \left(\frac{v_n - \langle v_n\rangle}{\langle v_n\rangle}\right)^j p(v_n)dv_n$$

$$\langle v_n^{2k}\rangle = \langle v_n\rangle^{2k} \int \left(1 + \frac{v_n - \langle v_n\rangle}{\langle v_n\rangle}\right)^{2k} p(v_n)dv_n = \langle v_n\rangle^{2k}\left(1 + \sum_{j=2}^{2k} C_{2k}^j \theta_j\right)$$

- Keep up to θ_4:

$$\begin{aligned}
v_n\{2\}/\langle v_n\rangle &\approx 1 + \frac{1}{2}\theta_2 - \frac{3}{8}\theta_2^2 \\
v_n\{4\}/\langle v_n\rangle &\approx 1 - \frac{1}{2}\theta_2 - \theta_3 - \frac{1}{4}\theta_4 + \frac{1}{8}\theta_2^2 \\
v_n\{6\}/\langle v_n\rangle &\approx 1 - \frac{1}{2}\theta_2 - \frac{2}{3}\theta_3 + \frac{1}{4}\theta_4 - \frac{11}{8}\theta_2^2 \\
v_n\{8\}/\langle v_n\rangle &\approx 1 - \frac{1}{2}\theta_2 - \frac{7}{11}\theta_3 + \frac{31}{132}\theta_4 - \frac{117}{88}\theta_2^2
\end{aligned}$$

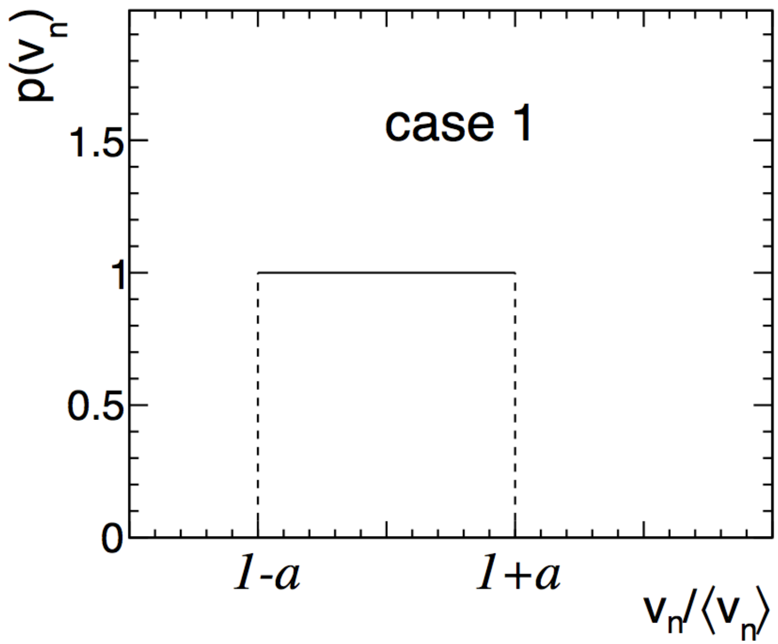
- Some useful relations (again only true for narrow distribution)

$$\frac{\sigma_n}{\langle v_n\rangle} \equiv \sqrt{\theta_2} \approx \sqrt{\frac{v_n^2\{2\} - v_n^2\{4\}}{v_n^2\{2\} + v_n^2\{4\}}}$$

Voloshin etc

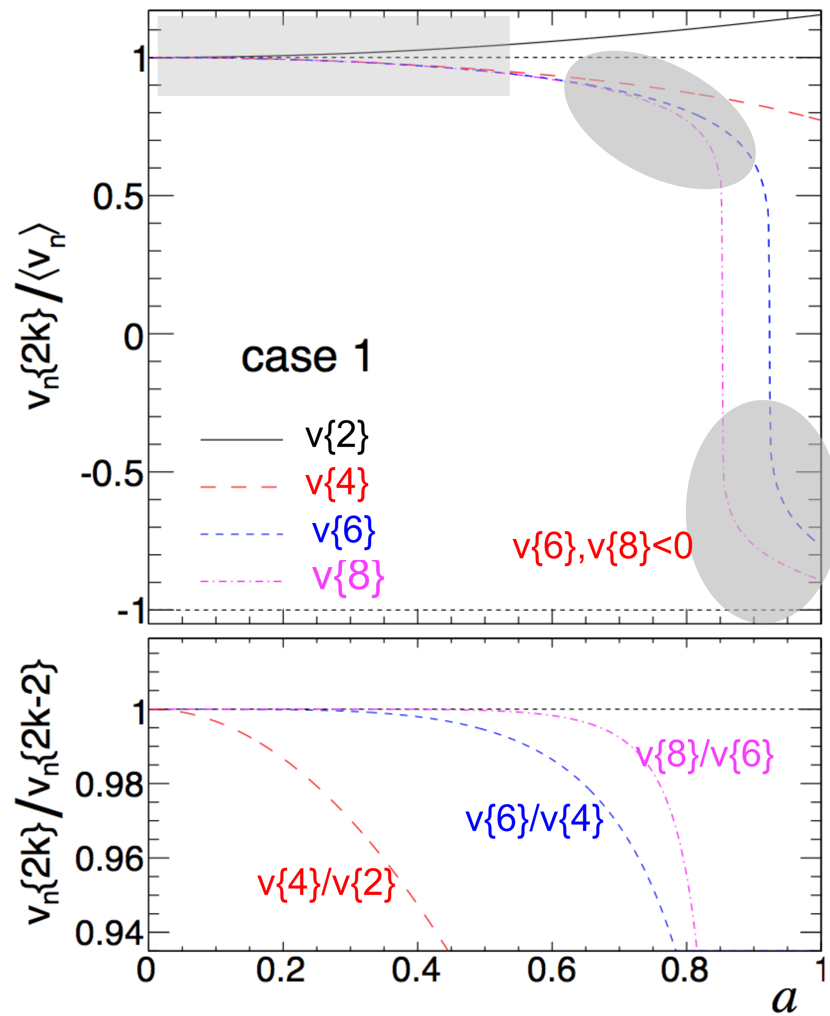
$$\begin{aligned}
v_n\{6\} - v_n\{4\} &\approx \langle v_n\rangle\left(\frac{1}{3}\theta_3 + \frac{\theta_4 - 3\theta_2^2}{2}\right) \\
v_n\{8\} - v_n\{6\} &\approx \langle v_n\rangle\left(\frac{1}{33}\theta_3 - \frac{\theta_4 - 3\theta_2^2}{66}\right)
\end{aligned}$$

Broad distribution -I



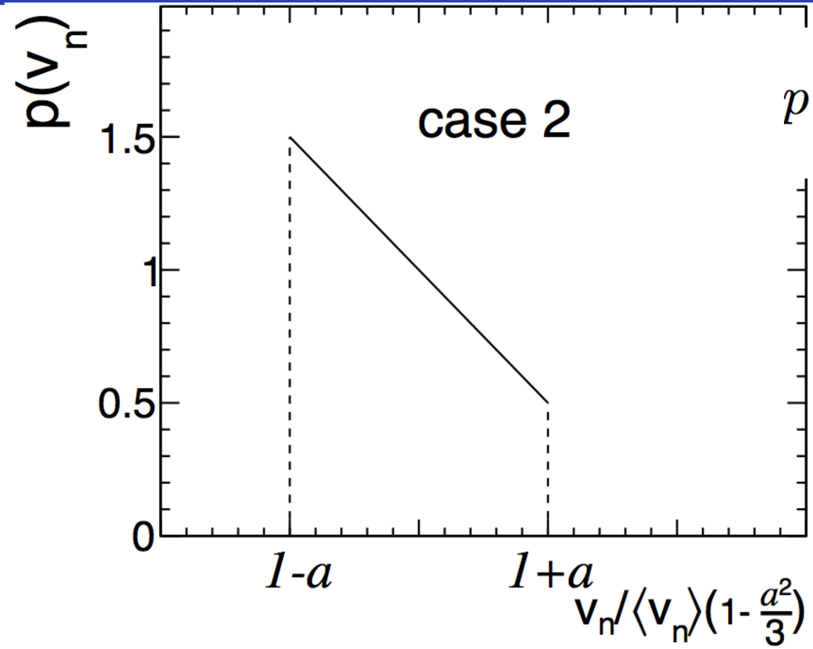
$$p(x;a) = \begin{cases} 1 & |x-1| \leq a \\ 0 & |x-1| > a \end{cases}, \qquad x \equiv \frac{v_n}{\langle v_n \rangle}$$

$$v_n\{2k\} \equiv \mathrm{sgn}(a_n\{2k\}) \sqrt[2k]{|a_n\{2k\}|}$$



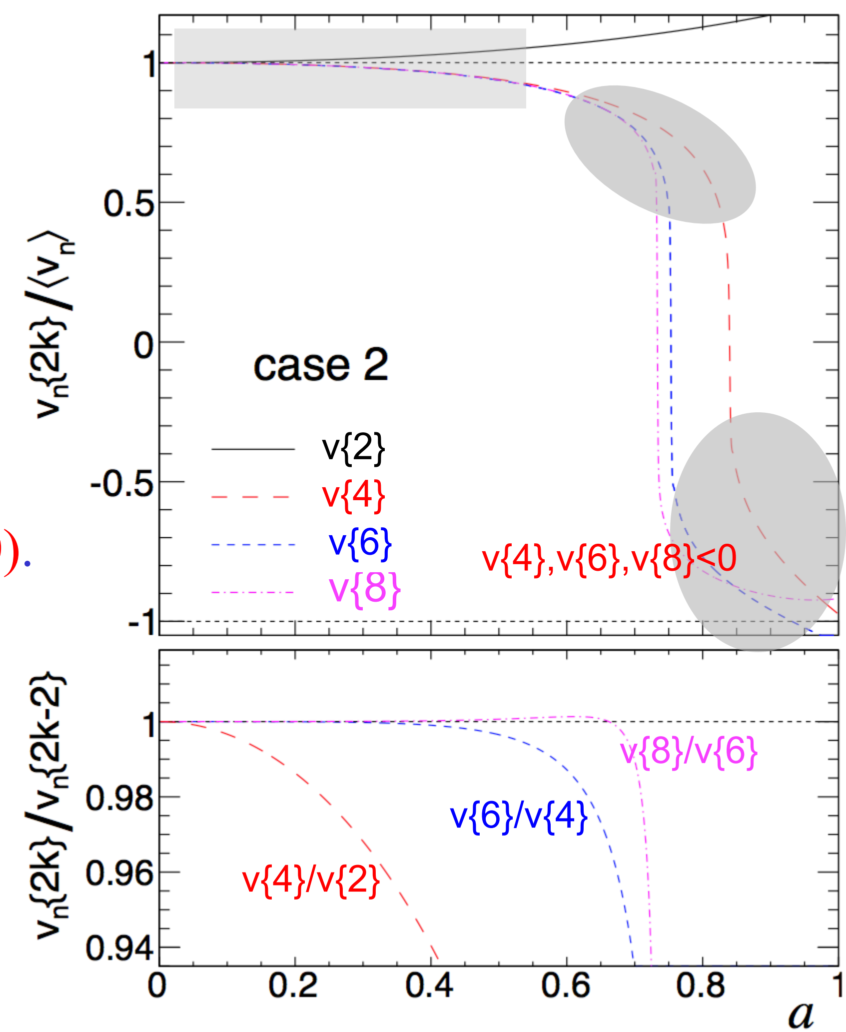
- Width controlled by *a*.
- v{2k}, k>1 close to each other for *a*<0.5. then deviates and flip sign (v{6},v{8}<0).

Broad distribution -II



$$p(x;a) = \begin{cases} 2-x & |x-1| \le a \\ 0 & |x-1| > a \end{cases}, x \equiv \frac{v_n}{\langle v_n \rangle}\left(1-a^2/3\right)$$

$$v_n\{2k\} \equiv \mathrm{sgn}(a_n\{2k\}) \sqrt[2k]{|a_n\{2k\}|}$$



- Width controlled by *a*.
- v{2k}, k>1 close to each other for *a*<0.5. then deviates and flip sign (v{4},v{6},v{8}<0).
- Unless have a-priori knowledge of its shape, i.e. close to BG, cumulants
  - Difficult to constrain p(v) beyond mean and width
  - In principle can't use $c_n\{4\}<0$ as sign of onset of collectivity

Summary

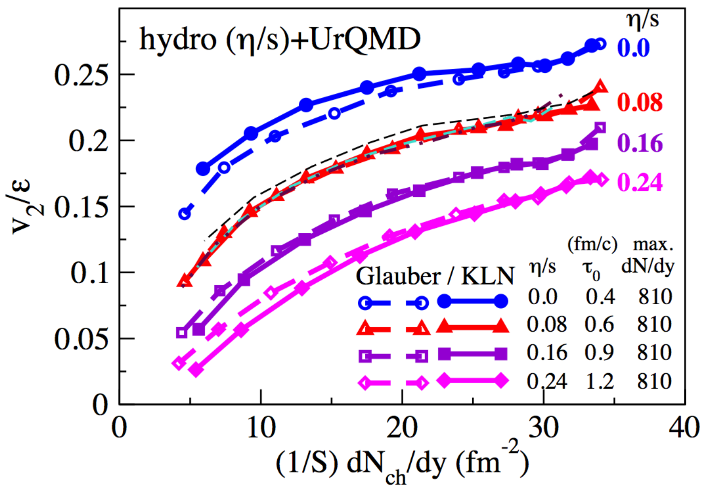
- Future measurements should provide direct physics insights
  - Obtain a complete picture at top √s, then apply to other systems and energies.
- Explore new observables sensitive to event-shape fluctuations:

$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{\mathrm{evts}}} \frac{dN_{\mathrm{evts}}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$

  - Augmented by the event-shape engineering technique
- Applications:
  - $p(v_2)$ is sensitive to non-linear response to $p(\varepsilon_2)$.
  - Event plane correlations and v_n-v_2 correlations allow quantitative separation of initial geometry and mode-mixing components
    - v_2 and v_3 are mainly controlled by ε_2 and ε_3
    - v_4, v_5, and v_6 contains both geometry and mode-mixing components, can be separated by data-driven methods.
  - Novel anti-correlation between v_3 and v_2 reflects the ε_3-ε_2 anti-correlation
  - Taking mode-mixing into account, conformal scaling explains the similarity of v_4 in pA and AA (in addition to v_2 and v_3)
- Cumulants not very sensitive to the $p(v_n)$ beyond its mean and width.

Backup

Eccentricity scaling: v_n/ε_n



Teaney and Yan

	Viscous correlation
Linear:	$-\frac{\Delta w_n}{w_n^{id}} \sim n^2 \frac{\eta}{s}$

Viscous damping expected
to be larger for larger n