



# Beam energy dependence of the viscous damping of anisotropic flow in relativistic heavy ion collisions

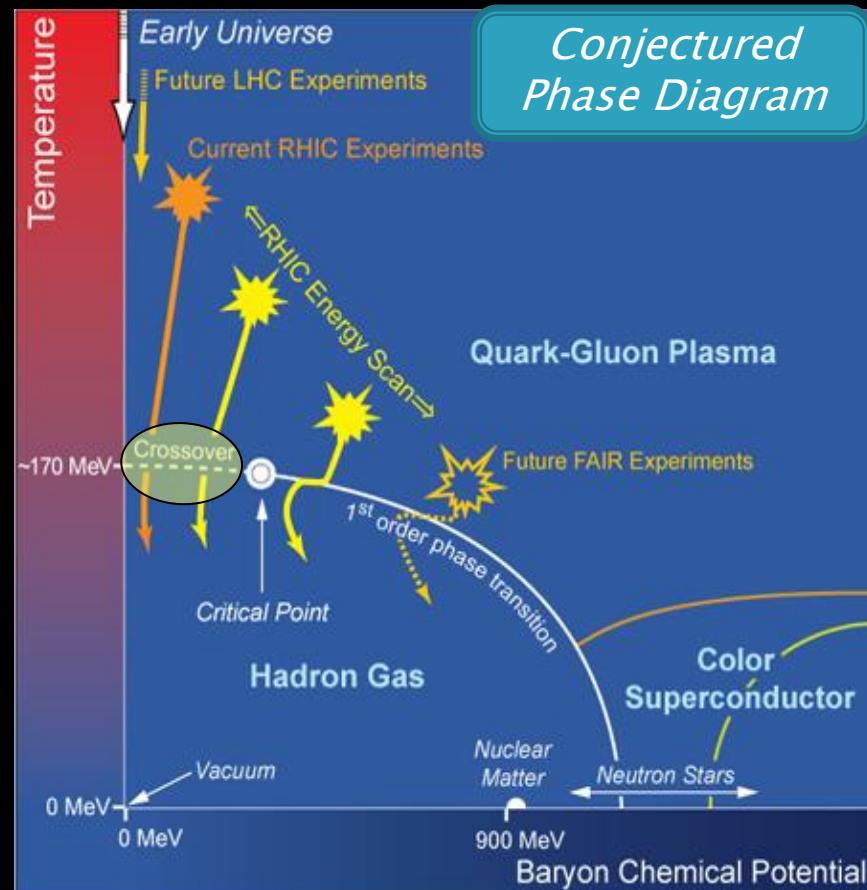
*Roy A. Lacey  
Stony Brook University*

## Outline:

- I. **Motivation.**
  - *Interest and observables*
- II. **Available data and scaling**
  - *Demonstration of scaling in visc. hydro*
- III. **Results**
- IV. **Conclusion**

**We now have strong indications for a change in the dynamics with  $\sqrt{s_{NN}}$**

# Quantitative study of the QCD phase diagram is a central current focus of our field



## A Known known

- *Spectacular achievement:*  
*Validation of the crossover transition leading to the QGP*  
→ *Necessary requirement for CEP*

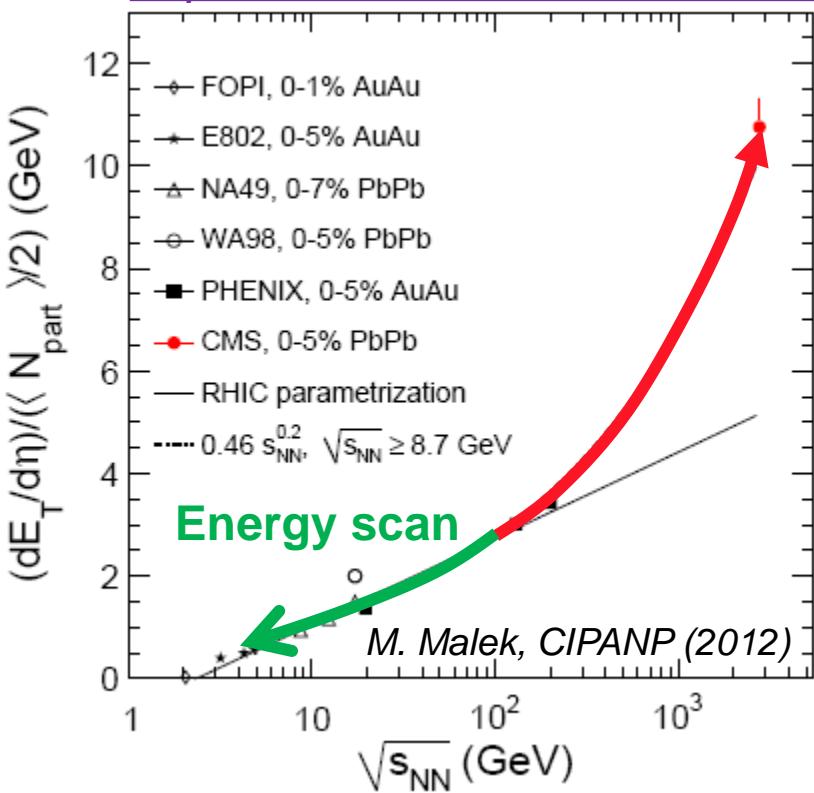
## Known unknowns

- **Location of the critical End point (CEP)?**
  - **Location of phase coexistence regions?**
  - **Detailed properties of each phase?**
- All are fundamental to the phase diagram of any substance**

*Measurements which span a broad range of the  $(T, \mu_B)$ -plane are essential for detailed studies of the phase diagram*

# A Current Strategy

Exploit the RHIC-LHC beam energy lever arm

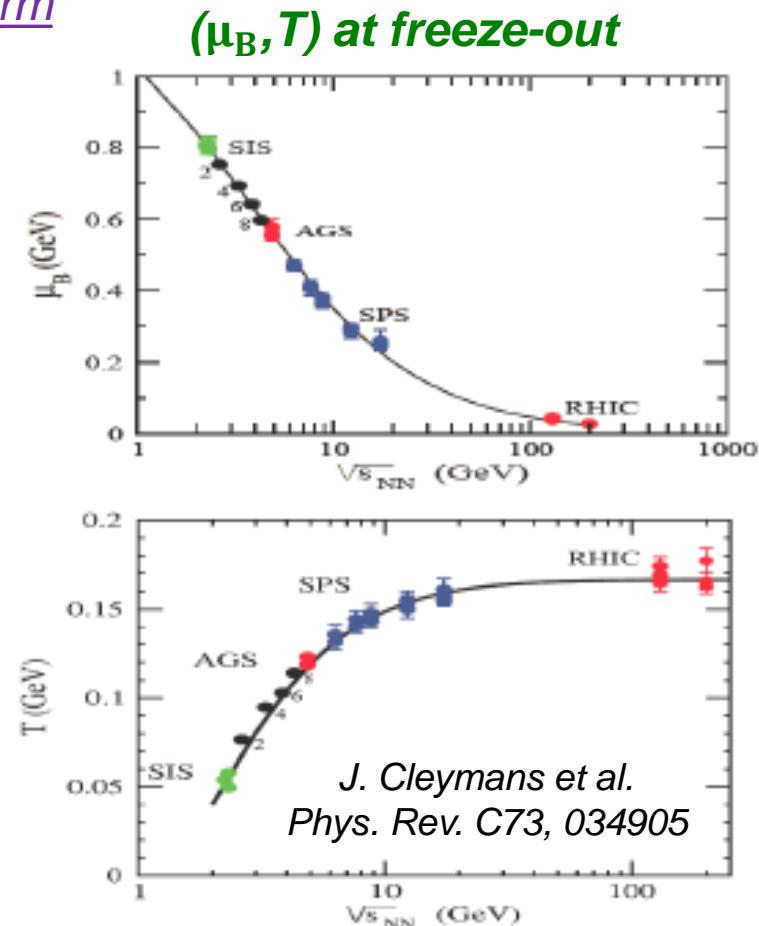


➤ LHC → access to high  $T$  and small  $\mu_B$

➤ RHIC → access to different systems and a broad domain of the  $(\mu_B, T)$ -plane

$RHIC_{BES}$  to LHC →  $\sim 360 \sqrt{s_{NN}}$  increase

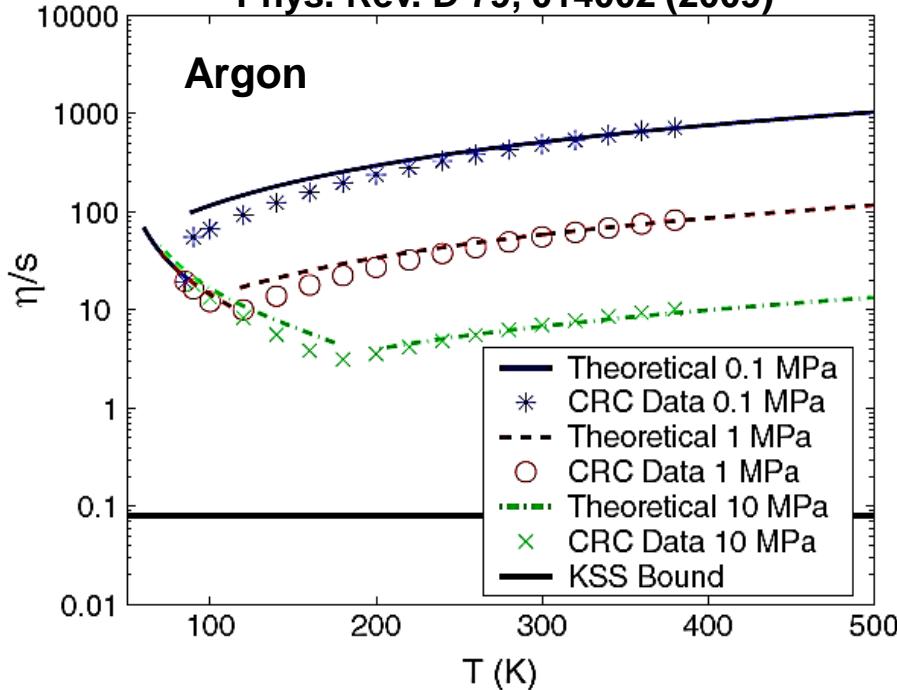
Challenge → identification of robust signals



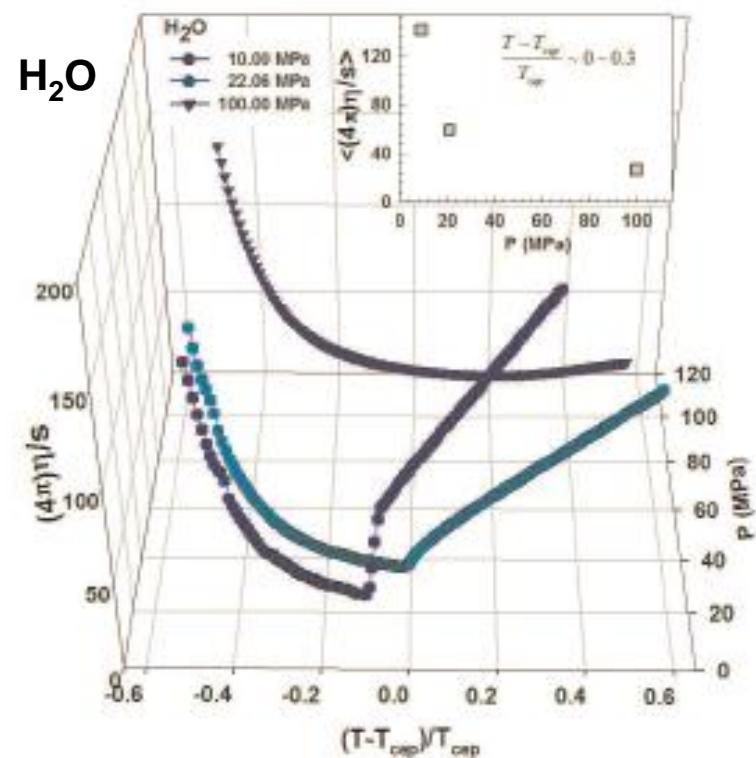
➤ LHC + BES → access to an even broader domain of the  $(\mu_B, T)$ -plane

# Possible signals

Csernai et. al,  
Phys.Rev.Lett. 97 (2006) 152303  
A. Dobado et. al,  
Phys. Rev. D 79, 014002 (2009)



Lacey et. al,  
Phys. Rev.Lett. 98 (2007) 092301  
[arXiv:0708.3512](https://arxiv.org/abs/0708.3512) (2008)



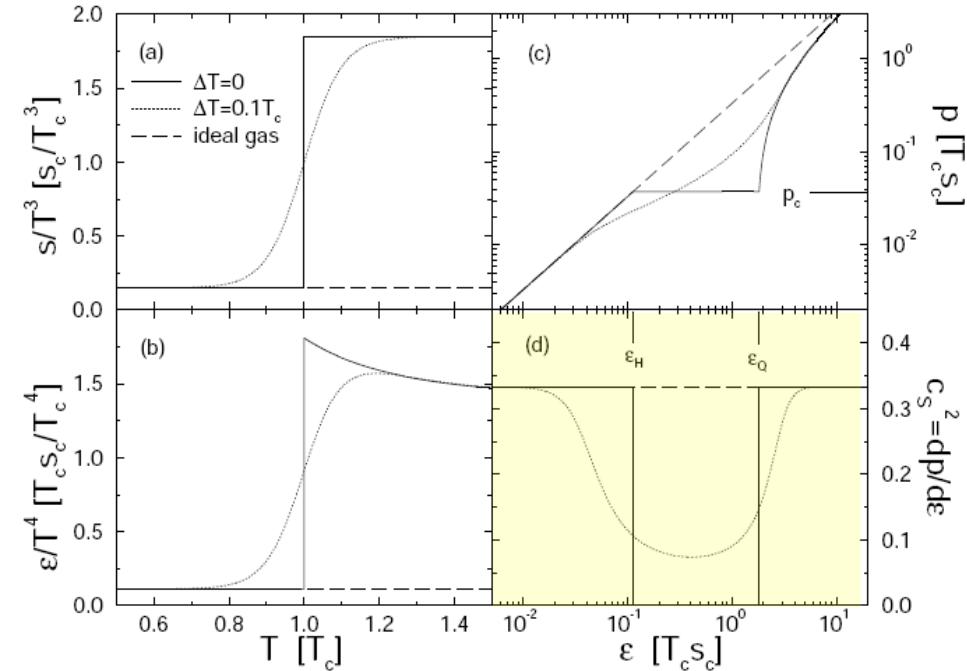
*At the CEP or close to it, anomalies in the dynamic properties of the medium can drive abrupt changes in transport coefficients*

*Anisotropic flow ( $v_n$ ) measurements are an invaluable probe*

# Possible signals

## Collapse of directed flow H. Stoecker, NPA 750, 121 (2005)

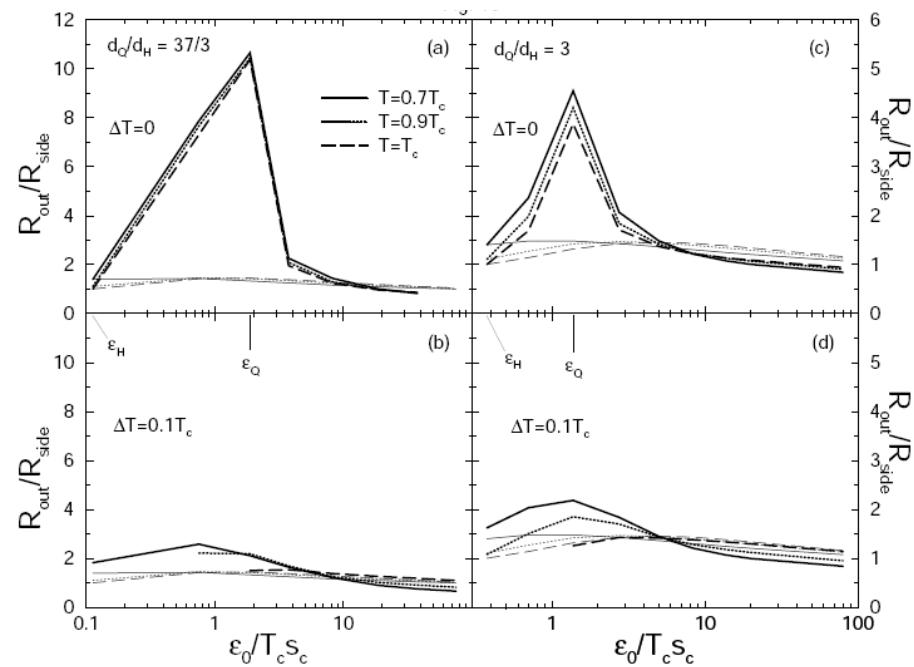
Dirk Rischke and Miklos Gyulassy  
Nucl.Phys.A608:479-512,1996



*In the vicinity of a phase transition or the CEP, the sound speed is expected to soften considerably.*

**v<sub>1</sub> and HBT measurements are invaluable probes**

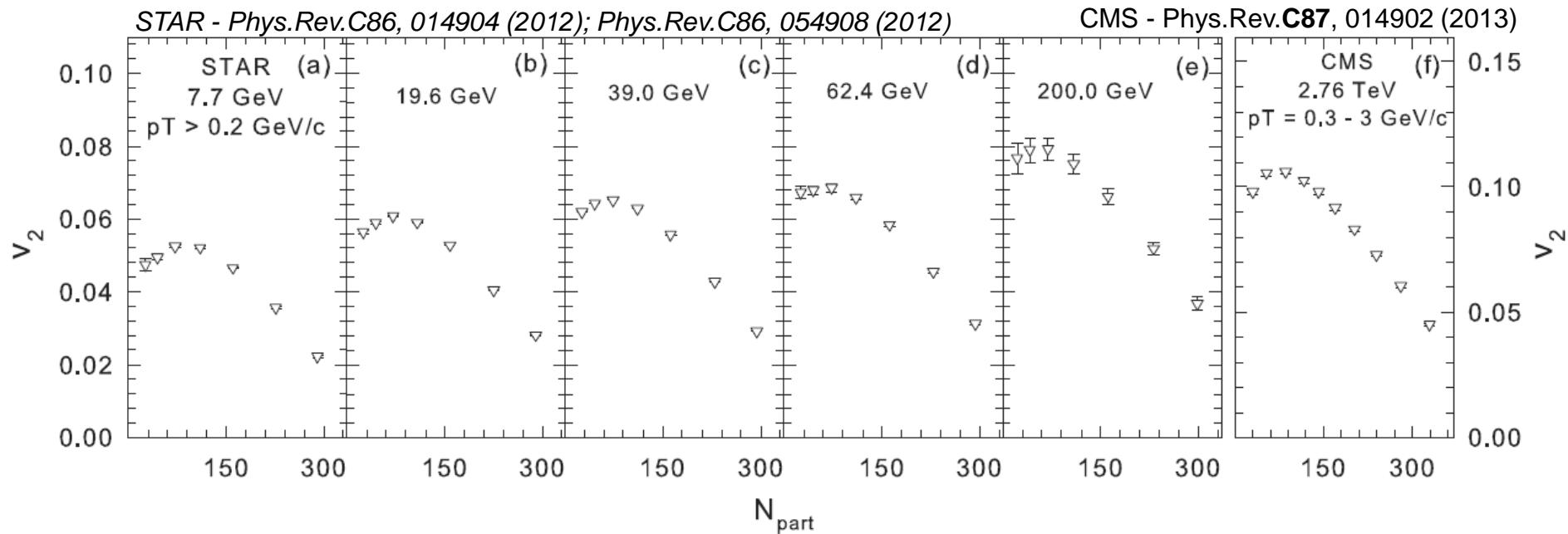
Dirk Rischke and Miklos Gyulassy  
Nucl.Phys.A608:479-512,1996



*In the vicinity of a phase transition or the CEP anomalies in the space-time dynamics can enhance the time-like component of emissions.*

# Anisotropic flow Measurements

Lacey et. al, Phys.Rev.Lett. 112 (2014) 082302



- An extensive set of flow measurements now span a broad range of beam energies ( $T, \mu_B$ ).

# HBT Measurements

For detailed presentations of PHENIX data

See talks by:

**Ron Soltz**

Tue.

15:00 - 15:20

**Helium**

Darmstadtium

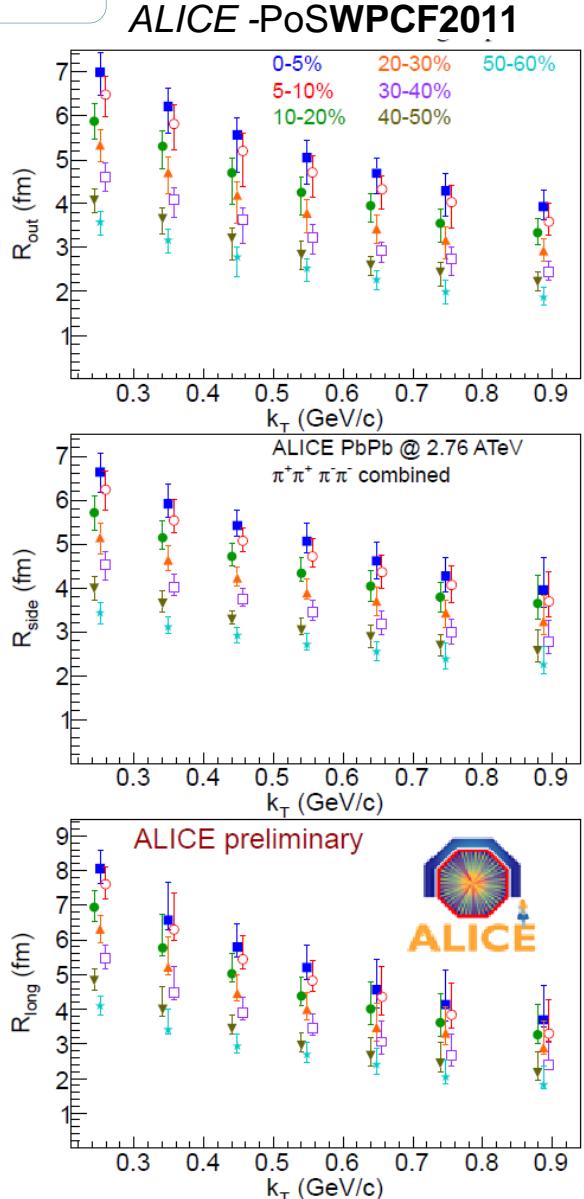
**Nuggehalli Ajitanand**

Wed.

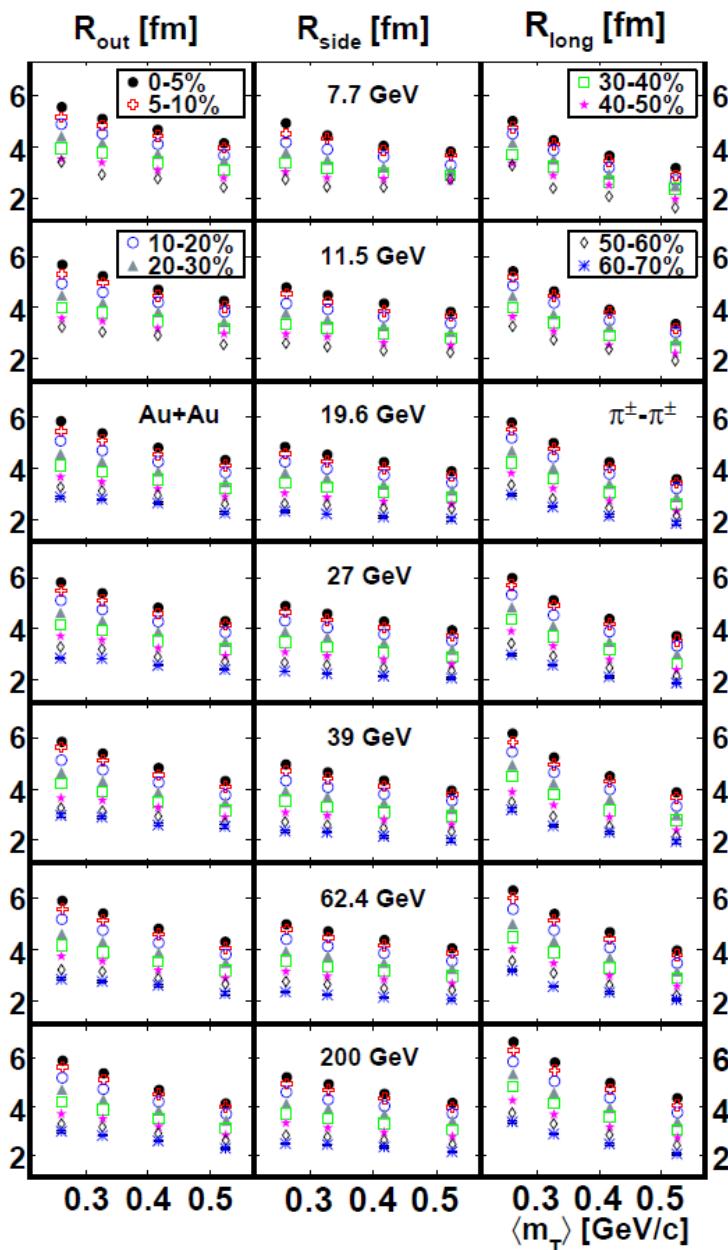
12:30 - 12:50

**Europium**

Darmstadtium



STAR - 1403.4972



Exquisite data set for combined RHIC-LHC results?

## Essential Questions

I. Can the wealth of data be understood in a consistent framework?

**YES!**

II. If it can, what new insight/s are we afforded?

- Do we see evidence for the CEP?

I. Expansion dynamics is pressure driven and is therefore acoustic!

- This acoustic property leads to several testable scaling predictions for anisotropic flow and HBT

$$\ln\left(\frac{v_n}{\epsilon_n}\right) \propto -\frac{\beta''}{R}$$

Staig & Shuryak  
arXiv:1008.3139  
[Lacey et. al.](#),  
arXiv:1301.0165

$$t \propto \bar{R}$$
$$R_{out}, R_{side}, R_{long} \propto \bar{R}$$

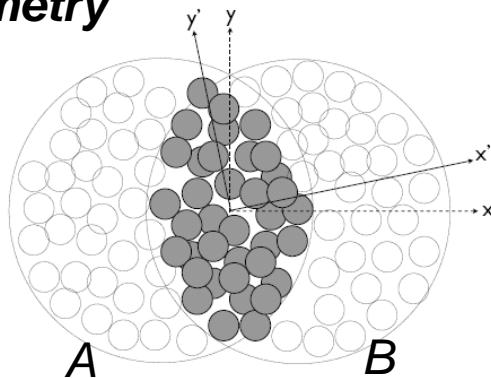
$^1/\bar{R}$  scaling for anisotropic flow

$\bar{R}$  scaling for the HBT radii

**These scaling properties are evidenced by viscous hydrodynamics**

# Geometric quantities for scaling

## Geometry



$$\Psi_n^* = \frac{1}{n} \tan^{-1} \left( \frac{S_{ny}}{S_{nx}} \right)$$

$$\varepsilon_n = \langle \cos n(\phi - \psi_n^*) \rangle$$

$$\frac{1}{\bar{R}} = \sqrt{\left( \frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2} \right)}$$

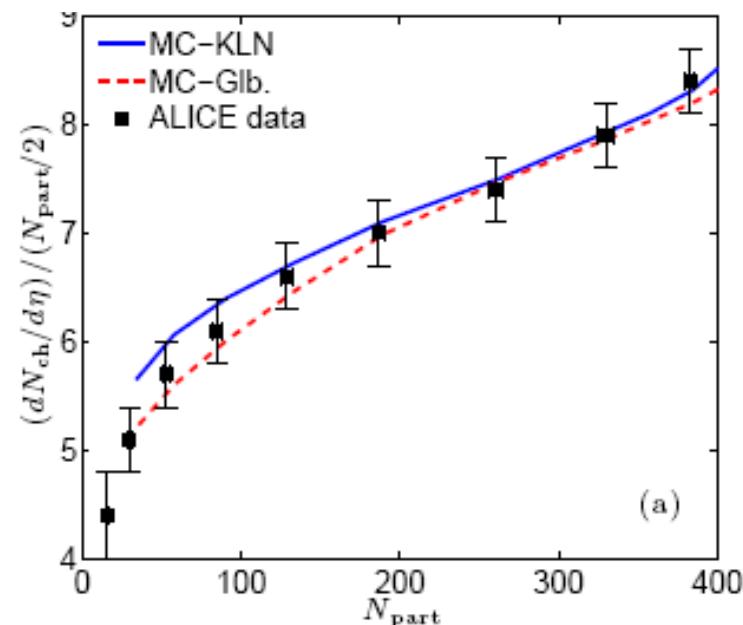
arXiv:1203.3605

$\sigma_x$  &  $\sigma_y \rightarrow$  RMS widths of density distribution

Phys. Rev. C 81, 061901(R) (2010)

$$S_{nx} \equiv S_n \cos(n\Psi_n^*) = \int d\mathbf{r}_\perp \rho_s(\mathbf{r}_\perp) \omega(\mathbf{r}_\perp) \cos(n\phi)$$

$$S_{ny} \equiv S_n \sin(n\Psi_n^*) = \int d\mathbf{r}_\perp \rho_s(\mathbf{r}_\perp) \omega(\mathbf{r}_\perp) \sin(n\phi),$$



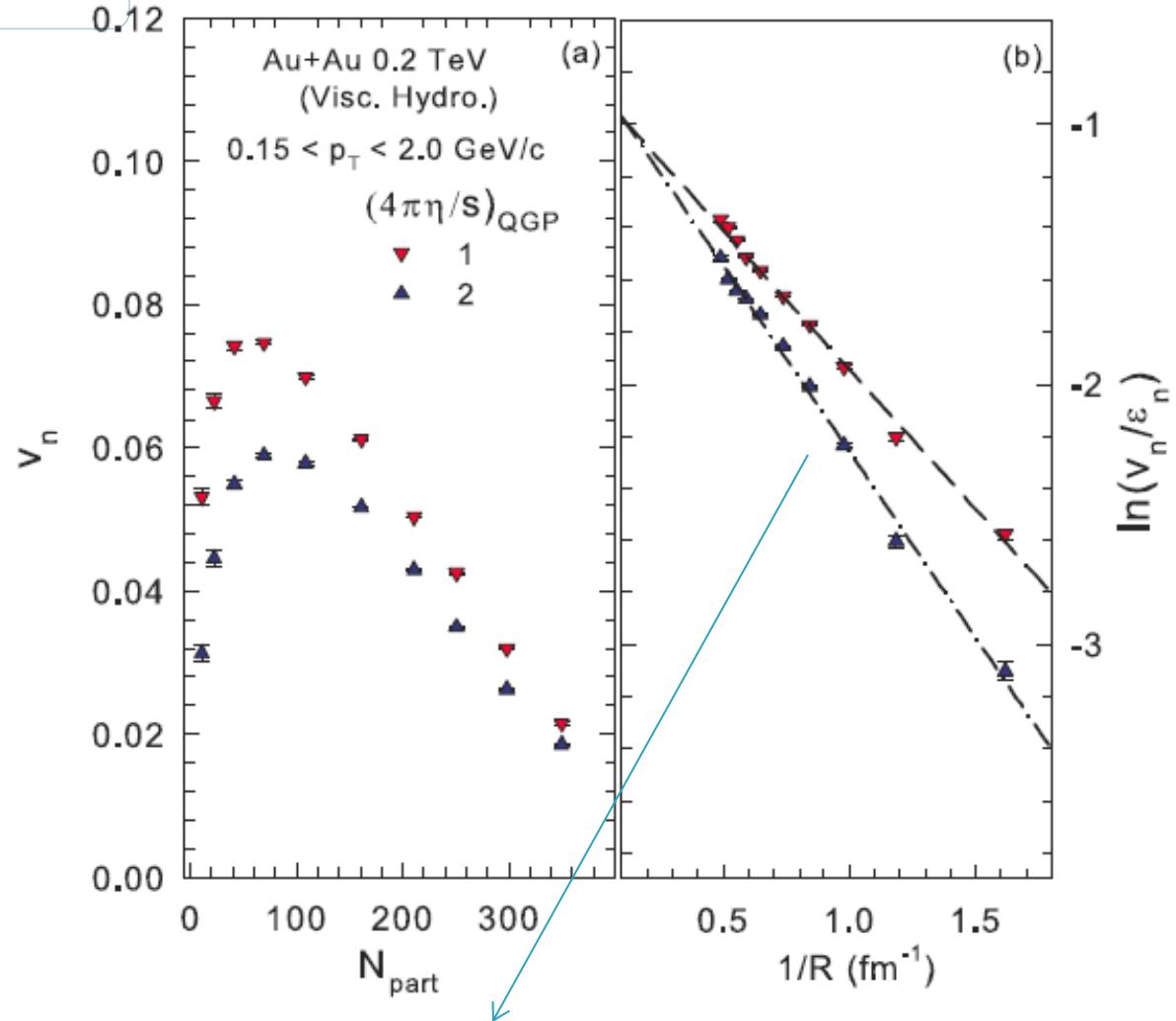
- Geometric fluctuations included
- Geometric quantities constrained by multiplicity density.

# $1/\bar{R}$ scaling of anisotropic flow

## - Viscous Hydrodynamics

$$\ln\left(\frac{v_n}{\varepsilon_n}\right) \propto \frac{-\beta''}{\bar{R}}$$

✓ Characteristic acoustic scaling validated for viscous hydrodynamics



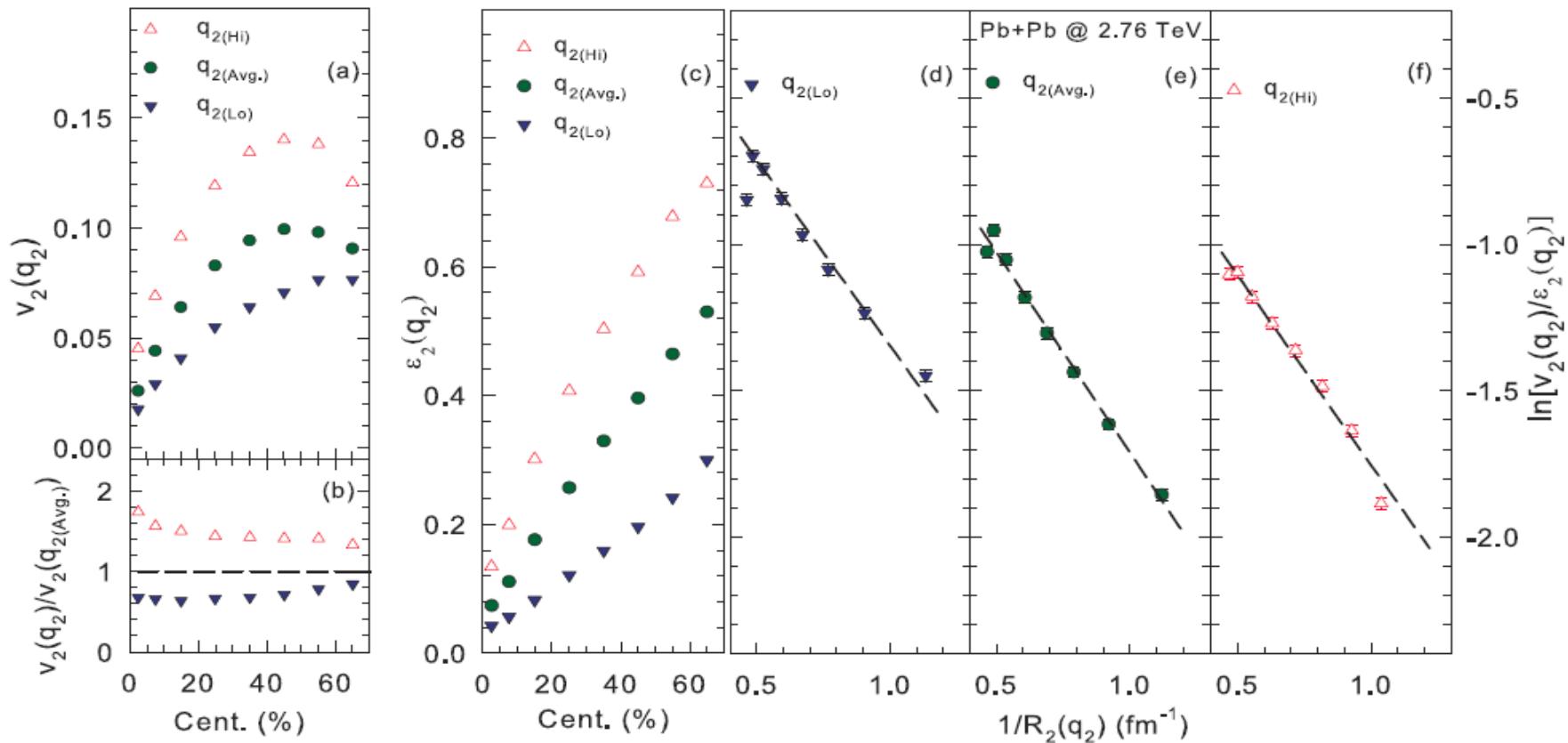
✓ viscous coefficient  $\beta''$  shows clear sensitivity to  $\eta/s$

# Scaling properties of flow

$$Q_{n,x} = \sum_i^M \cos(n\phi_i); Q_{n,y} = \sum_i^M \sin(n\phi_i)$$

$$q_n = Q_n/\sqrt{M},$$

## Acoustic Scaling of shape-engineered events



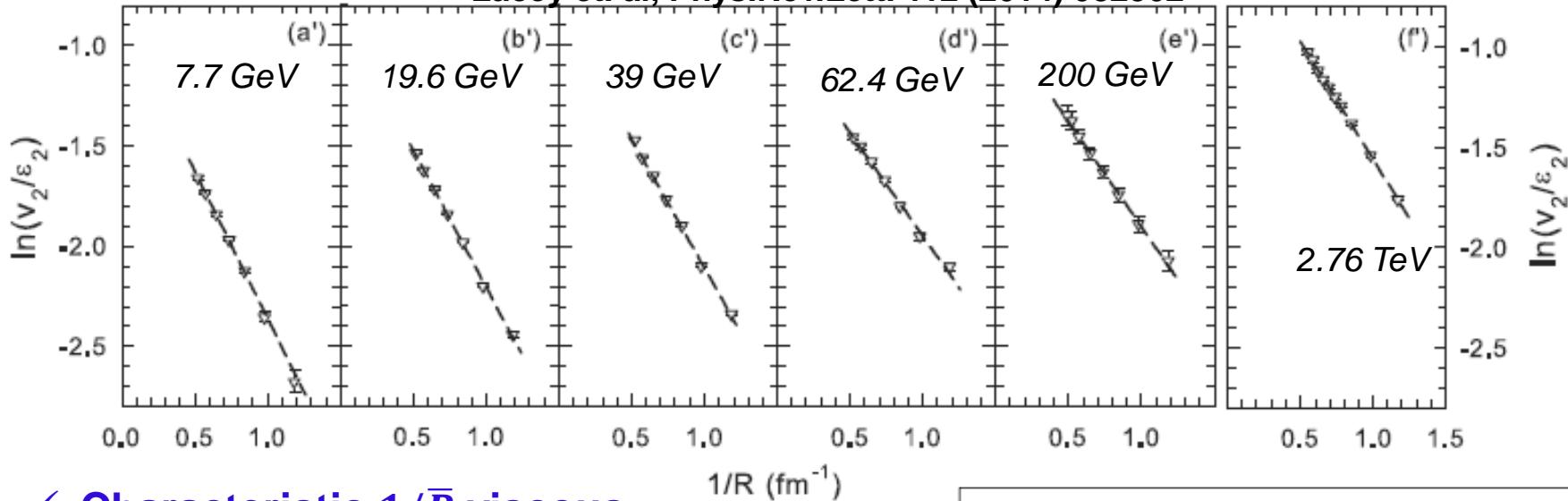
- ✓ Characteristic  $1/R$  viscous damping validated for different event shapes at the same centrality
  - ✓ A strong constraint for initial-state models and  $\eta/s$ 
    - ✓  $4\pi\eta/s$  for RHIC plasma  $\sim 1.3 \pm 0.2$
    - ✓  $4\pi\eta/s$  for LHC plasma  $\sim 2.2 \pm 0.2$
- } Following calibration

## Scaling properties of flow

### Acoustic Scaling – $\frac{1}{R}$ Scaling for the Beam Energy Scan

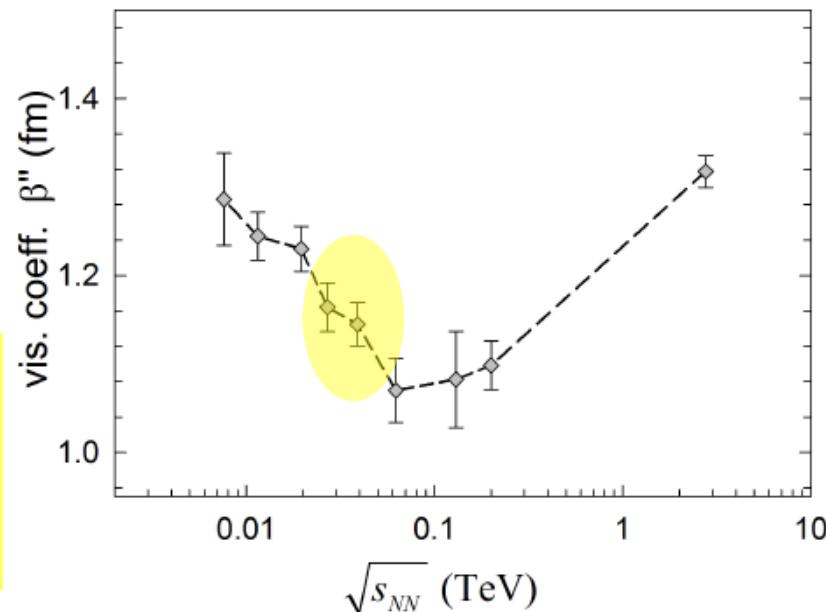
Lacey et. al, Phys.Rev.Lett. 112 (2014) 082302

$$\ln\left(\frac{v_n}{\epsilon_n}\right) \propto \frac{-\beta''}{R}$$

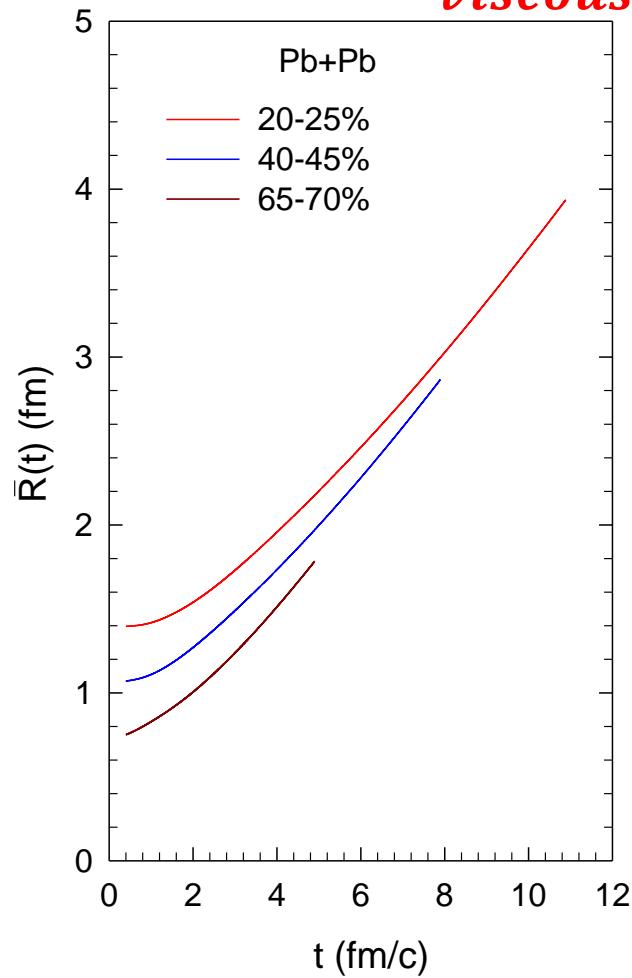


- ✓ Characteristic  $1/\bar{R}$  viscous damping validated across beam energies
- ✓ First experimental indication for  $\eta/s$  variation in the  $(T, \mu_B)$ -plane
- ✓ CEP?

HBT should show complimentary signals for similar  $\sqrt{s_{NN}}$

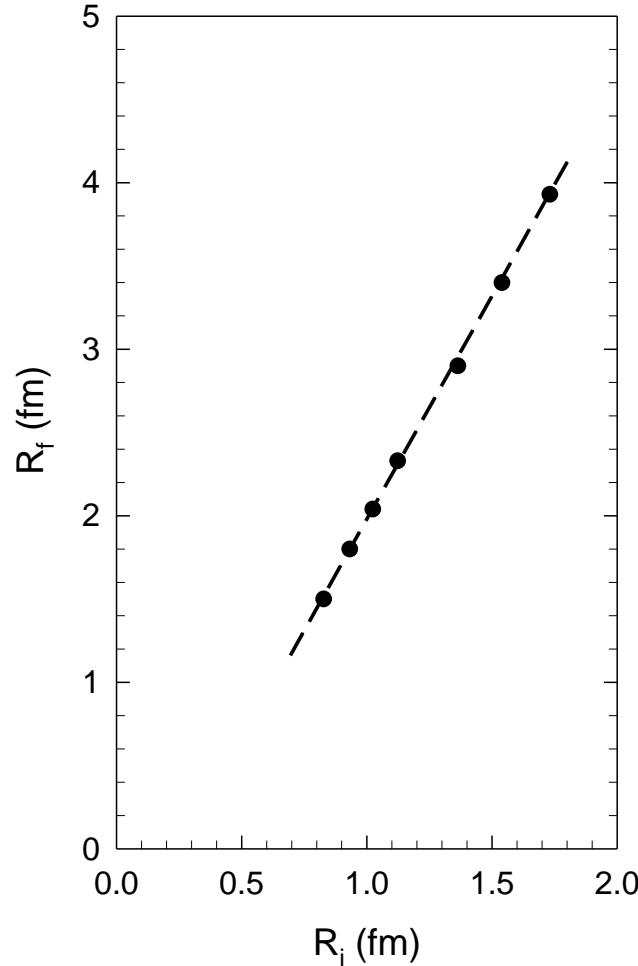


✓ Characteristic acoustic scaling validated for viscous hydrodynamics



Freeze-out time varies with initial size

$$t \propto \bar{R}$$

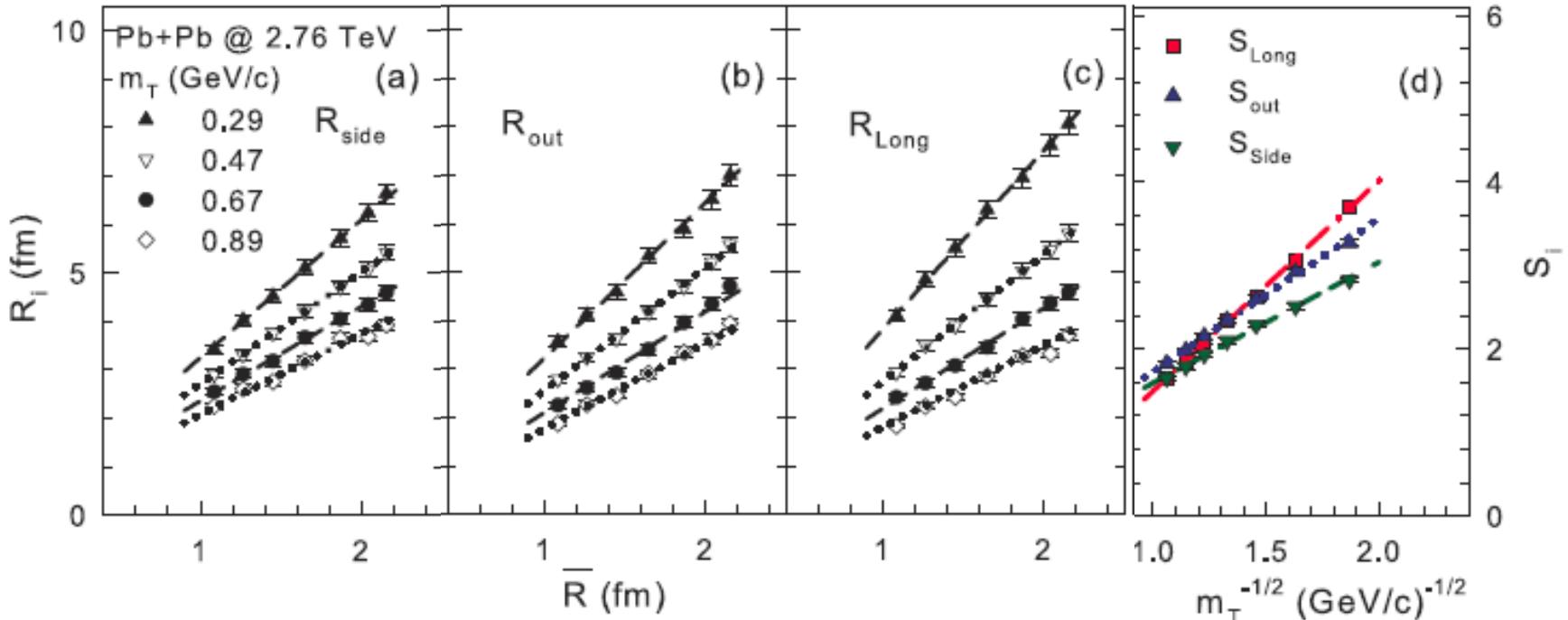
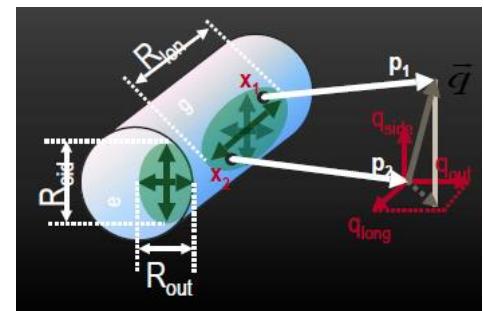


Freeze-out transverse size proportional to initial transverse size

# Acoustic Scaling of HBT Radii

$$t \propto \bar{R}$$

$$R_{out}, R_{side}, R_{long} \propto \bar{R}$$



- $\bar{R}$  and  $m_T$  scaling of the full RHIC and LHC data sets
- The centrality and  $m_T$  dependent data scale to a single curve for each radii.
- Qualitatively similar expansion dynamics at RHIC & LHC

Nuggehalli

Ajitanand

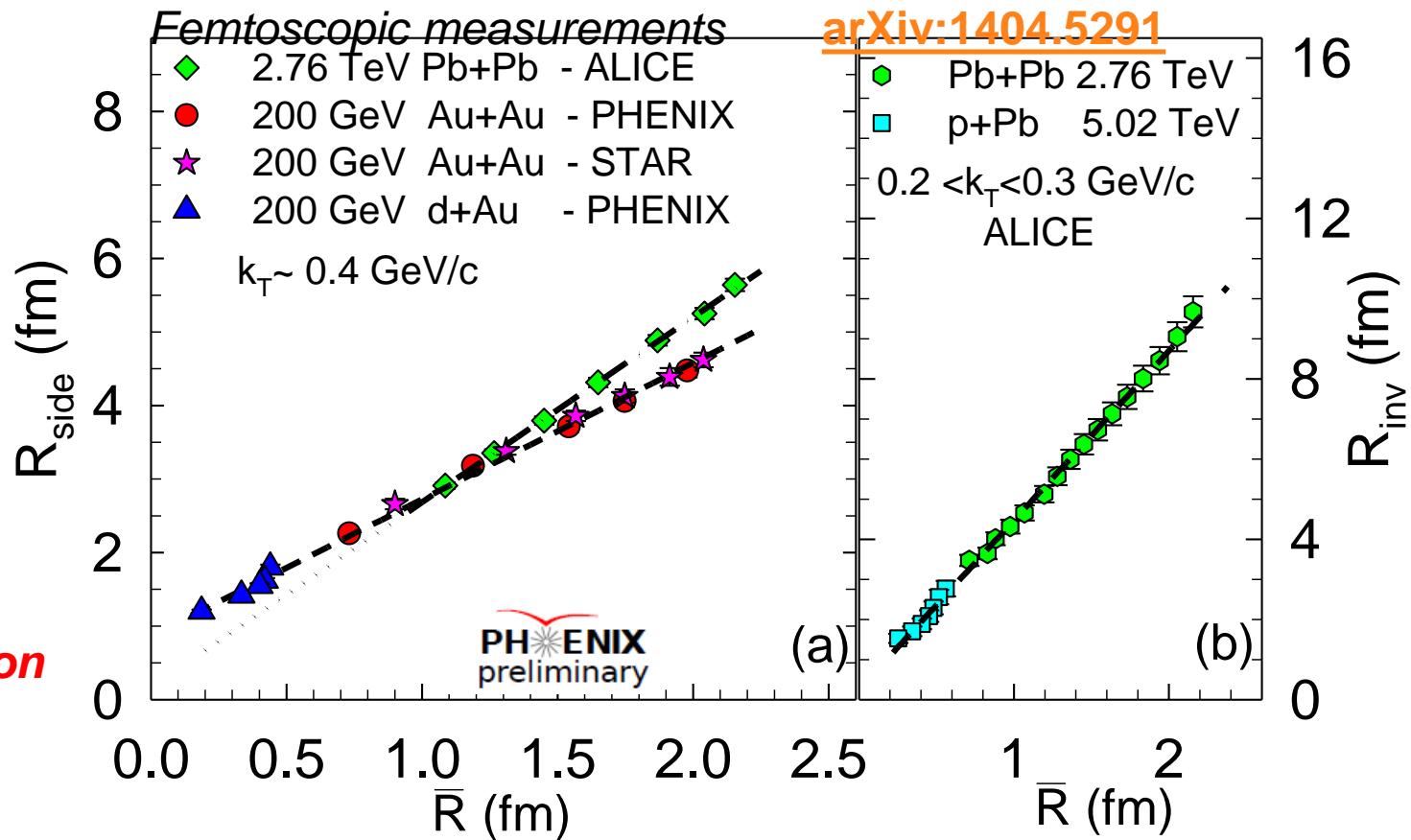
Wed.

12:30 - 12:50

*Europium*

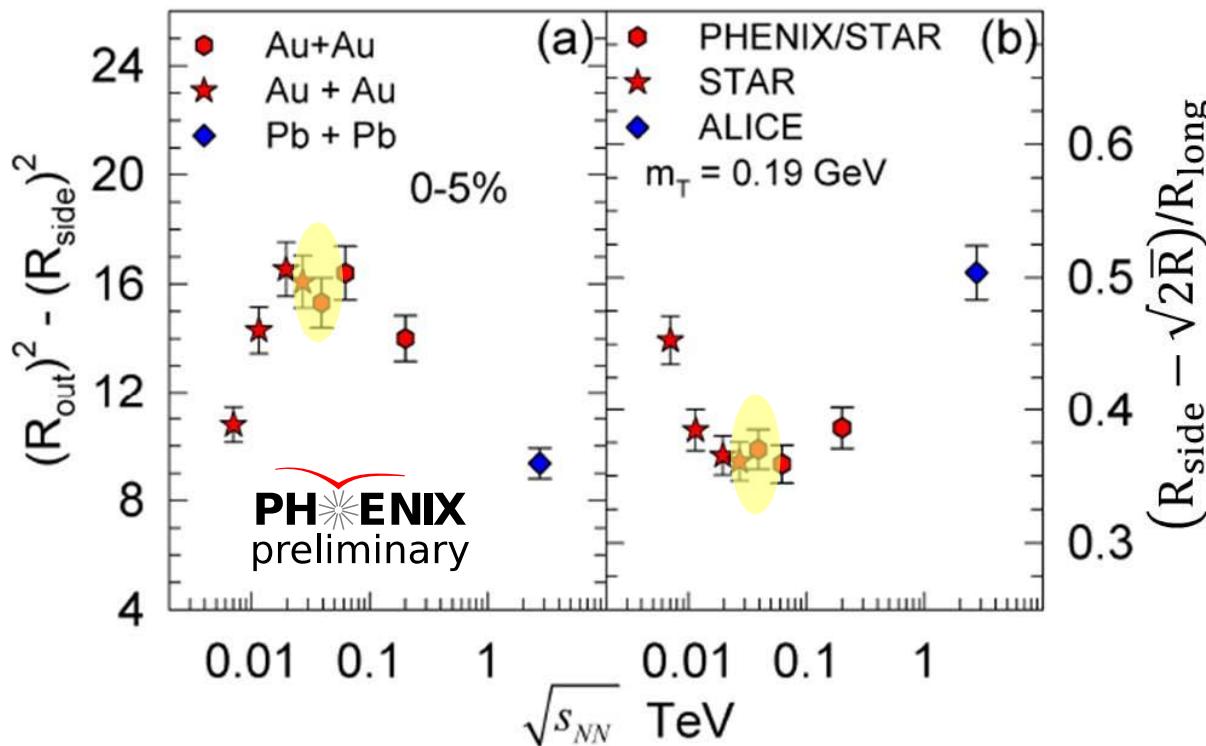
Darmstadtium

➤ Larger expansion  
rate at the LHC



$t \propto \bar{R}$  **exquisitely demonstrated via HBT measurement for several systems**

# $\sqrt{s_{NN}}$ dependence of HBT signals



$$R_{long} \propto \tau$$

$$(R_{out}^2 - R_{side}^2) \propto \Delta\tau$$

$$(R - R_i) / R_{long} \propto u$$

$$R_i = \sqrt{2\bar{R}}$$

Ron Soltz

Tue.

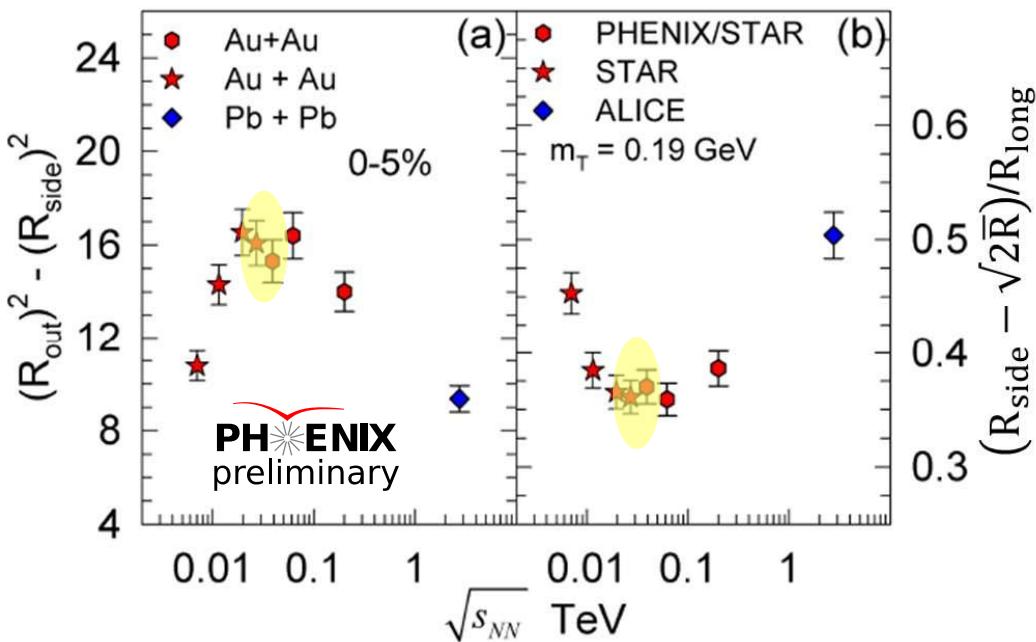
15:00 - 15:20

Helium

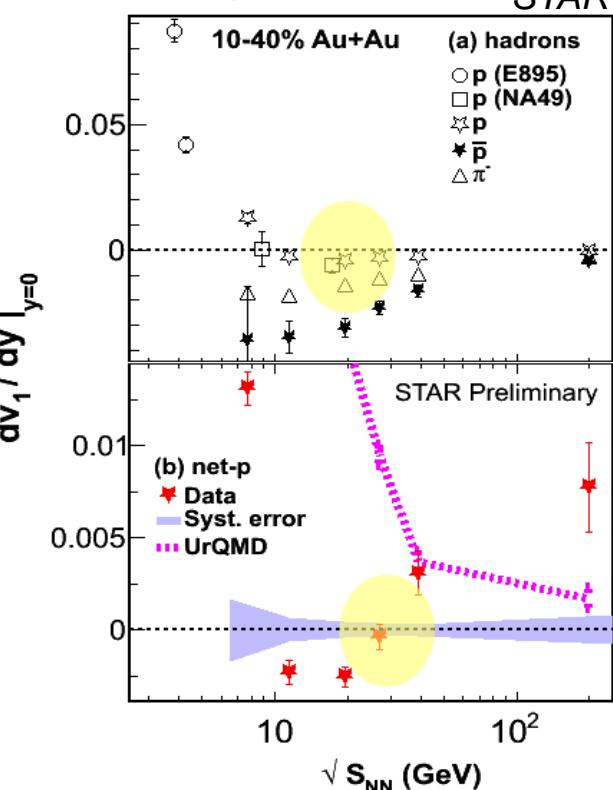
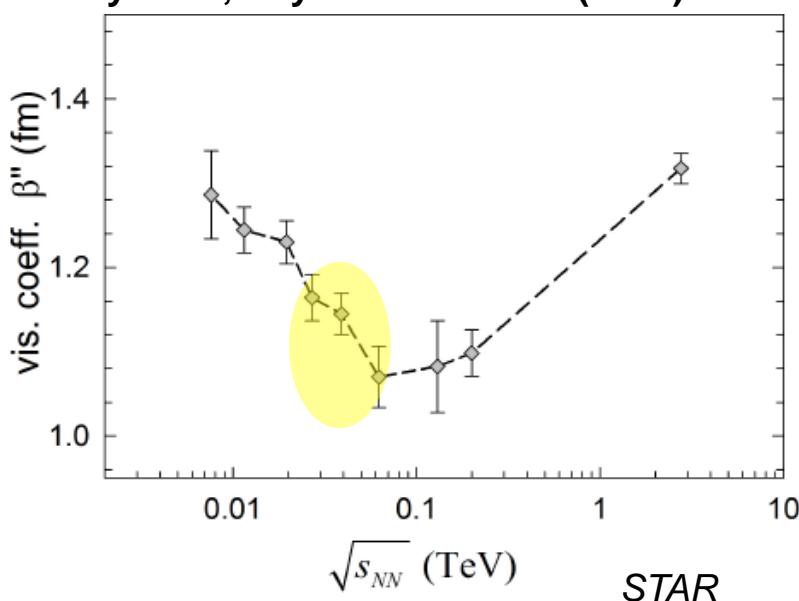
Darmstadtium

These characteristic patterns signal an important change in the reaction dynamics  
CEP? Phase transition?

# $\sqrt{s_{NN}}$ dependence of HBT signals



Combined results  
→ Strongest indications for  
the CEP to date!



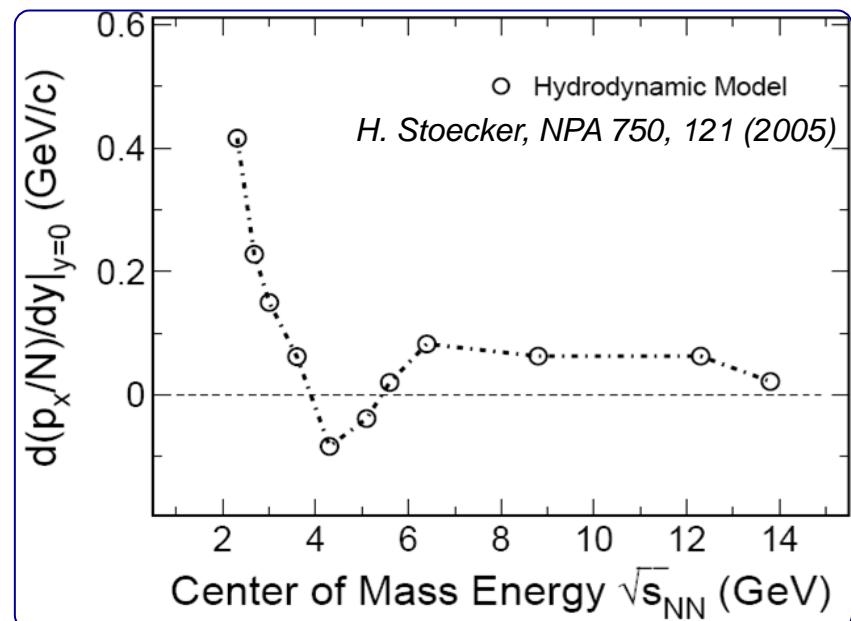
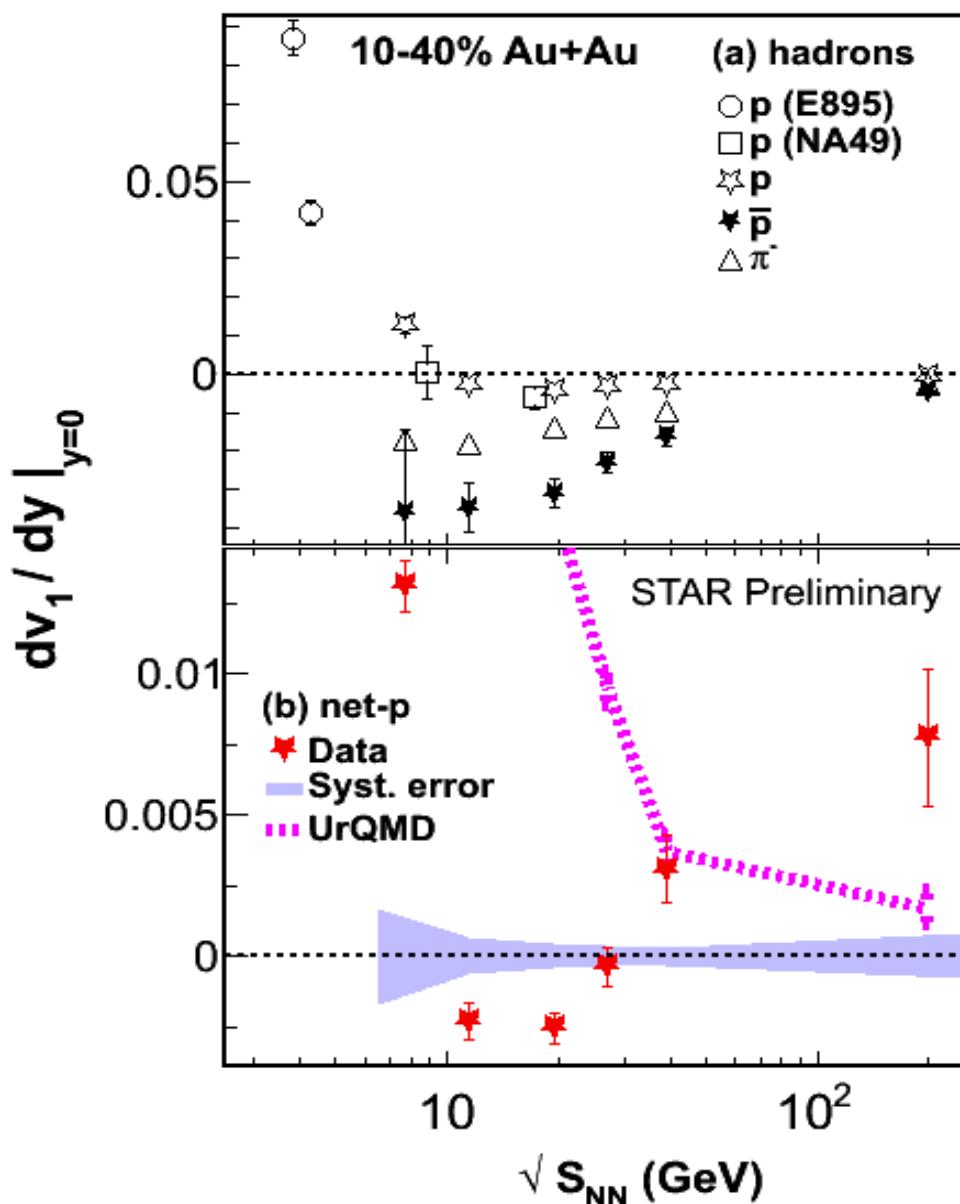
**Acoustic scaling of anisotropic flow and HBT radii lend profound mechanistic insights, as well as new constraints for key observables**

### What do we learn?

- **The expansion dynamics is acoustic – “as it should be”**
  - **Validates expected acoustic scaling of flow and HBT radii**
    - ✓ **constraints for  $4\pi\eta/s$  & viable initial-state models**
    - ✓  **$4\pi\eta/s$  for RHIC plasma  $\sim 1.3 \pm 0.2$  ~ my 2006 estimate**
    - ✓  **$4\pi\eta/s$  for LHC plasma  $\sim 2.2 \pm 0.2$**
    - ✓ **Extraction insensitive to initial geometry model**
  - **Characteristic dependence of viscous coefficient  $\beta$ ” and  $v_1$ , as well as “ $c_s$ ” and  $\Delta\tau$  on  $\sqrt{s_{NN}}$  give new constraints which could be an indication for reaction trajectories in close proximity to the CEP?**

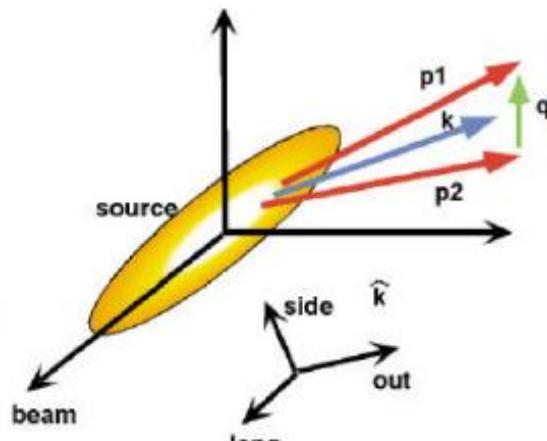
*End*

## Directed flow of transported protons



*Data qualitatively resembles predictions of a hydrodynamic model with a first-order phase transition*

Chapman, Scotto, and Heinz



$$R_I^2 = \tau_0^2 \frac{T}{m_\perp} \left[ 1 + \left( \frac{1}{2} + \frac{1}{1 + \frac{m_\perp}{T} v^2} \right) \frac{T}{m_\perp} \right]$$

$$R_{long} \propto \tau$$

$$R_s^2 = \frac{R^2}{1 + (m_\perp / T)v^2}$$

$$R_o^2 = \frac{R^2}{1 + (m_\perp / T)v^2} + \frac{1}{2} \left( \frac{T}{m_\perp} \right)^2 \beta_\perp^2 \tau_0^2$$

$$(R_{out}^2 - R_{side}^2) \propto \Delta \tau$$

proxy for the sound speed  
 $c_s$

estimate for initial size in central events

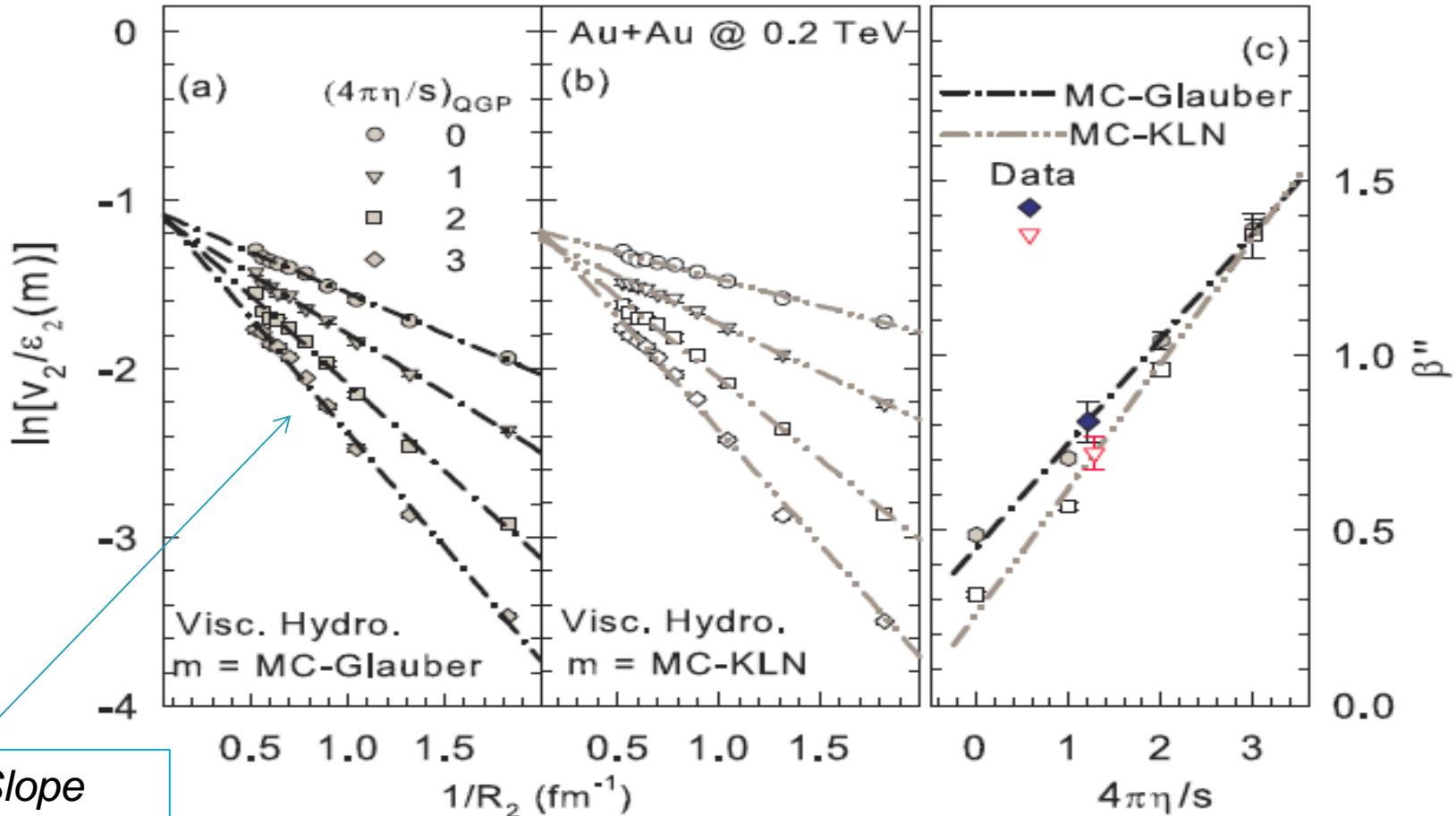
$$(R - R_i) / R_{long} \propto u$$

$$R_i = \sqrt{2} \bar{R}$$

**Study HBT observables as a function of  $\sqrt{s_{NN}}$**

## Extraction of $\eta/s$

$$\ln\left(\frac{v_n}{\epsilon_n}\right) \propto \frac{-\beta''}{R}$$

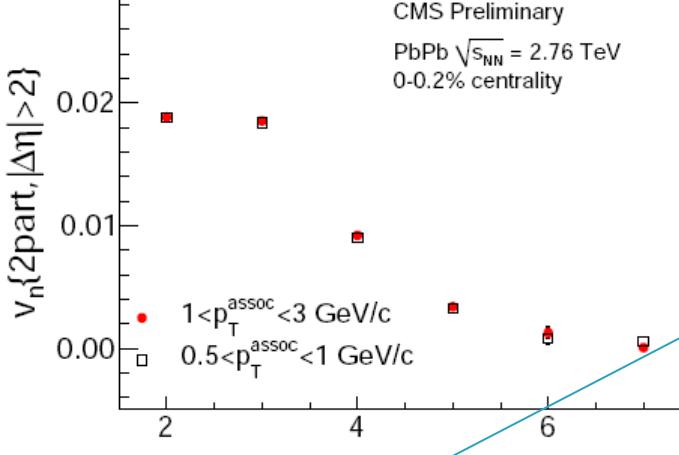


Slope  
sensitive  
to  $4\pi\eta/s$

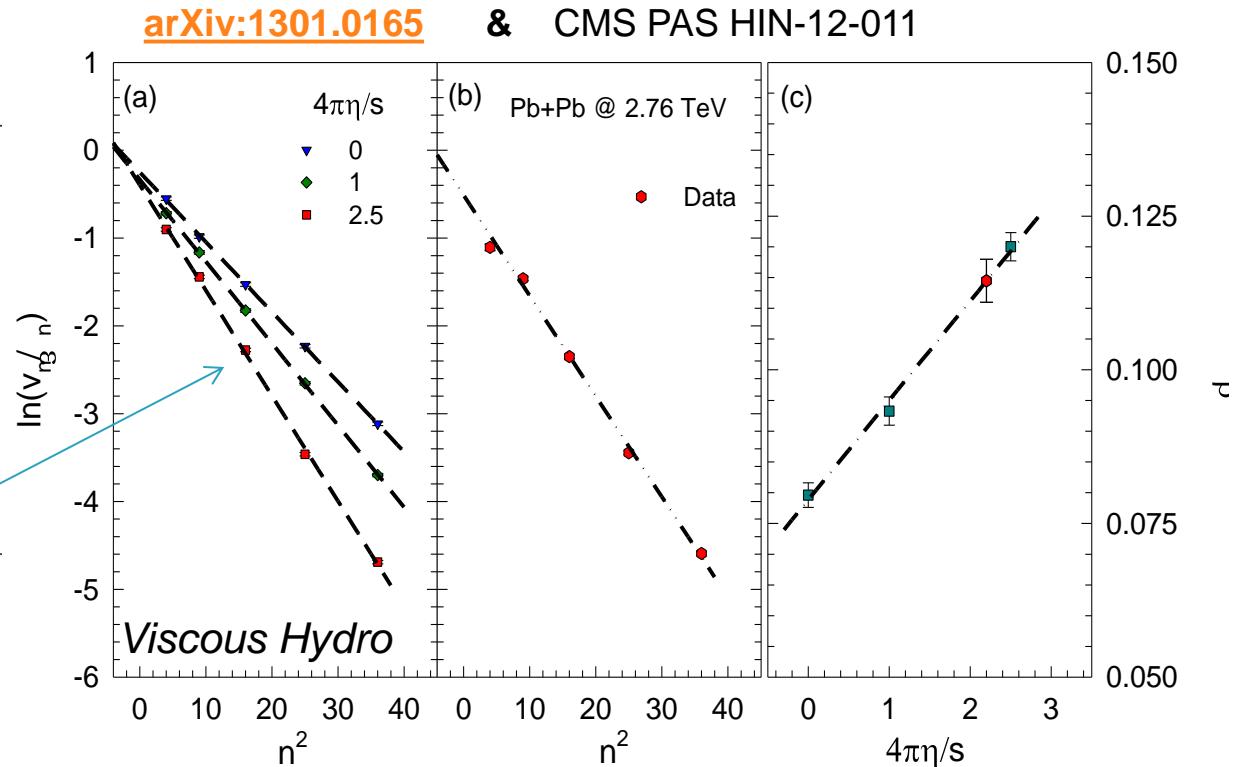
**Characteristic  $1/\bar{R}$  viscous damping validated in viscous hydrodynamics; calibration  $\rightarrow 4\pi\eta/s \sim 1.3 \pm 0.2$**   
**Extracted  $\eta/s$  value insensitive to initial conditions**

## Extraction of $\eta/s$

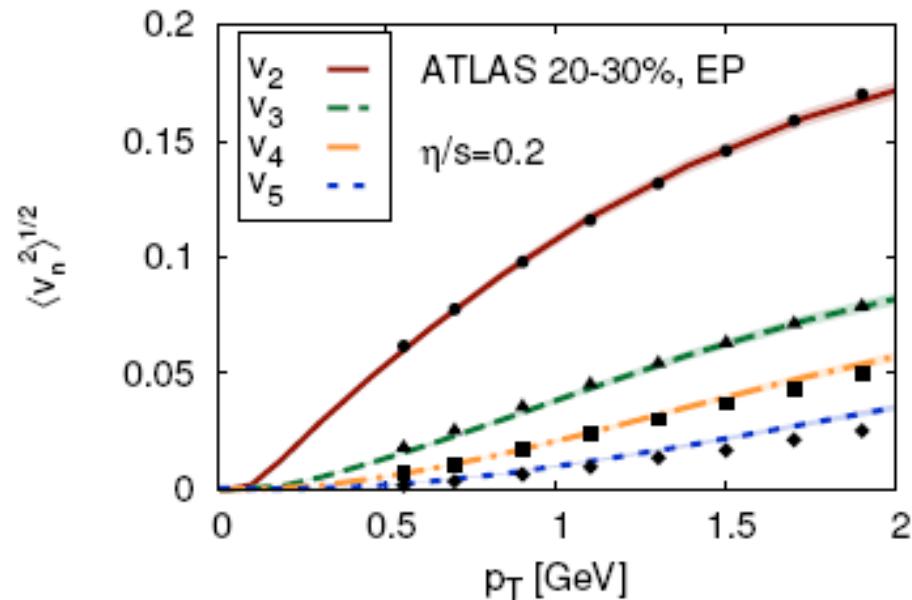
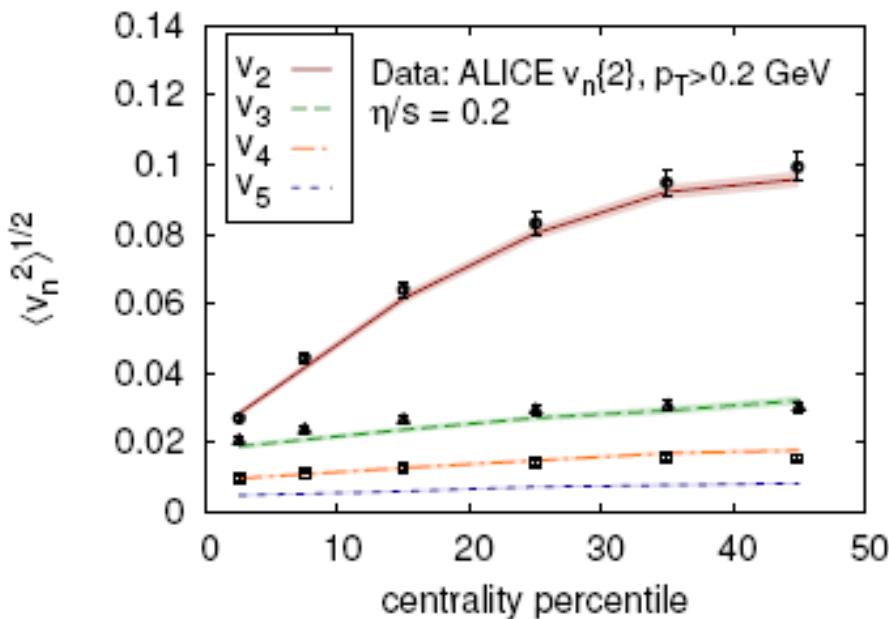
$$\frac{v_n(p_T)}{\varepsilon_n} \propto \exp(-\beta' n^2)$$



Slope sensitive  
to  $\eta/s$



**$n^2$  scaling validated in experiment and viscous hydrodynamics;  
 calibration  $\rightarrow 4\pi\eta/s \sim 2.2 \pm 0.2$   
 Note agreement with Schenke et al.**



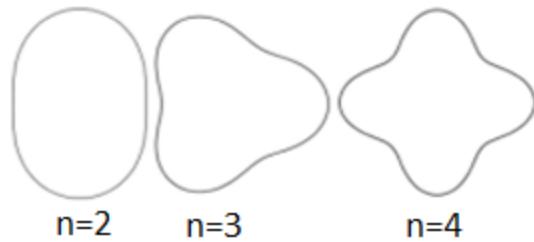
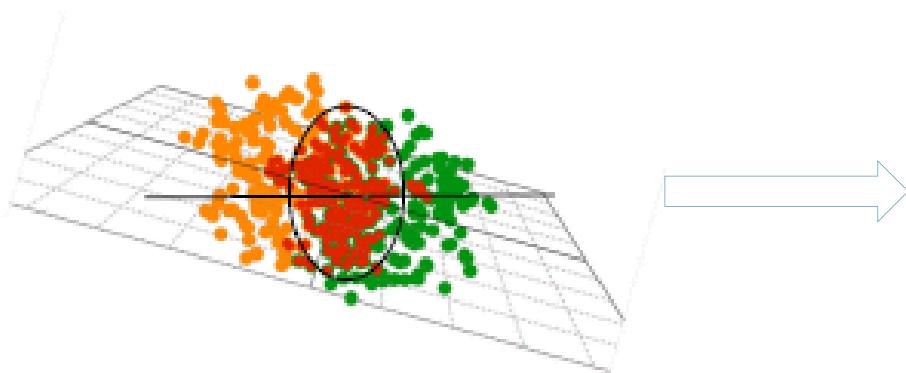
$$4\pi\eta/s \text{ for LHC plasma } \sim 2.5$$

*Higher harmonics provide important constraints!*

*Can we find methodologies and constraints  
which are insensitive to the initial-state geometry?*

# Scaling properties of flow

**Initial Geometry characterized by many shape harmonics ( $\varepsilon_n$ )  $\rightarrow$  drive  $v_n$**



**Acoustic viscous modulation of  $v_n$**

$$\delta T_{\mu\nu}(t, k) = \exp\left(\frac{2\eta}{3s}k^2\frac{t}{T}\right)\delta T_{\mu\nu}(0)$$

Staig & Shuryak arXiv:1008.3139

$$v_n \propto \varepsilon_n$$

$$k = n / \bar{R}$$

$$t \propto \bar{R}$$

$$\delta T_{\mu\nu}(n, t) = \exp(-\beta n^2) \delta T_{\mu\nu}(0), \quad \beta = \frac{2\eta}{3s} \frac{1}{\bar{R}^2} \frac{t}{T}$$

**Scaling expectations:**

**$n^2$  dependence**

$$\frac{v_n(p_T)}{\varepsilon_n} \propto \exp(-\beta' n^2)$$

**$v_n$  is related to  $v_2$**

$$\frac{v_n(p_T)}{v_2(p_T)} = \frac{\varepsilon_n}{\varepsilon_2} \cdot \exp(-\beta'(n^2 - 4))$$

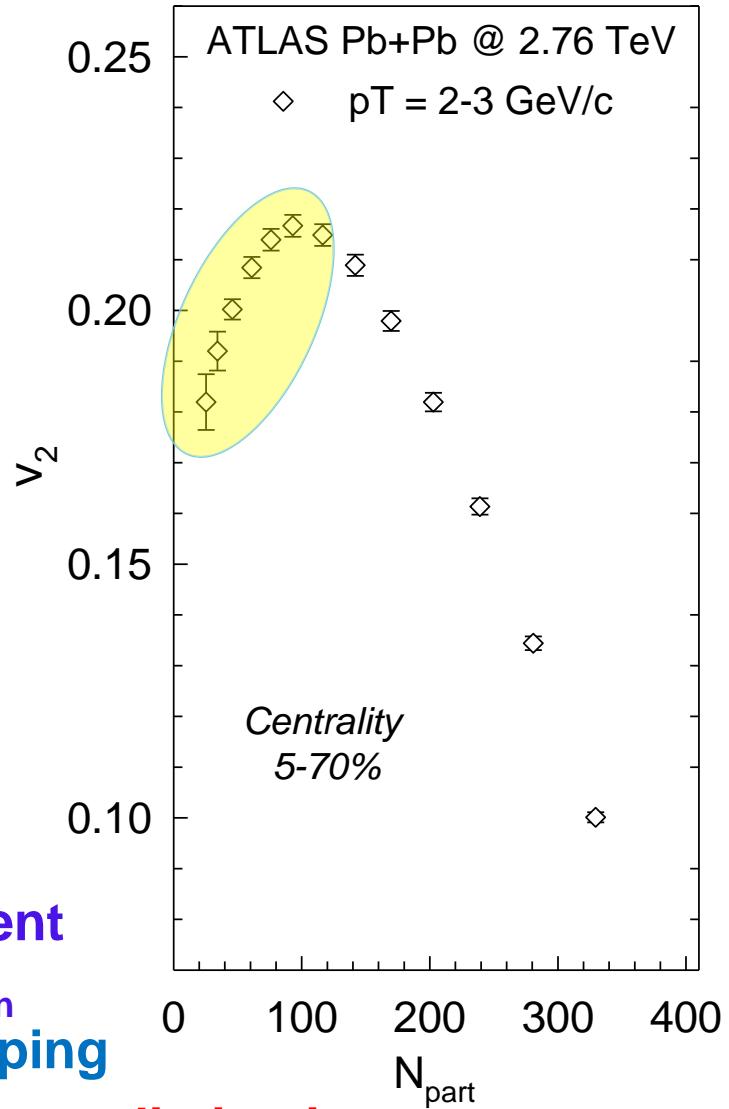
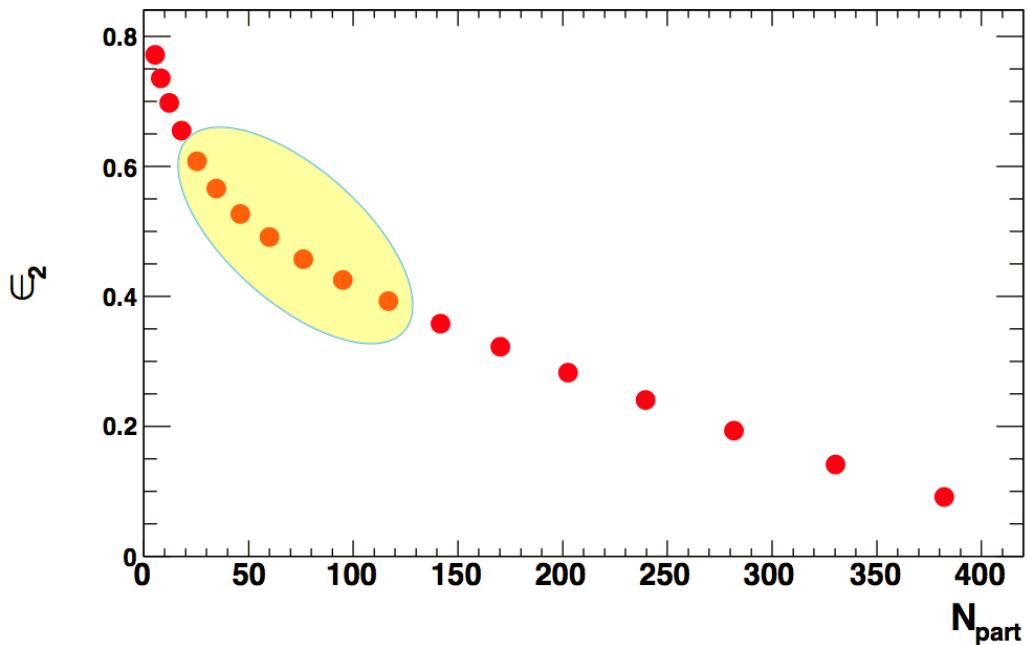
**System size dependence**

$$\ln\left(\frac{v_n}{\varepsilon_n}\right) \propto \frac{-\beta''}{\bar{R}}$$

**Each of these scaling expectations can be validated**  
 $\eta/s \propto \beta', \beta''$

**Acoustic Scaling** –  $\frac{1}{R}$

$$\ln\left(\frac{v_n}{\varepsilon_n}\right) \propto -\frac{\beta''}{\bar{R}}$$



➤ Eccentricity change alone is not sufficient

To account for the N<sub>part</sub> dependence of v<sub>n</sub>

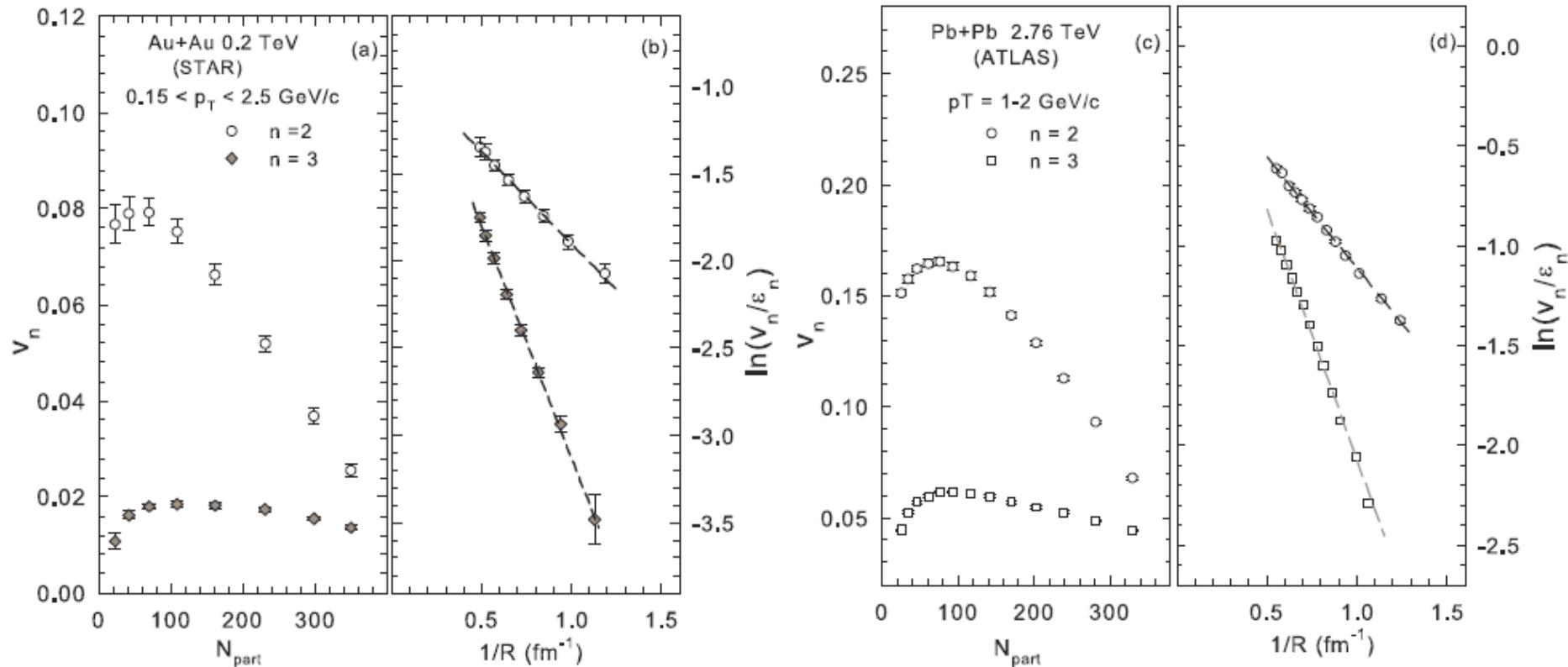
Transverse size ( $\bar{R}$ ) influences viscous damping

✓ Characteristic  $1/\bar{R}$  scaling prediction is non-trivial

## Scaling properties of flow

**Acoustic Scaling –  $\frac{1}{R}$**

$$\ln\left(\frac{v_n}{\varepsilon_n}\right) \propto \frac{-\beta''}{R}$$



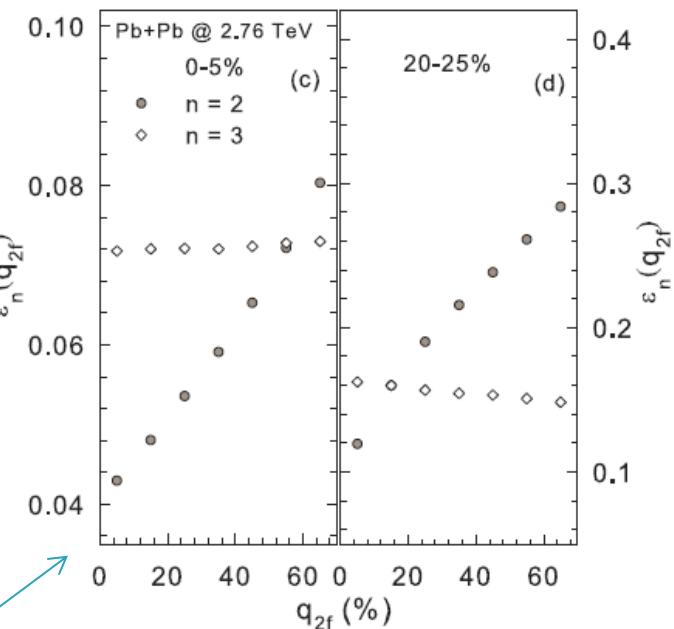
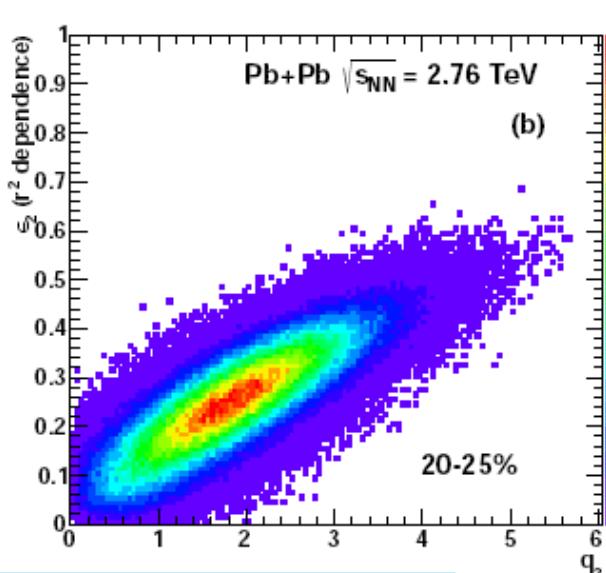
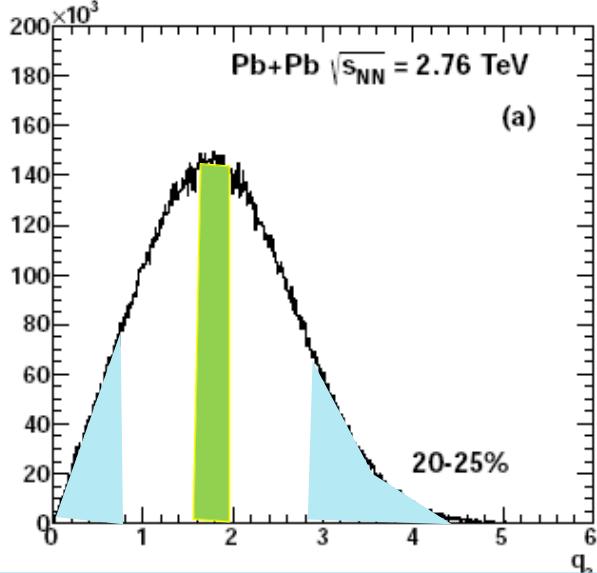
- ✓ Characteristic  $1/R$  viscous damping validated at RHIC & the LHC
- ✓ A further constraint for  $\eta/s$

## Shape-engineered events

**Shape fluctuations lead to  
a distribution of the Q vector  
at a fixed centrality**

$$Q_{n,x} = \sum_i^M \cos(n\phi_i); Q_{n,y} = \sum_i^M \sin(n\phi_i)$$

$$q_n = Q_n/\sqrt{M},$$



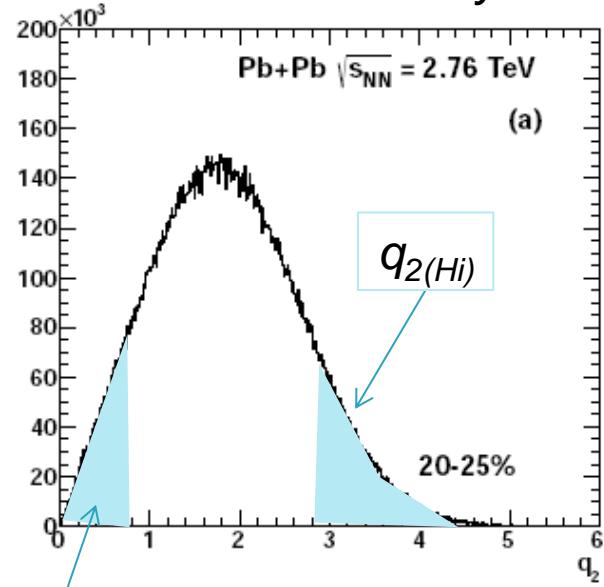
- Cuts on  $q_n$  should change the magnitudes  $\langle \epsilon_n \rangle$ ,  $\langle v_n \rangle$ ,  $\langle R_n \rangle$  at a given centrality due to fluctuations
- These magnitudes can influence scaling

- Note characteristic anti-correlation predicted for  $v_3(q_2)$  in mid-central events

➤ Crucial constraint for initial-geometry models

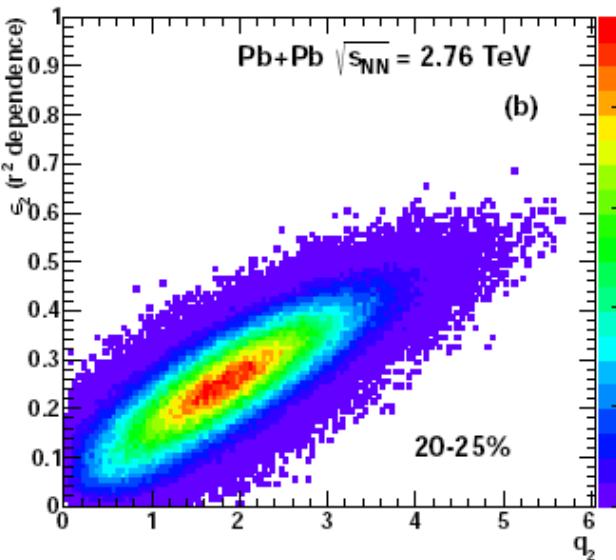
## Shape-engineered events

Shape fluctuations lead to a distribution of the Q vector at a fixed centrality

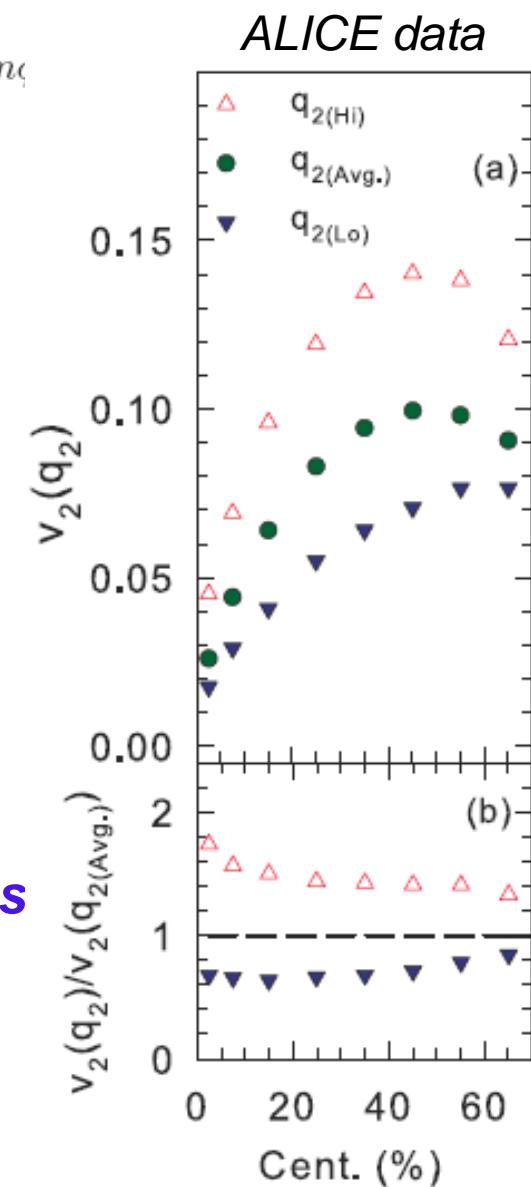


$$Q_{n,x} = \sum_i^M \cos(n\phi_i); \quad Q_{n,y} = \sum_i^M \sin(n\phi_i)$$

$$q_n = Q_n / \sqrt{M},$$

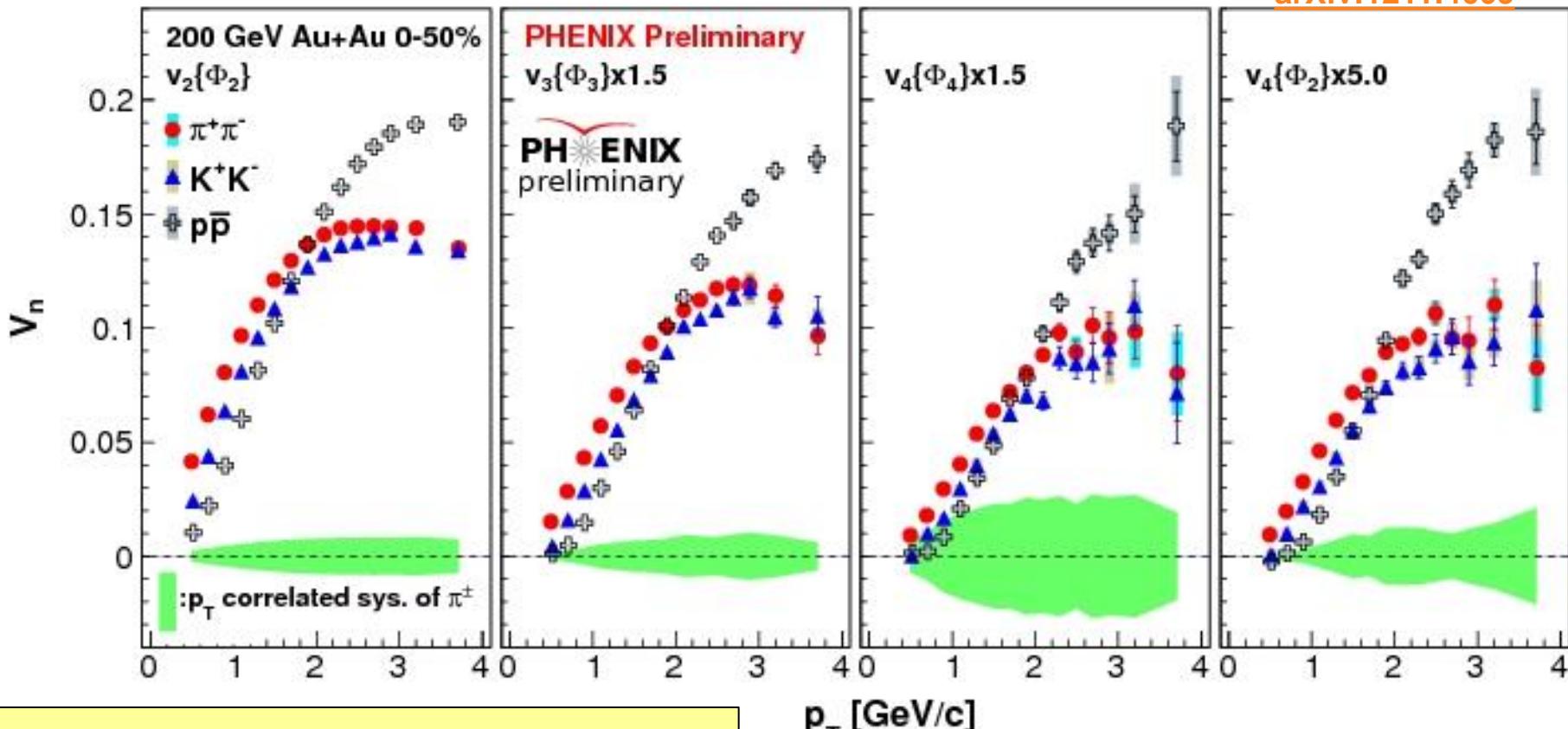


- Cuts on  $q_n$  should change the magnitudes  $\langle \epsilon_n \rangle$ ,  $\langle v_n \rangle$ ,  $\langle R_n \rangle$  at a given centrality due to fluctuations
- Viable models for initial-state fluctuations should still scale



# Flow is partonic & Acoustic?

arXiv:1211.4009



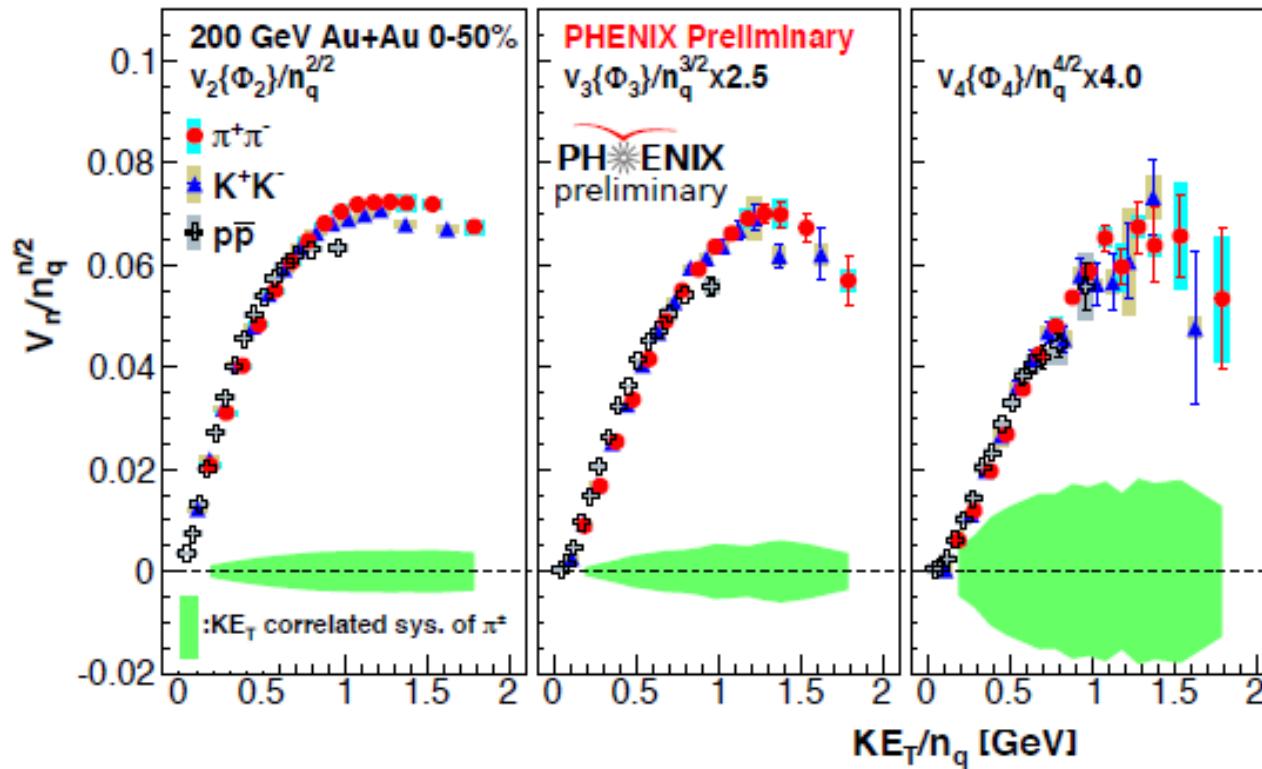
Expectation:  $v_n(KE_T) \sim v_2^{n/2}$  or  $\frac{v_n}{(n_q)^{n/2}}$

**Note species dependence for all  $v_n$**

**For partonic flow, quark number scaling expected  
→ single curve for identified particle species  $v_n$**

## Acoustic Scaling – Ratios

### $v_n$ PID scaling

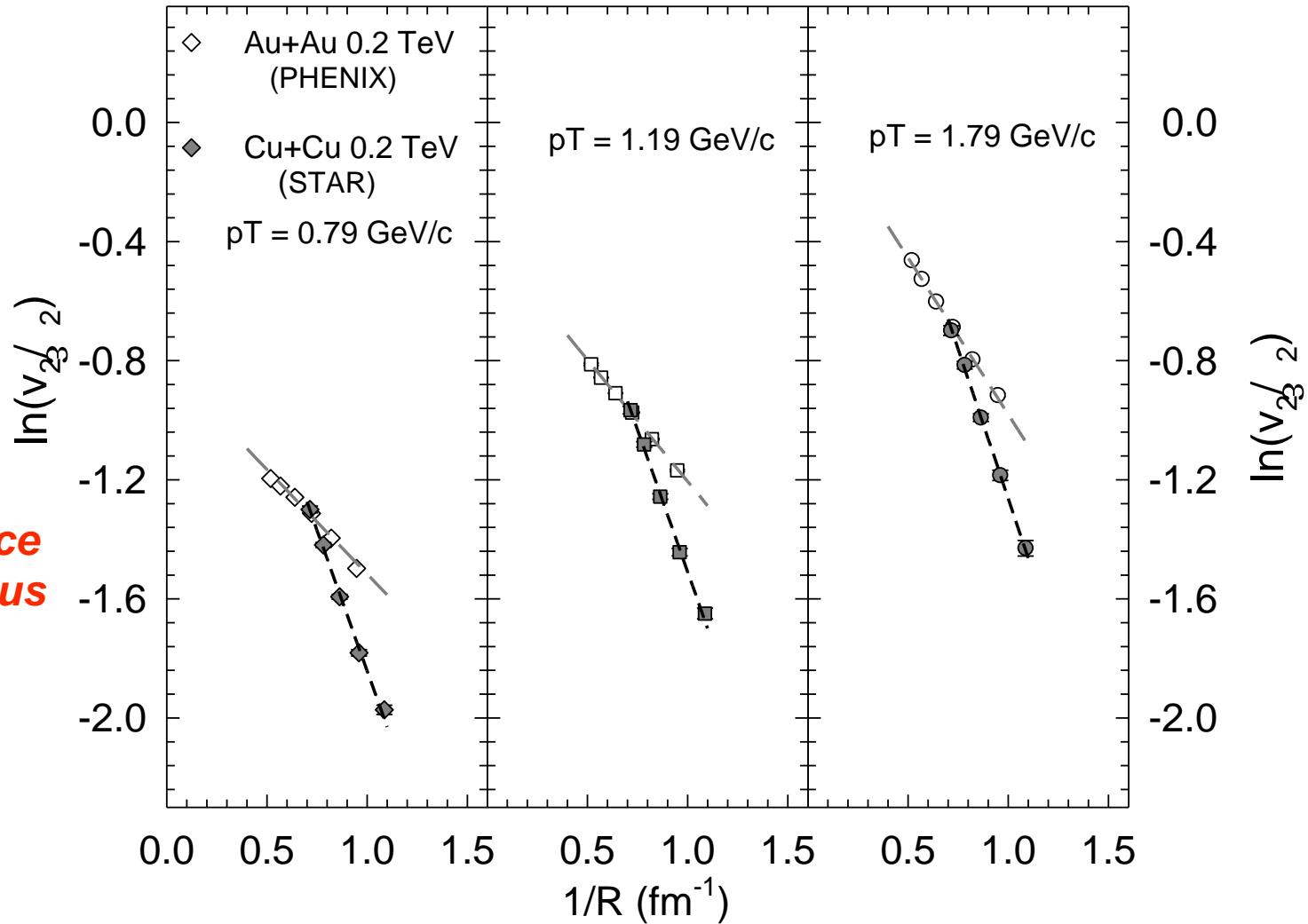


Expectation validated:  $v_n(KE_T) \sim v_2^{n/2}$  or  $\frac{v_n}{(n_q)^{n/2}}$

# Acoustic Scaling – $1/R$

Compare system size @ RHIC

$$\ln\left(\frac{v_n}{\epsilon_n}\right) \propto \frac{-\beta''}{R}$$



✓ Viscous coefficient larger for more dilute system