Indications for a critical end point in the phase diagram for hot and dense nuclear matter

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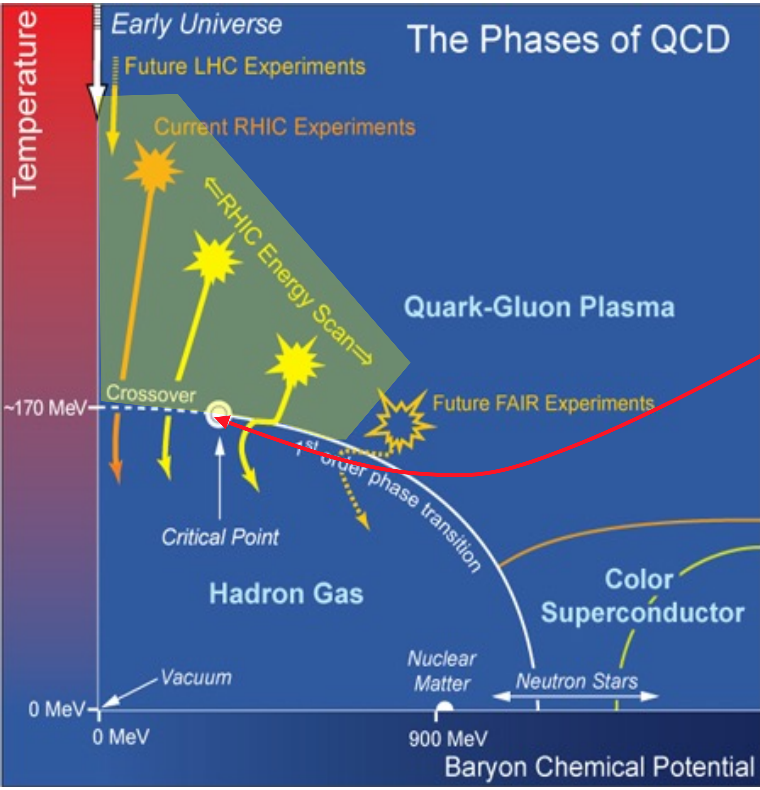
Outline

- ***Introduction***
  - ✓ *Phase Diagram*
- ***Search strategy***
  - ✓ *Theoretical guidance*
  - ✓ *Femtoscopic probe*
- ***Analysis***
  - ✓ *Finite-Size-Scaling*
  - ✓ *Dynamic Finite-Size-Scaling*
- ***Epilogue***

The QCD Phase Diagram

A central goal of the worldwide program in relativistic heavy ion collisions, is to chart the QCD phase diagram



Essential Question

What new insights do we have on:

The CEP "landmark"?

- ✓ *Location* (T^{cep}, μ_B^{cep}) *values?*
- ✓ *Static critical exponents -* ν, γ*?*
  - *Static universality class?*
  - *Order of the transition*
- ✓ *Dynamic critical exponent – z?*
  - *Dynamic universality class?*

All are required to fully characterize the CEP & drives the ongoing search

(New) measurements, analysis techniques and theory efforts which investigate a broad range of the (T, μ_B)***-plane are currently underway***

Theoretical Guidance

Theory consensus on the static universality class for the CEP

3D-Ising Z(2)

- ✓ $\nu \sim 0.63$
- ✓ $\gamma \sim 1.2$

M. A. Stephanov
Int. J. Mod. Phys. A 20, 4387 (2005)

Dynamic Universality class for the CEP less clear

➢ **One slow mode**

- ✓ ***z ~ 3 - Model H***

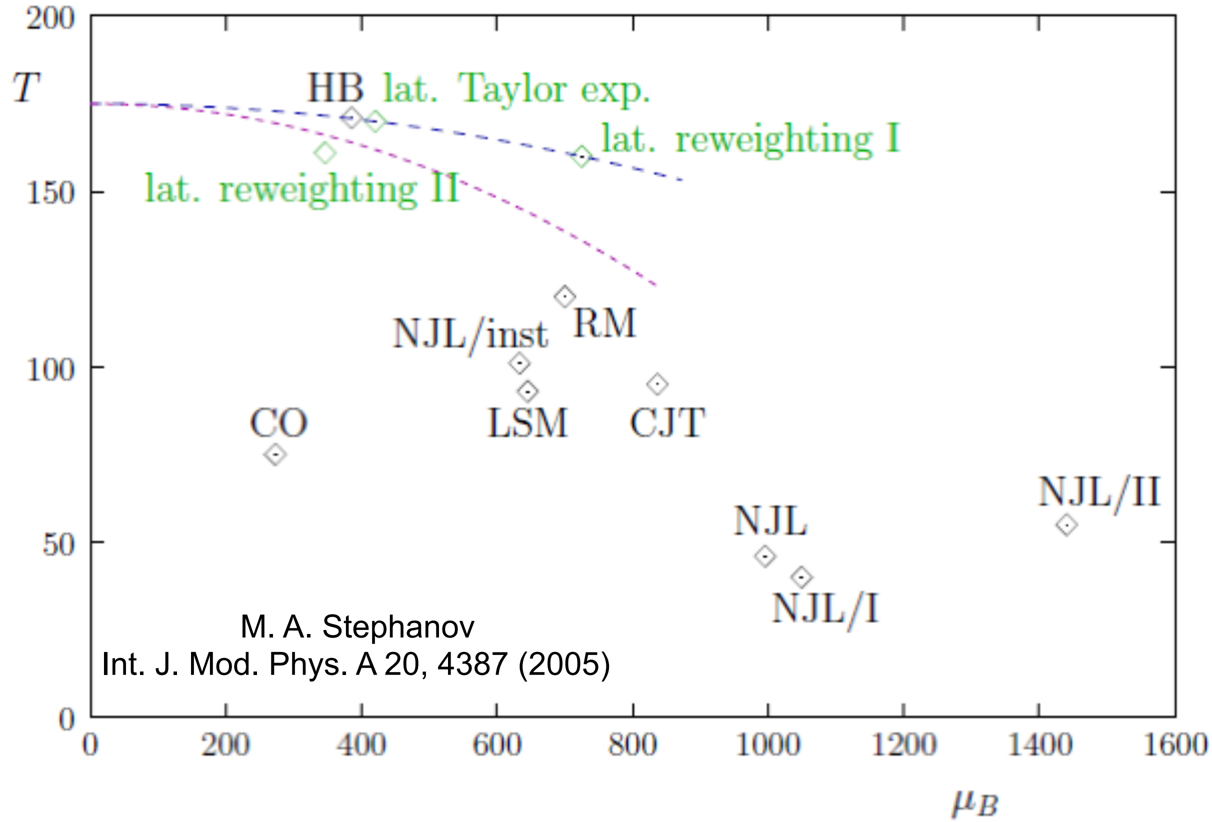
Son & Stephanov
Phys.Rev. D70 (2004) 056001
Moore & Saremi ,
JHEP 0809, 015 (2008)

➢ **Three slow modes**

- ✓ $z_T \sim 3$
- ✓ $z_v \sim 2$
- ✓ $z_s \sim -0.8$

Y. Minami - **Phys.Rev. D83 (2011) 094019**

The predicted location (T^{cep}, μ_B^{cep}) of the CEP is even less clear!



Experimental verification and characterization of the CEP is a crucial ingredient

✓ ***We use femtoscopic measurements to perform our search***

Femtoscopy as a susceptibility probe

S. Afanasiev et al. (PHENIX)
PRL 100 (2008) 232301

Hanbury Brown & Twist (HBT) radii obtained from two-pion correlation functions

$$C(\mathbf{q}) = \frac{dN_2 / d\mathbf{p}_1 d\mathbf{p}_2}{(dN_1 / d\mathbf{p}_1)(dN_1 / d\mathbf{p}_2)}$$

<u>3D Koonin Pratt Eqn.</u>

$$R(\vec{q}) = C(\vec{q}) - 1 = 4\pi \int dr r^2 K_0(\vec{q}, r) S(\vec{r}) \quad (1)$$

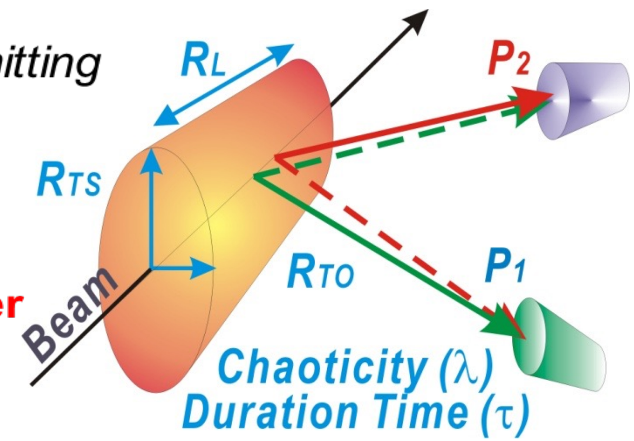
Correlation function

Encodes FSI

Source function
(Distribution of pair separations)

The expansion of the emitting source (R_L, R_{TO}, R_{TS}) produced in HI collisions is driven by c_s

χ of the order parameter diverges at the CEP



Inversion of this integral equation ➔ Source Function
(R_L, R_{TO}, R_{TS})

$$c_s^2 = \frac{1}{\rho\kappa}$$

Susceptibility (χ)

In the vicinity of a phase transition or the CEP, the divergence of κ leads to anomalies in the expansion dynamics

<u>Strategy</u>

Search for non-monotonic patterns for HBT radii combinations that are sensitive to the divergence of κ

Interferometry Probe

Hung, Shuryak, PRL. 75,4003 (95)
Chapman, Scotto, Heinz, PRL.74.4400 (95)

Makhlin, Sinyukov, ZPC.39.69 (88)

The measured HBT radii encode space-time information for the reaction dynamics

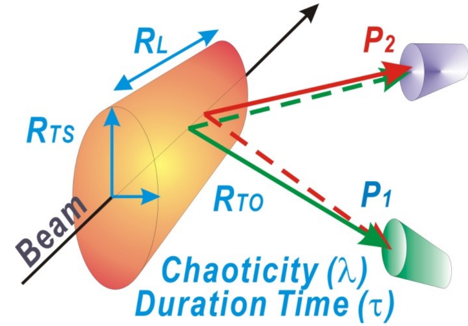
$$R_{side}^2 = \frac{R^2_{geo}}{1+\frac{m_T}{T}\beta_T^2}$$

emission duration

$$R_{out}^2 = \frac{R^2_{geo}}{1+\frac{m_T}{T}\beta_T^2} + \beta_T^2(\Delta\tau)^2$$

$$R_{long}^2 \approx \frac{T}{m_T}\tau^2$$

emission lifetime



$$c_s^2 = \frac{1}{\rho\kappa}$$

The divergence of the susceptibility κ

- ✓ *"softens" the sound speed c_s*
- ✓ *extends the emission duration*

(R^2_{out} - R^2_{side}) sensitive to the κ

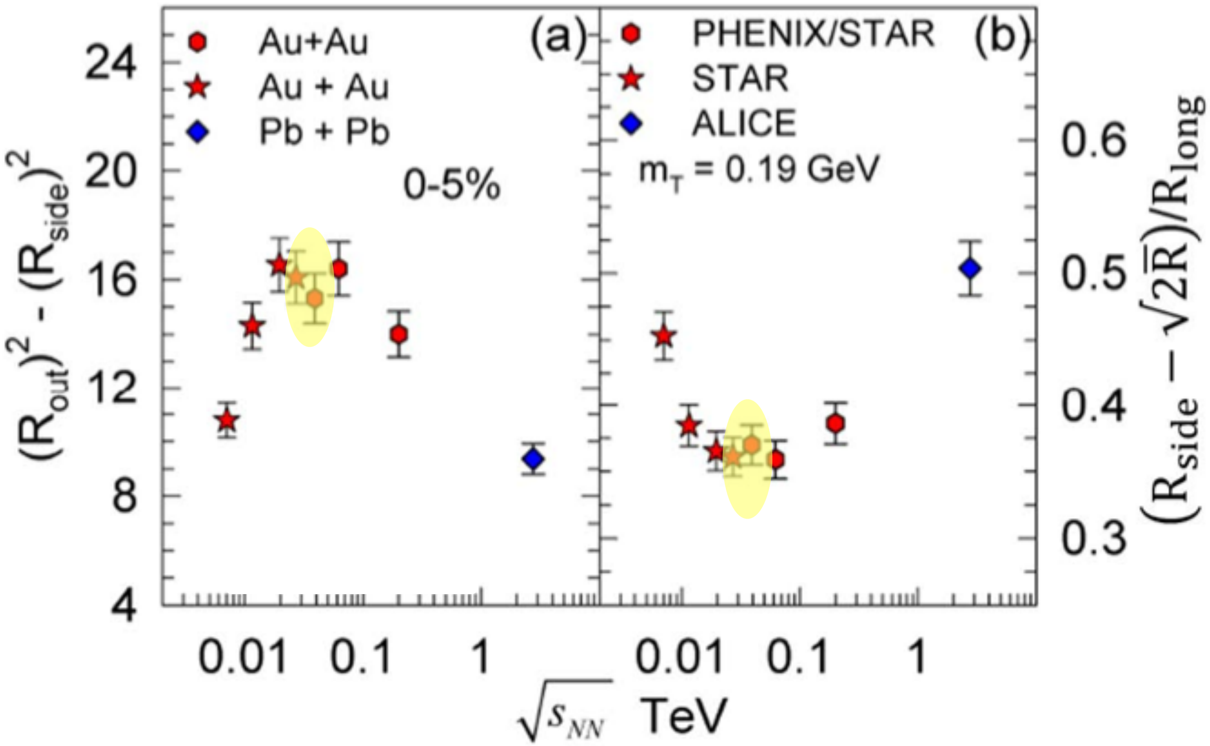
(R_{side} - R_{init})/ R_{long} sensitive to c_s

Specific non-monotonic patterns expected as a function of $\sqrt{s_{NN}}$

- ➢ **A maximum for (R^2_{out} - R^2_{side})**
- ➢ ***A minimum for*** **(R_{side} - $R_{initial}$)/R_{long}**

$\sqrt{s_{NN}}$ dependence of interferometry signal

Lacey QM2014
Adare et. al. (PHENIX)
arXiv:1410.2559



$$R_{long} \propto \tau$$

$$\left(R_{out}^2 - R_{side}^2\right) \propto \Delta\tau^2$$

$$\left(R_{side} - R_i\right)/R_{long} \propto u$$

$$R_{initial} = \sqrt{2}\bar{R}$$

The measurements validate the expected non-monotonic patterns!

à **Reaction trajectories spend a fair amount of time near a "soft point" in the EOS that coincides with the CEP!**

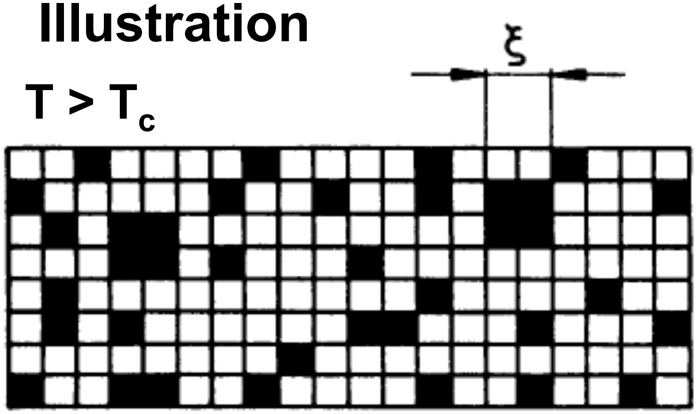
**** Note that R_{long}, R_{out} and R_{side} [all] increase with $\sqrt{s_{NN}}$ ****

<u>Finite-Size Scaling</u> (FSS) is used for further validation of the CEP, as well as to characterize its static and dynamic properties

Basis of Finite-Size Effects

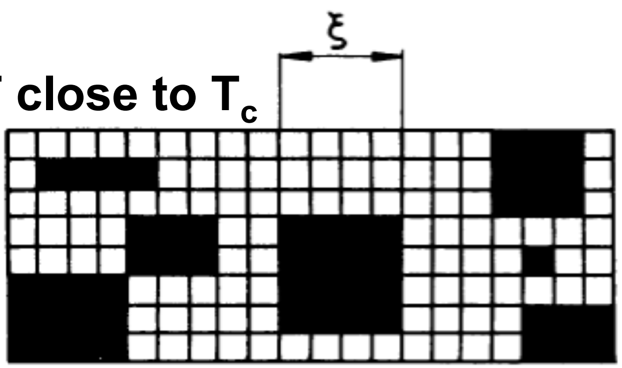
Illustration

T > T_c

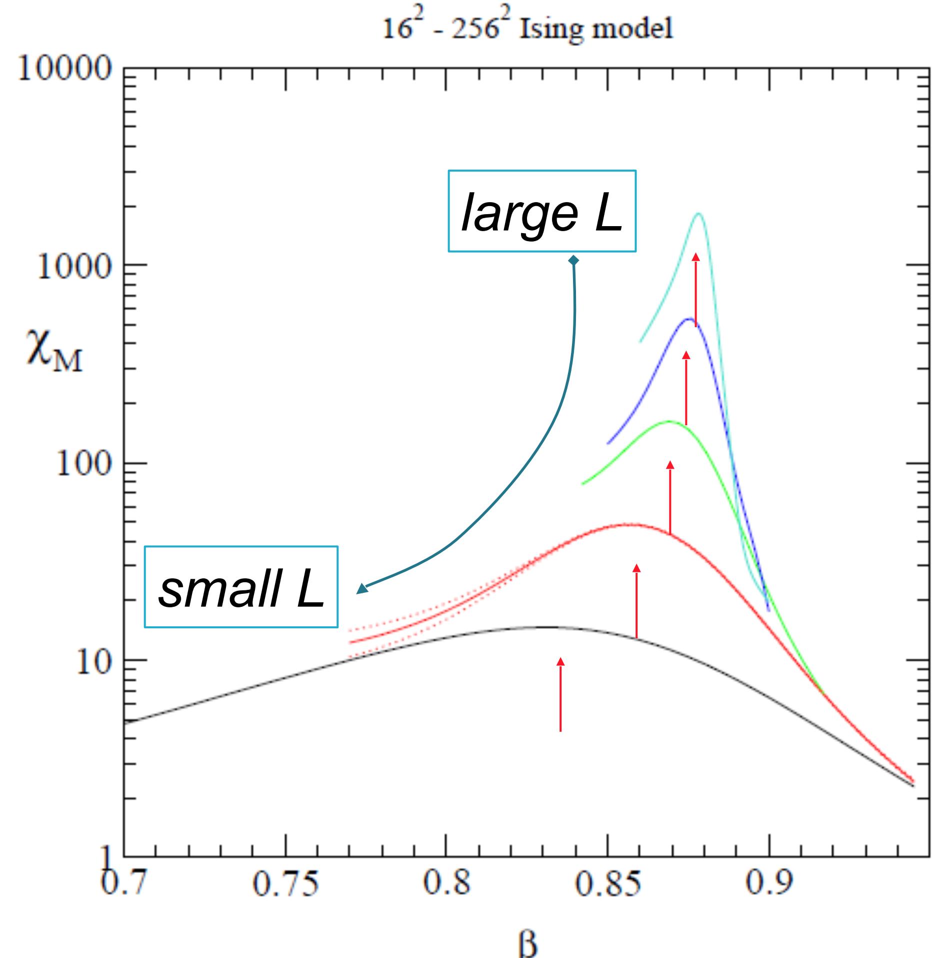


L characterizes the system size

T close to T_c



$$\xi \sim |T - T_c|^{-\nu} \leq L$$



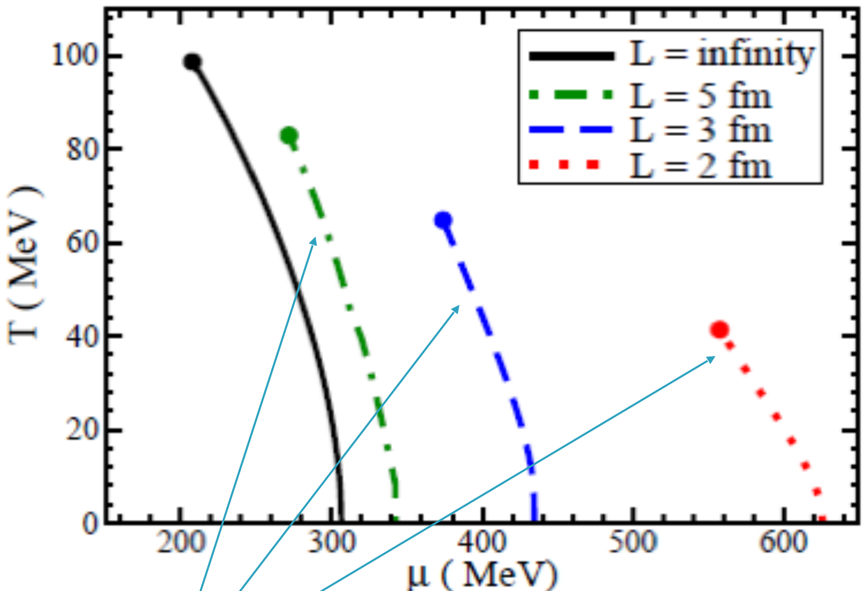
note change in peak heights, positions & widths

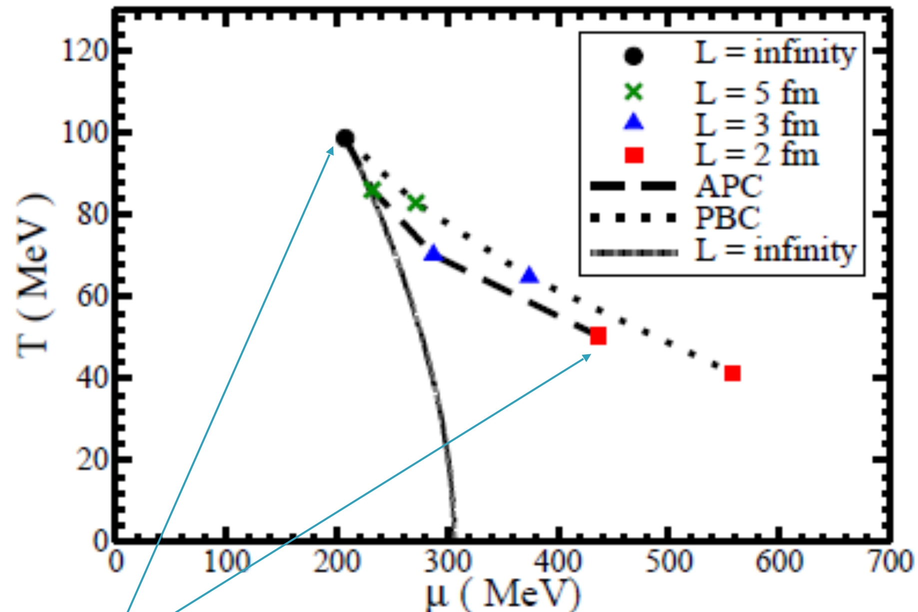
→ A curse of Finite-Size Effects (FSE)

→ Only a pseudo-critical point is observed → shifted from the genuine CEP

The curse of Finite-Size effects

E. Fraga et. al.
J. Phys.G 38:085101, 2011





Displacement of pseudo-first-order transition lines and CEP due to finite-size

Finite-size shifts both the pseudo-critical point and the transition line

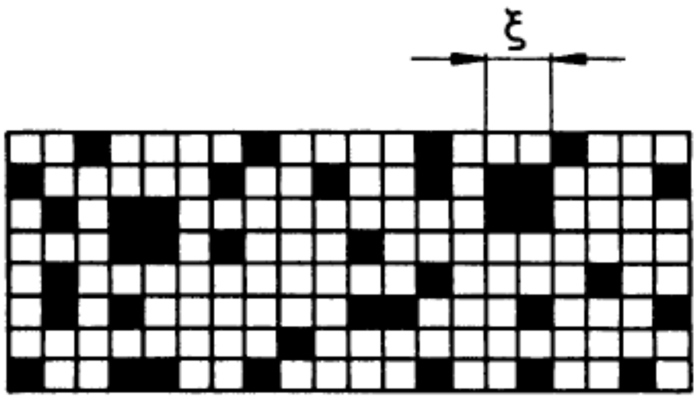
à *A flawless measurement, sensitive to FSE,* ***can*** ***<u>not</u>*** *give the precise location of the CEP*

The Blessings of Finite-Size

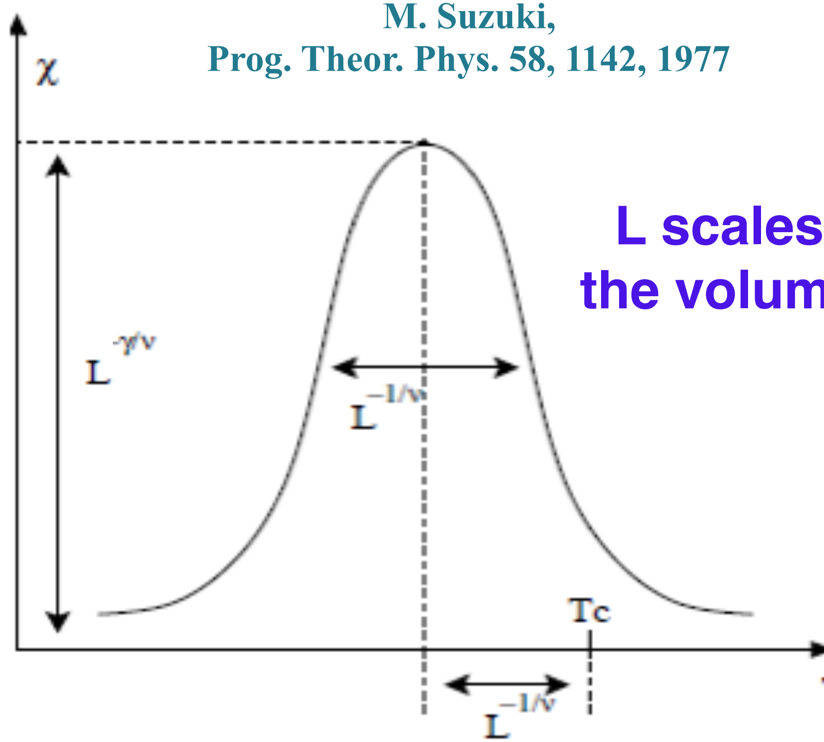
$$\chi_T^{\max}(V) \sim L^{\gamma/\nu},$$
$$\delta T(V) \sim L^{-\frac{1}{\nu}},$$
$$\tau_T(V) \sim T^{\text{cep}}(V) - T^{\text{cep}}(\infty) \sim L^{-\frac{1}{\nu}},$$

$$\chi(T,L) = L^{\gamma/\nu} P_\chi(tL^{1/\nu}) \qquad t = (T - T_c)/T_c$$

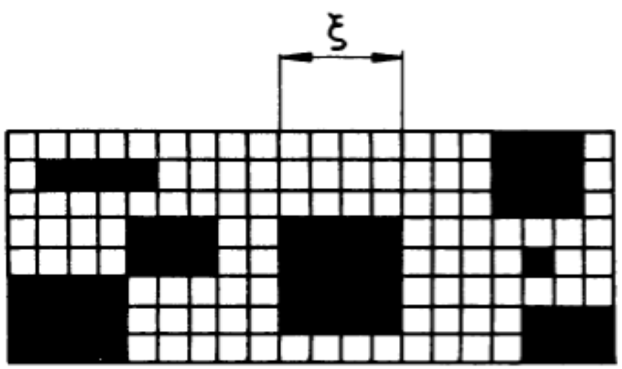
M. Suzuki,
Prog. Theor. Phys. 58, 1142, 1977



Finite-size effects have a specific L dependence



L scales the volume



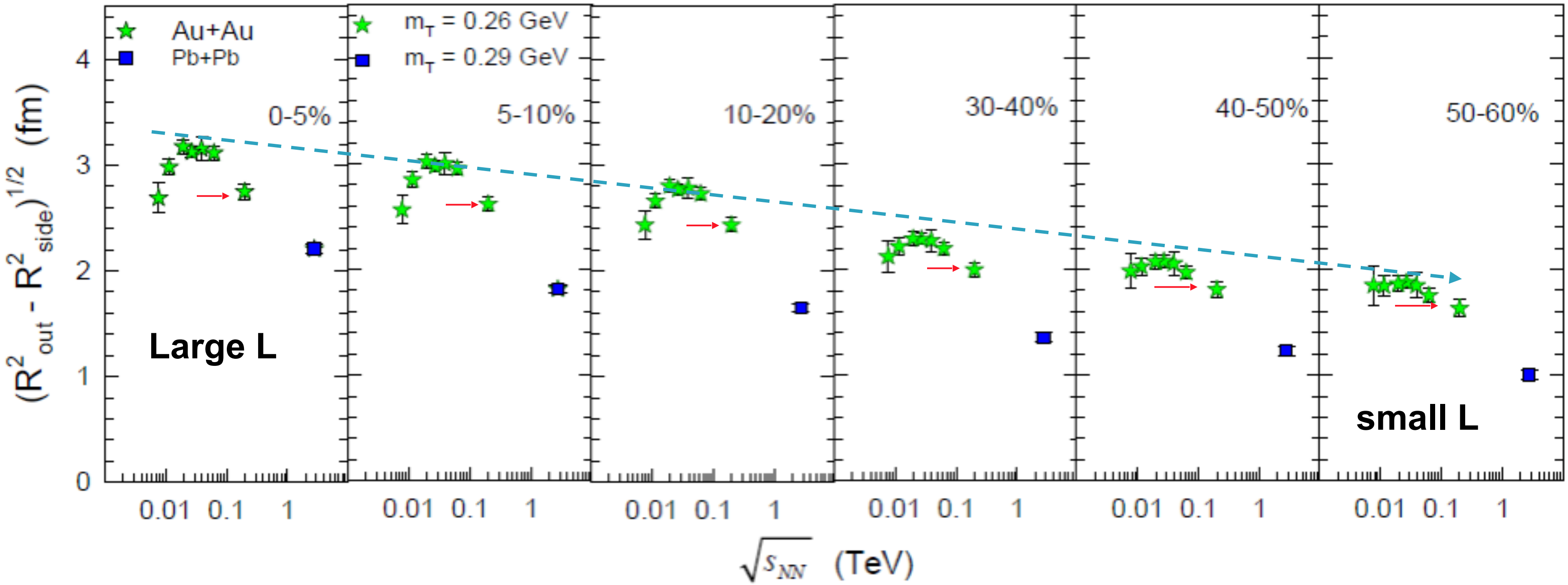
$$\xi \sim |T - T_c|^{-\nu} \leq L$$

- ✓ **Finite-size effects have specific identifiable dependencies on size (L)**
- ✓ **The scaling of these dependencies give access to the CEP's location and it's critical exponents**

Size dependence of HBT excitation functions

Roy A. Lacey
Phys.Rev.Lett. 114 (2015) 14, 142301

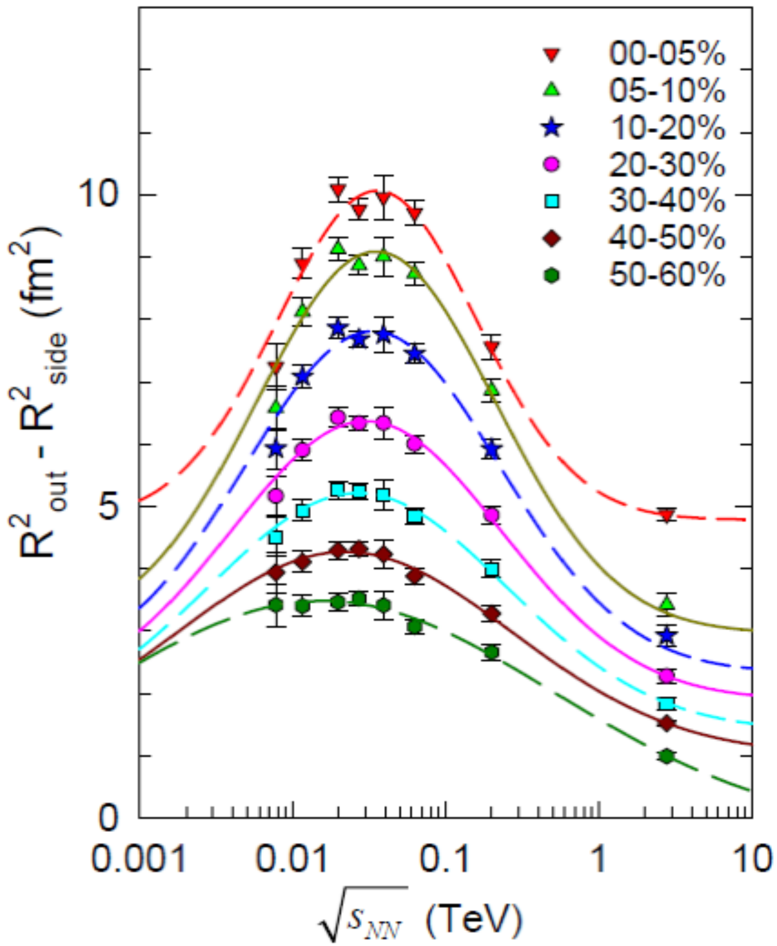
Data from L. Adamczyk et al. (STAR)
Phys.Rev. C92 (2015) 1, 014904

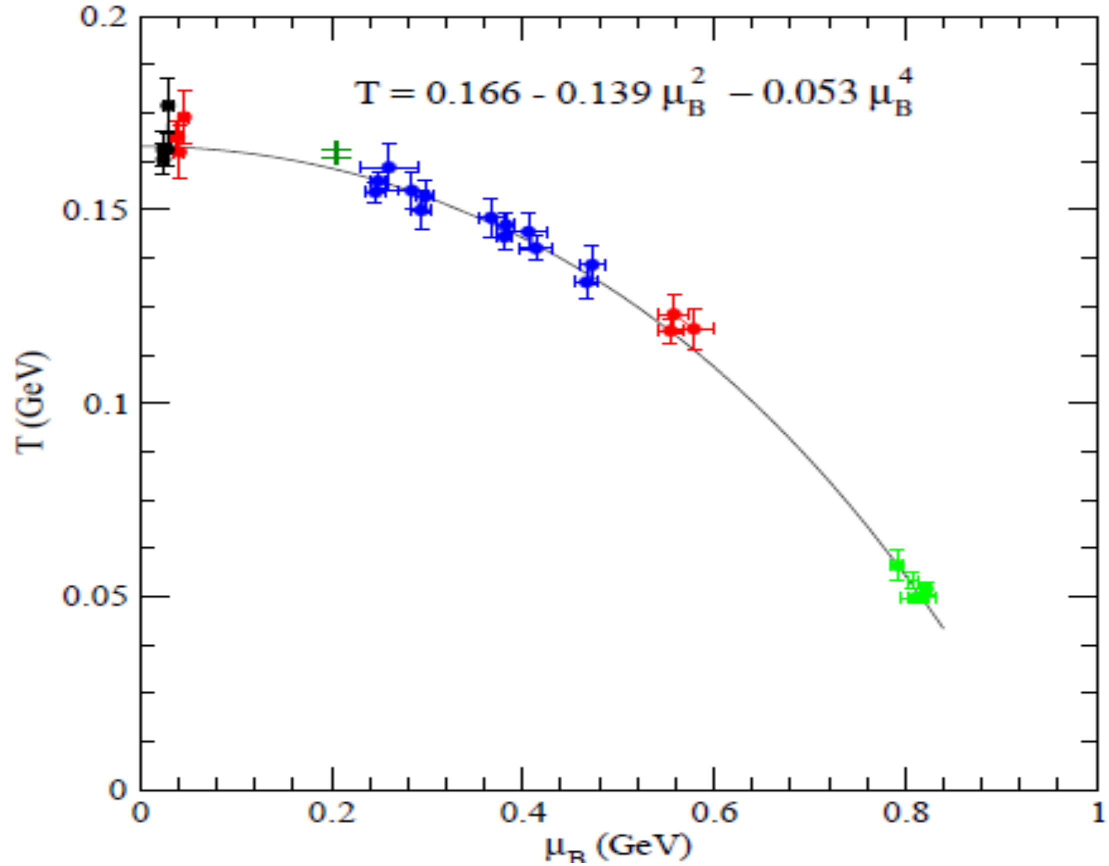


The data validate the expected patterns for Finite-Size Effects

- ✓ **Max values decrease with decreasing system size**
- ✓ **Peak positions shift with decreasing system size**
- ✓ **Widths increase with decreasing system size**

Size dependence of HBT excitation functions





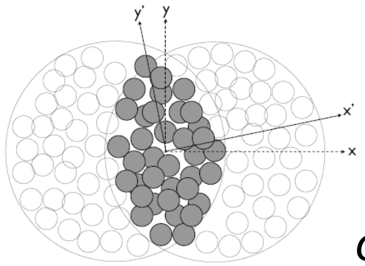
characteristic patterns signal the effects of finite-size

II. Parameterize distance to the CEP by $\sqrt{s_{NN}}$

$$\tau_s = \left(\sqrt{s} - \sqrt{s_{CEP}}\right)/\sqrt{s_{CEP}}$$

III. Perform Finite-Size Scaling analysis with length scale $L = \bar{R}$

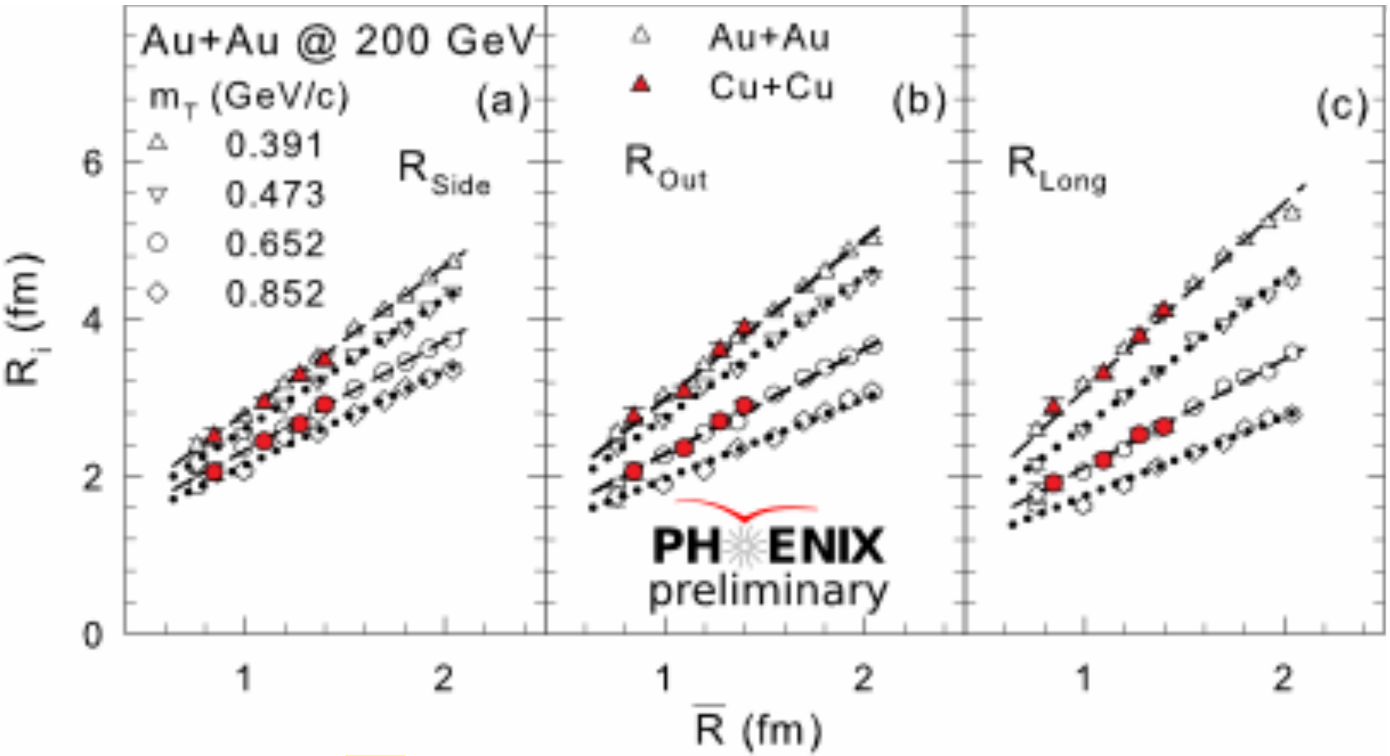
Length Scale for Finite Size Scaling

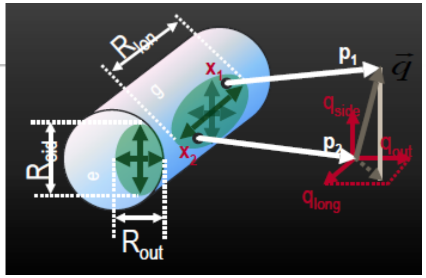


$$\frac{1}{\bar{R}} = \sqrt{\left(\frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2}\right)}$$

$\bar{R}$ is a characteristic length scale of the initial-state transverse size,

σ_x & σ_y → *RMS widths of density distribution*





$$R_{out}, R_{side}, R_{long} \propto \bar{R}$$

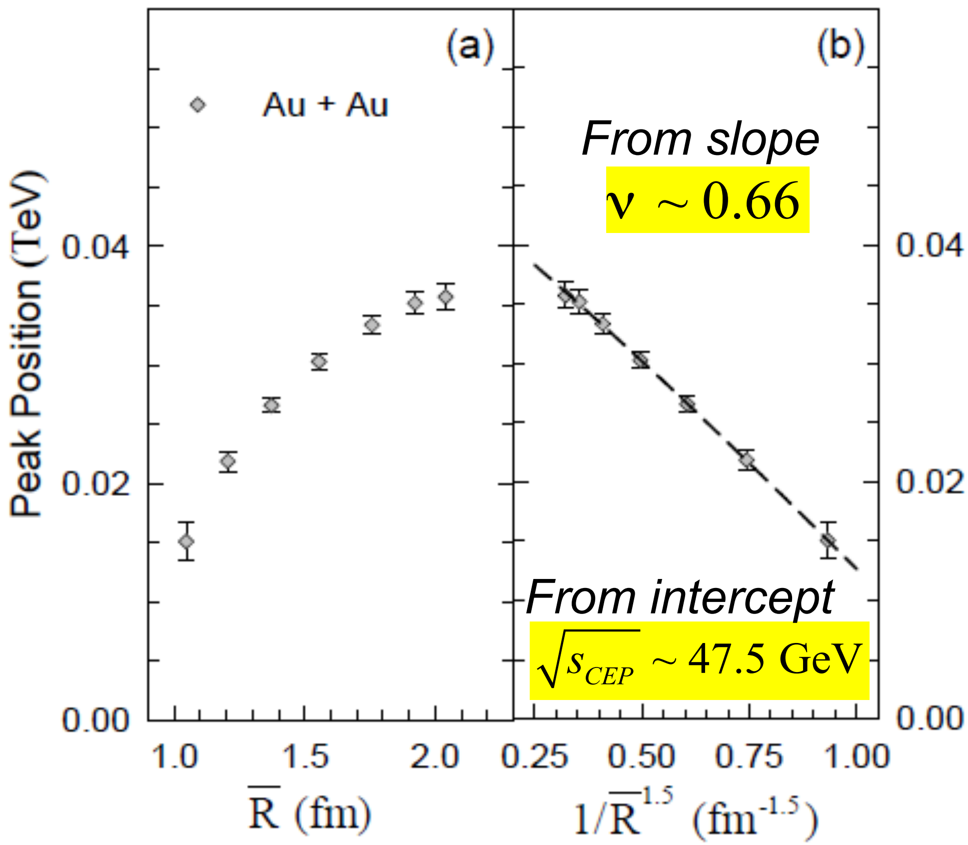
$\bar{R}$ **scales the volume**

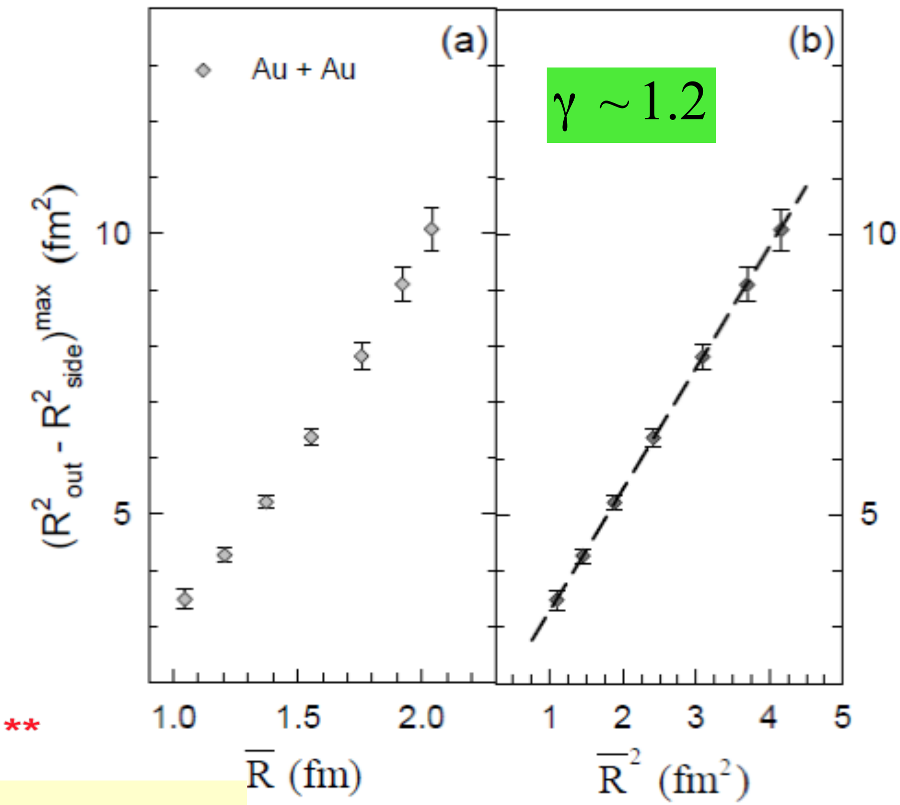
➢ $\bar{R}$ ***scales the full RHIC and LHC data sets***

Finite – Size Scaling

$$\sqrt{s_{NN}}(V) = \sqrt{s_{NN}}(\infty) - k \times \bar{R}^{-1/\nu}$$

$$\left(R^2_{out} - R^2_{side}\right)^{max} \propto \bar{R}^{\gamma/\nu}$$





**** Same ν value from analysis of the widths ****

- ➢ ***The critical exponents validate***
  - ✓ ***the 3D Ising model (static) universality class***
  - ✓ ***2nd order phase transition for CEP***

$$T^{cep} \sim 165 \text{ MeV}, \mu_B^{cep} \sim 95 \text{ MeV}$$

($\sqrt{s_{CEP}}$ & chemical freeze-out systematics)

Closurer test for FSS

- ***2^{nd} order phase transition***
- ***3D Ising Model (static) universality class for CEP***

$\nu \sim 0.66$ $\gamma \sim 1.2$

$T^{cep} \sim 165$ MeV, $\mu_B^{cep} \sim 95$ MeV

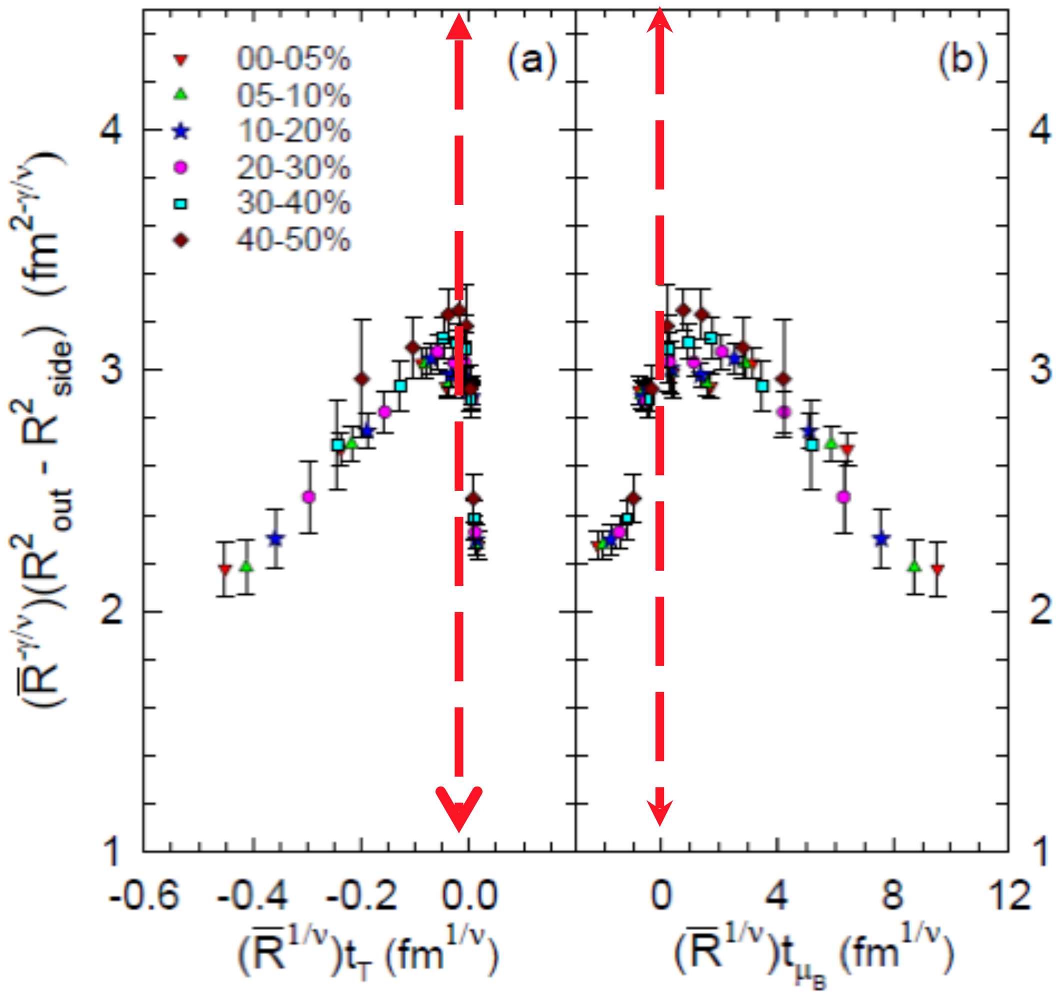
$$\chi(T,L) = L^{\gamma/\nu} P_\chi(tL^{1/\nu})$$

M. Suzuki,
Prog. Theor. Phys. 58, 1142, 1977

Use T^{cep}, μ_B^{cep}, ν and γ to obtain Scaling Function P_χ

$$R^{-\gamma/\nu} \times (R^2_{\rm out} - R^2_{\rm side}) \text{ vs. } R^{1/\nu} \times t_T,$$
$$\bar{R}^{-\gamma/\nu} \times (R^2_{\rm out} - R^2_{\rm side}) \text{ vs. } \bar{R}^{1/\nu} \times t_{\mu_B},$$
$$t_T = (T - T^{\rm cep})/T^{\rm cep}$$
$$t_{\mu_B} = (\mu_B - \mu_B^{\rm cep})/\mu_B^{\rm cep}$$

T anf μ_B *are* from $\sqrt{s_{NN}}$



*****A further validation of the location of the CEP and the (static) critical exponents*****

What about Finite-Time Effects (FTE)?

χ_{op} diverges at the CEP
so relaxation of the order parameter could be anomalously slow

$\tau \sim \xi^{z}$ ← dynamic critical exponent

$z > 0$ - Critical slowing down

Multiple slow modes?

$z_T \sim 3$, $z_v \sim 2$, $z_s \sim -0.8$

$z < 0$ - Critical speeding up

Y. Minami - **Xiv:1201.6408**

An important consequence

$$\xi \sim \tau^{1/z}$$

Significant signal attenuation for short-lived processes with $z_T \sim 3$ or $z_v \sim 2$

eg. $\langle(\delta n)\rangle \sim \xi^2$ (without FTE)
$\langle(\delta n)\rangle \ll \xi^2$ (with FTE)

The value of the dynamic critical exponent/s is crucial for HIC

Dynamic Finite-Size Scaling (DFSS) is used to estimate the dynamic critical exponent z

Dynamic Finite – Size Scaling

➢ **2nd order phase transition**

$\nu \sim 0.66$ $\gamma \sim 1.2$

$T^{cep} \sim 165$ MeV, $\mu_B^{cep} \sim 95$ MeV

DFSS ansatz

at time τ when T is near T_{cep}

$$\chi(L,T,\tau) = L^{\gamma/\nu} f\left(L^{1/\nu} t_T, \tau L^{-z}\right)$$

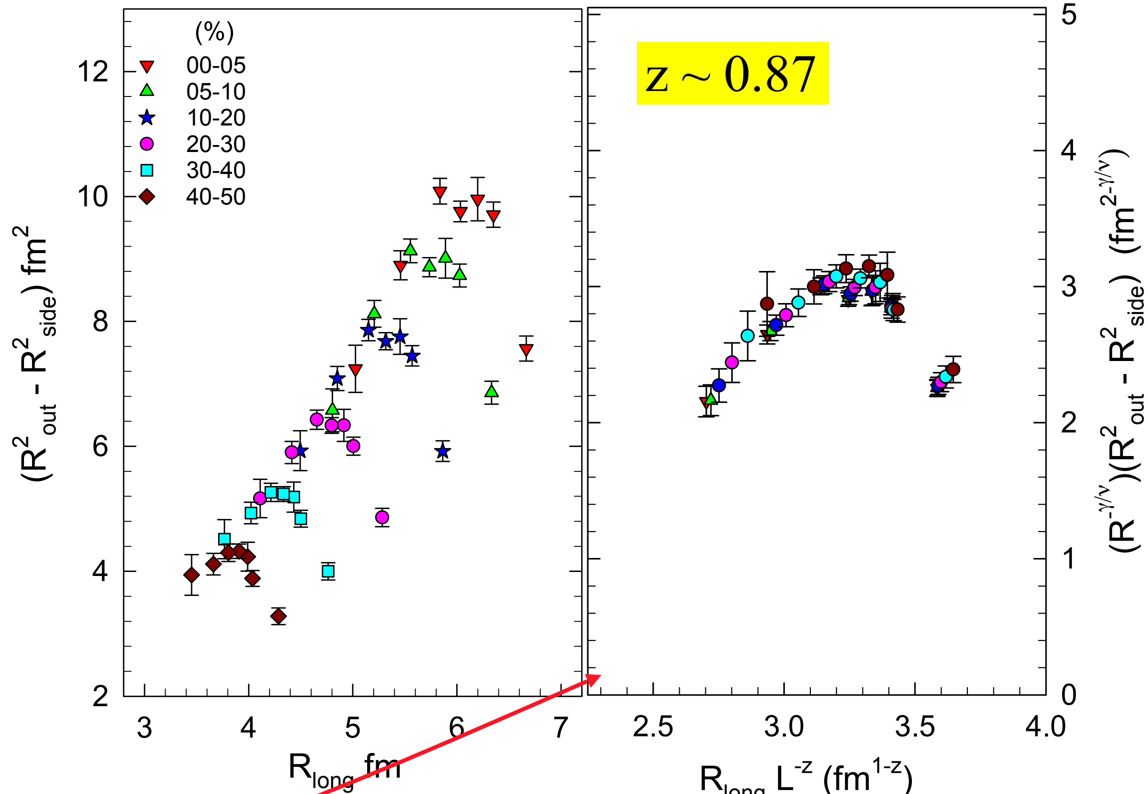
For $T = T_c$

$$\chi(L,T_c,\tau) = L^{\gamma/\nu} f\left(\tau L^{-z}\right)$$

M. Suzuki,
Prog. Theor. Phys. 58, 1142, 1977

$R_{long} \propto \tau$

****Experimental estimate of the dynamic critical exponent****



The magnitude of z is similar to the predicted value for z_s but the sign is opposite

Epilogue

Strong experimental indication for the CEP and its location

(Dynamic) Finite-Size Scalig analysis

- **3D Ising Model (static) universality class for CEP**
- **2nd order phase transition**

$T^{cep} \sim 165$ MeV, $\mu_B^{cep} \sim 95$ MeV

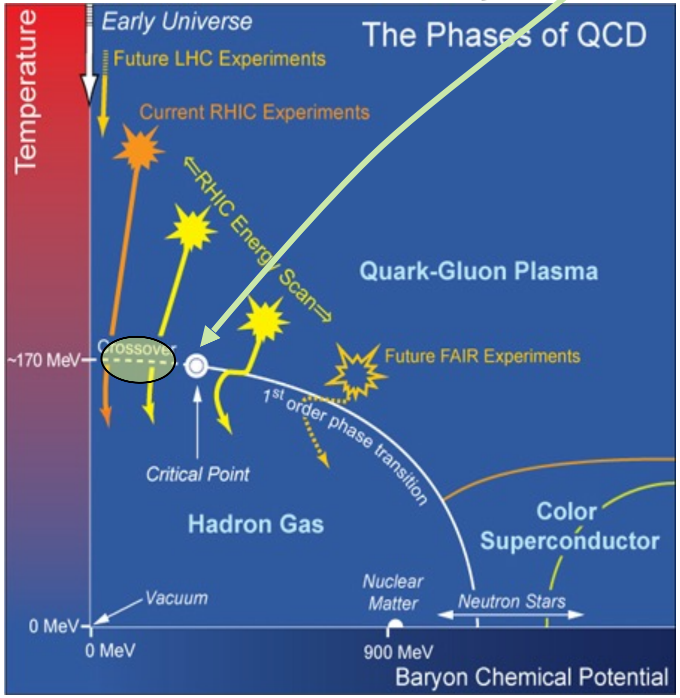
$\nu \sim 0.66$

$\gamma \sim 1.2$

$z \sim 0.87$

New Data from RHIC (BES-II) together with theoretical modeling, will provide crucial validation tests for the coexistence regions, as well as to firm-up characterization of the CEP!

✓ Landmark validated
✓ Crossover validated
✓ Deconfinement validated
✓ (Static) Universality class validated
✓ Model H Universality class invalidated?
✓ Other implications!



Much additional work required to get to "the end of the line"

End

Finite – Size Scaling Analysis

(only two exponents are independent)

$$\chi_T^{\max}(V) \sim L^{\gamma/\nu},$$

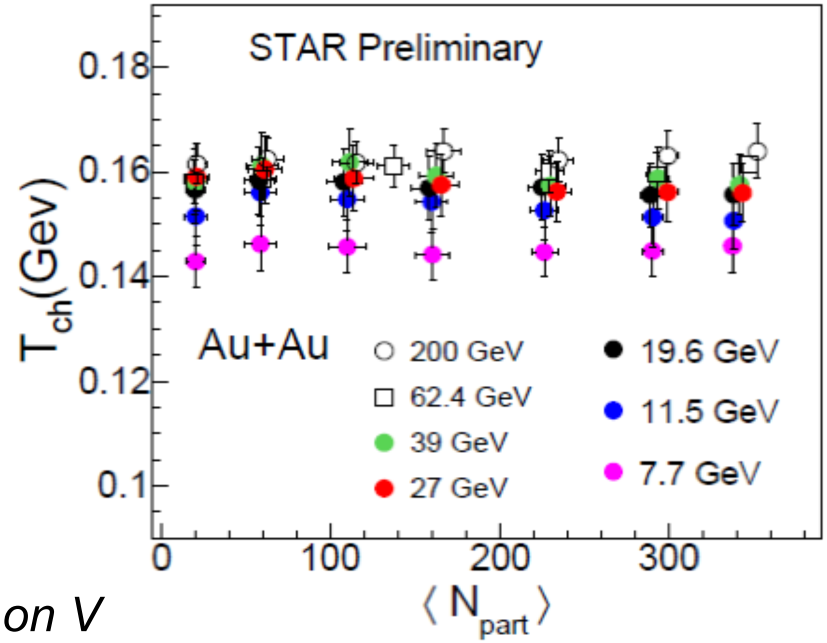
$$\delta T(V) \sim L^{-\frac{1}{\nu}},$$

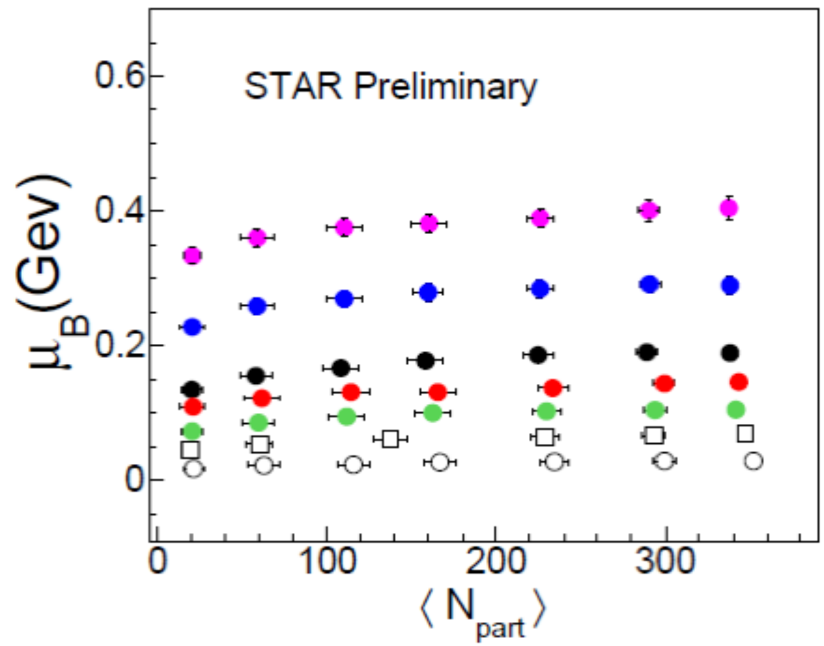
$$\tau_T(V) \sim T^{\text{cep}}(V) - T^{\text{cep}}(\infty) \sim L^{-\frac{1}{\nu}},$$

$$(R^2_{\text{out}} - R^2_{\text{side}})^{\max} \propto \bar{R}^{\gamma/\nu},$$

$$\sqrt{s_{NN}}(V) = \sqrt{s_{NN}}(\infty) - k \times \bar{R}^{-\frac{1}{\nu}},$$

Note that (μ_B^f, T^f) *is not strongly dependent on V*





Phys.Rev.Lett.100:232301,2008)

Source breakup dynamics in Au+Au Collisions at $\sqrt{s_{NN}}$=200 GeV via three-dimensional two-pion source imaging

S. Afanasiev,[17] C. Aidala,[7] N.N. Ajitanand,[43] Y. Akiba,[37,38] J. Alexander,[43] A. Al-Jamel,[33] K. Aoki,[23,37] L. Aphecetche,[45] R. Armendariz,[33] S.H. Aronson,[3] R. Averbeck,[44] T.C. Awes,[34] B. Azmoun,[3] V. Babintsev,[14] A. Baldisseri,[8] K.N. Barish,[4] P.D. Barnes,[26] B. Bassalleck,[32] S. Bathe,[4] S. Batsouli,[7] V. Baublis,[36] F. Bauer,[4] A. Bazilevsky,[3] S. Belikov,[3,16,*] R. Bennett,[44] Y. Berdnikov,[40] M.T. Bjorndal,[7] J.G. Boissevain,[26] H. Borel,[8]

Phys.Lett. B685 (2010) 41-46

Three-dimensional two-pion source image from Pb+Pb collisions at $\sqrt{s_{NN}}$=17.3 GeV: new constraints for source breakup dynamics

C. Alt[9], T. Anticic[23], B. Baatar[8],D. Barna[4], J. Bartke[6], L. Betev[10], H. Białkowska[20], C. Blume[9], B. Boimska[20], M. Botje[1], J. Bracinik[3], P. Bunčić[10], V. Cerny[3], P. Christakoglou[1], P. Chung[19], O. Chvala[14], J.G. Cramer[16], P. Csató[4], P. Dinkelaker[9], V. Eckardt[13], D. Flierl[9], Z. Fodor[4], P. Foka[7], V. Friese[7], J. Gál[4], M. Gaździcki[9,11], V. Genchev[18], E. Gładysz[6], K. Grebieszkow[22], S. Hegyi[4], C. Höhne[7], K. Kadija[23], A. Karev[13], S. Kniege[9], V.I. Kolesnikov[8], R. Korus[11], M. Kowalski[6], M. Kreps[3], A. Laszlo[4], R. Lacey[19], M. van Leeuwen[1], P. Lévai[4], L. Litov[17], B. Lungwitz[9], M. Makariev[17], A.I. Malakhov[8], M. Mateev[17], G.L. Melkumov[8],

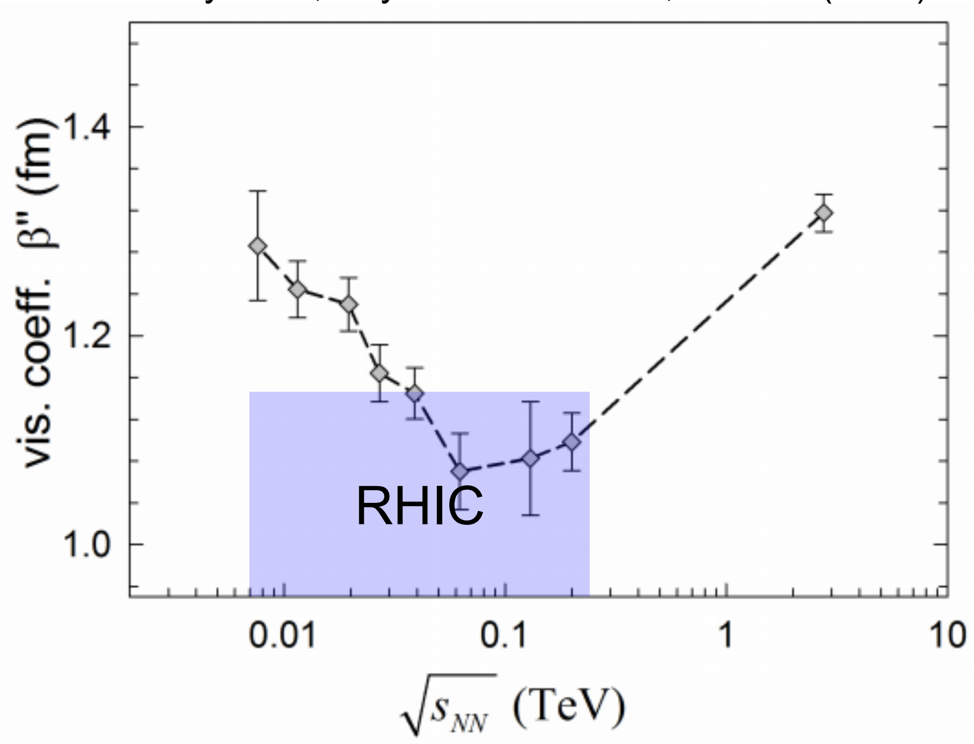
$$\tau = \tau_0 + a\rho$$

Space-time correlation parameter

Non Monotonic behavior of the viscous coefficient

- ✓ **Initial experimental indication for *η/s variation in the* (T, μ_B)-plane**
- ✓ **CEP?**

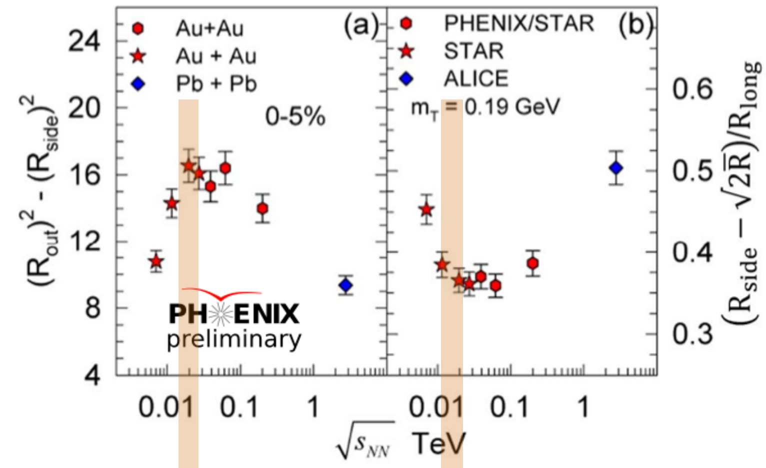
R. Lacey *et al.*, Phys. Rev. Lett. 112, 082302 (2014)

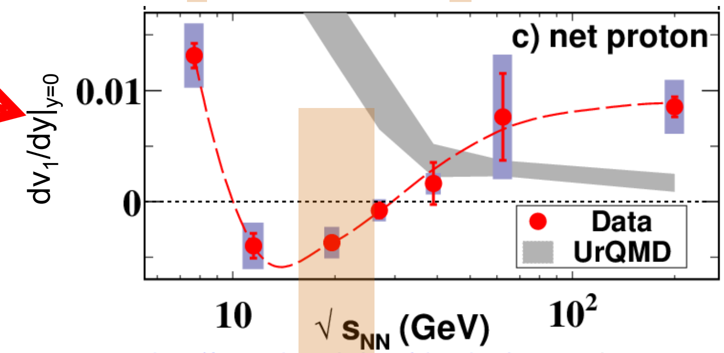


$\sqrt{s_{NN}}$ *dependence of various signals*

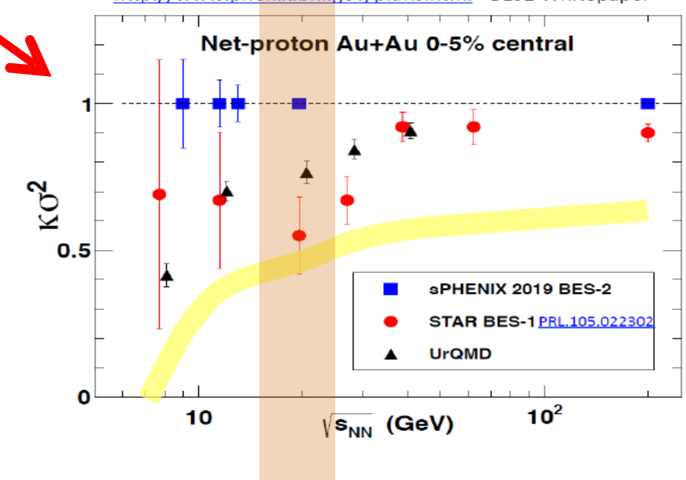
- Systematic study of various quantities as function of $\sqrt{s}$:
  - Source radii
  - Directed flow v_1
  - Scaled kurtosis (baryon fluctuations)
- Suggestive non-monotonic behavior at a ~common $\sqrt{s}$

☞ *Compelling* need for RHIC Beam Energy Scan II
  - Greatly increase statistical precision
  - Additional $\sqrt{s}$ points





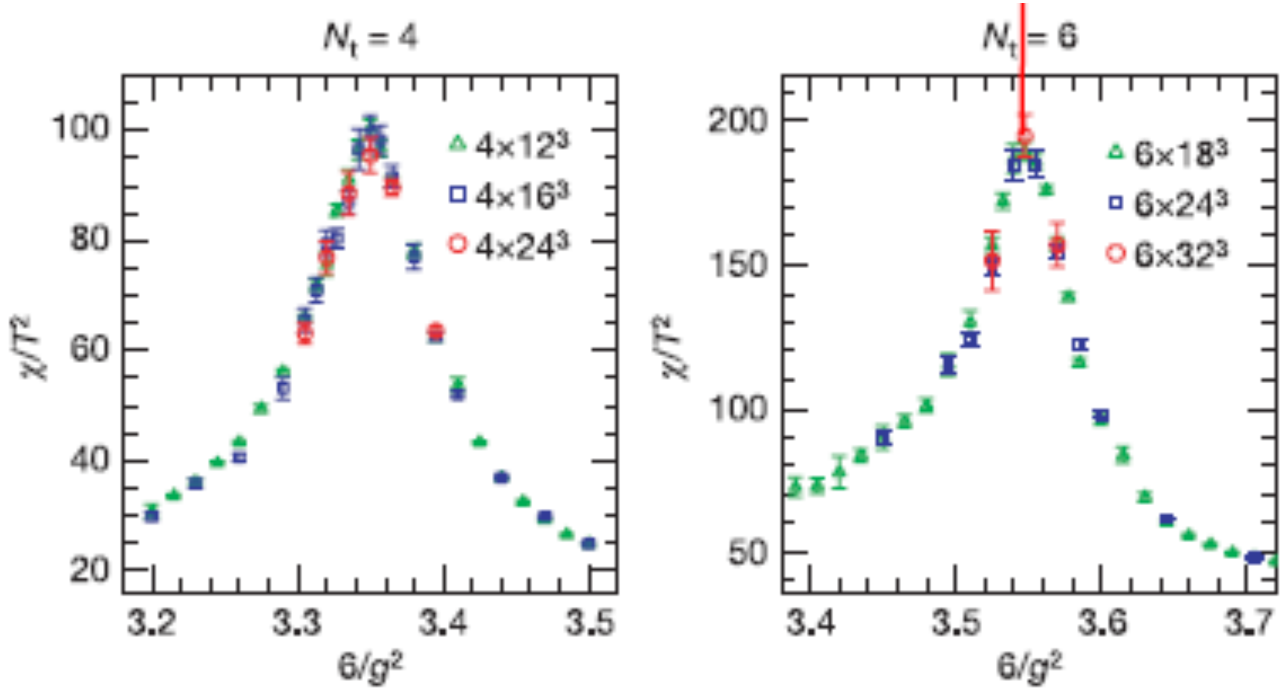
http://www.phenix.bnl.gov/plans.html - BES2 Whitepaper



Finite size scaling and the Crossover Transition

Finite size scaling played an essential role for identification of the crossover transition!

Y. Aoki, et. Al.,Nature , 443, 675(2006).



Reminder

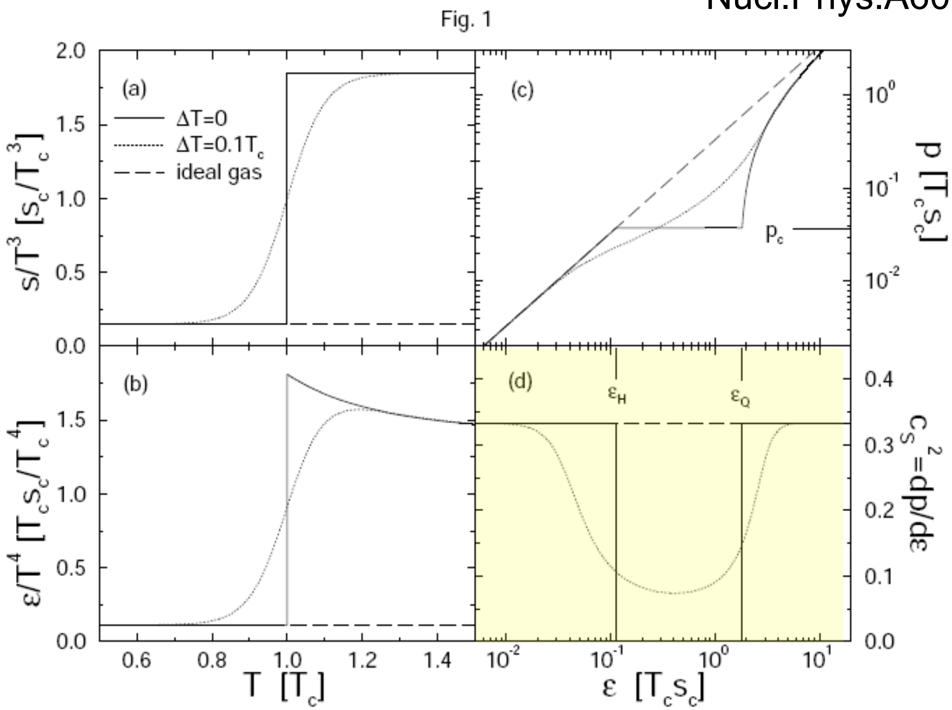
Crossover: size independent.

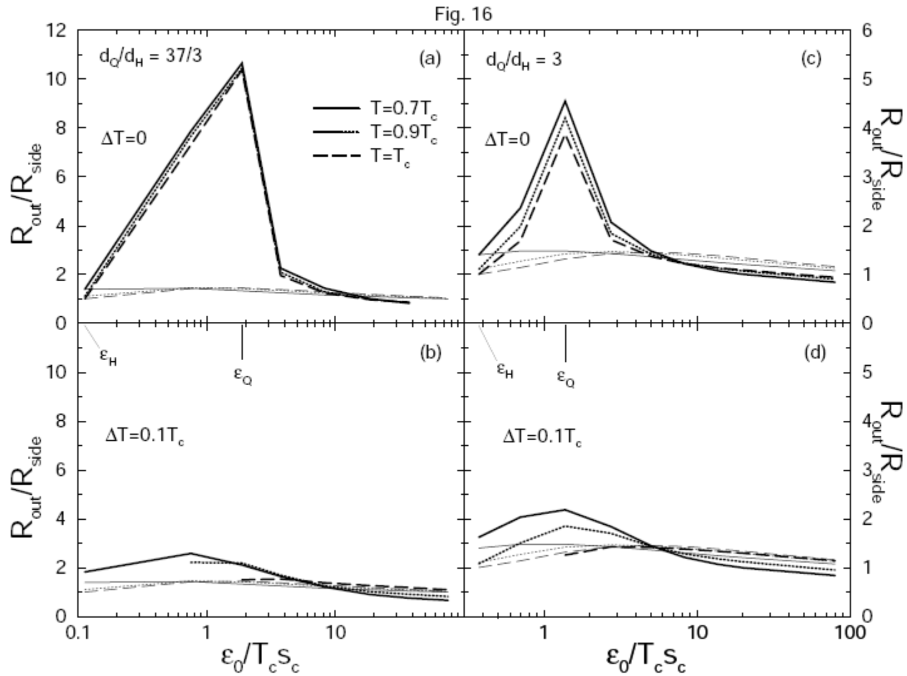
1st-order: finite-size scaling function, and scaling exponent is determined by spatial dimension (integer).

2nd-order: finite-size scaling function $\chi(T,L) = L^{\gamma/\nu} P_{\chi}(tL^{1/\nu})$

Interferometry as a susceptibility probe

Dirk Rischke and Miklos Gyulassy
Nucl.Phys.A608:479-512,1996





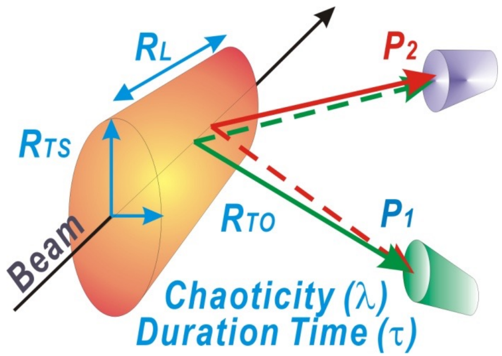
In the vicinity of a phase transition or the CEP, the sound speed is expected to soften considerably.

$$c_s^2 = \frac{1}{\rho \kappa_S}$$

Divergence of the compressibility (κ)

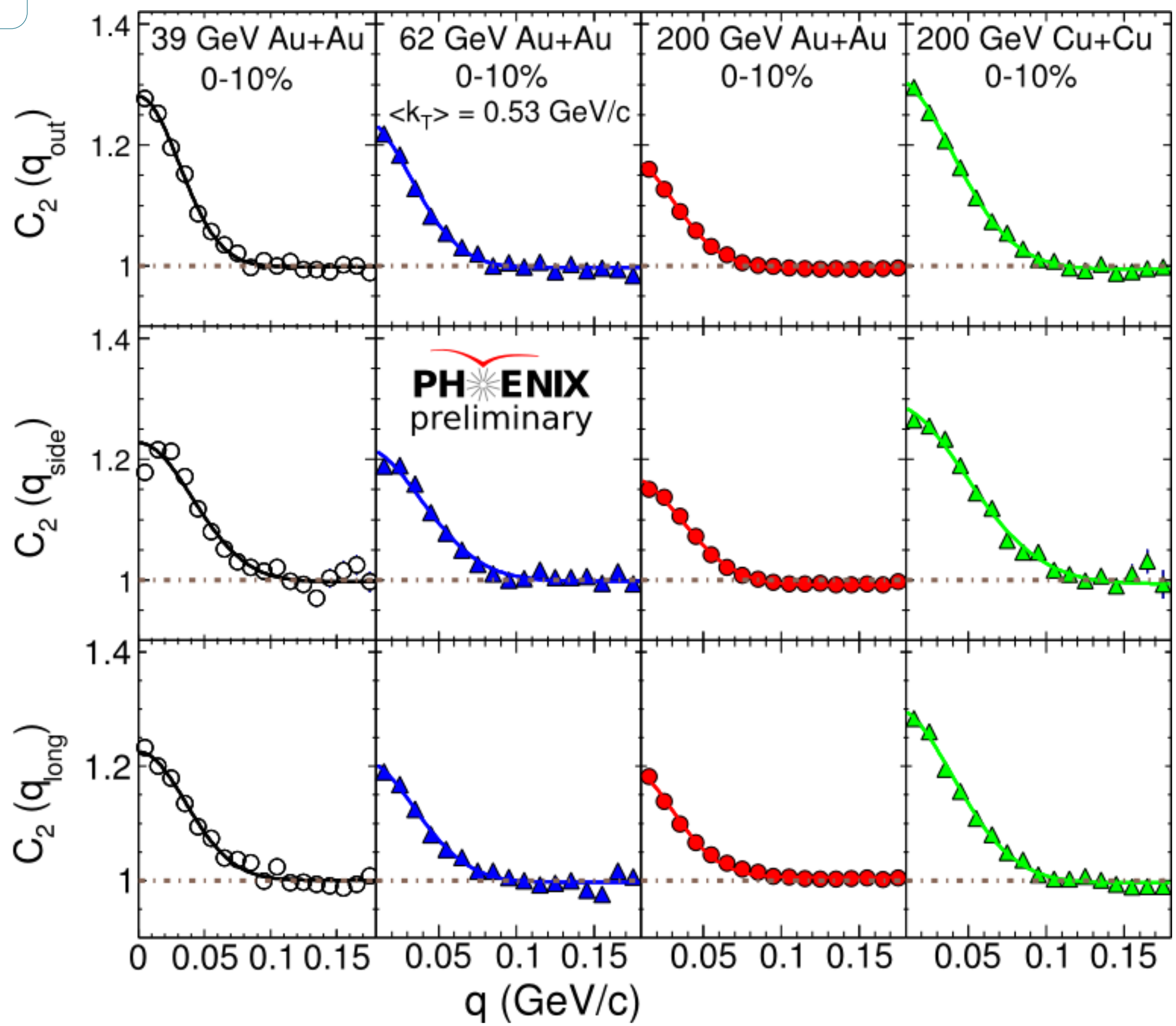
→ non-monotonic excitation function for (R^2_{out} - R^2_{side}) *due to an enhanced emission duration*

Interferometry signal



$$C(\mathbf{q}) = \frac{dN_2 / d\mathbf{p}_1 d\mathbf{p}_2}{(dN_1 / d\mathbf{p}_1)(dN_1 / d\mathbf{p}_2)}$$

Adare et. al. (PHENIX)
arXiv:1410.2559

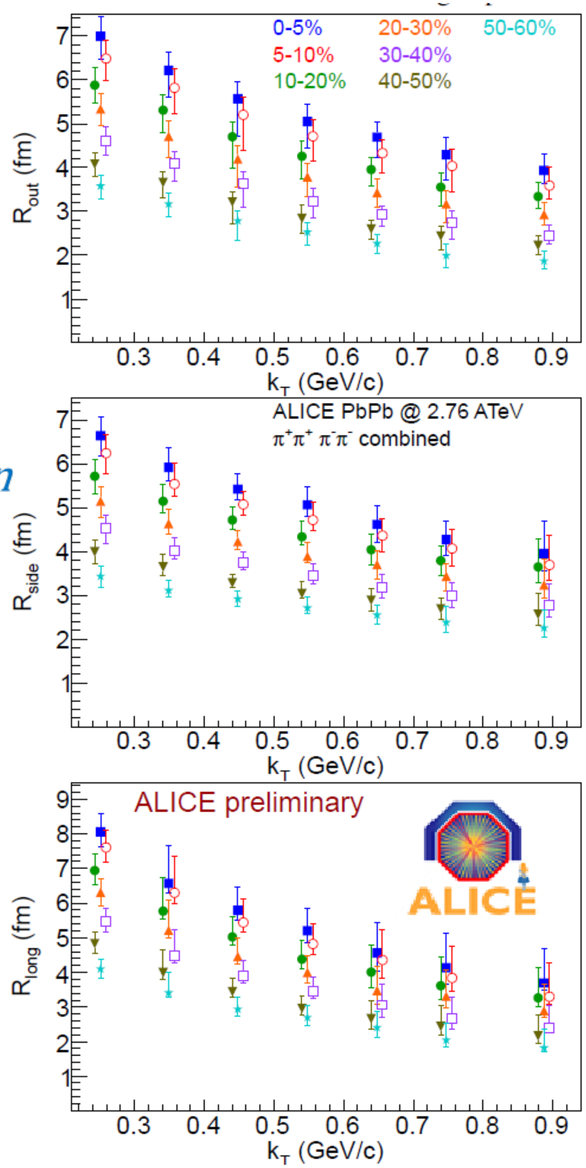


$$C_2(\mathbf{q}) = N[(\lambda(1 + G(\mathbf{q})))F_c + (1 - \lambda)],$$
$$G(\mathbf{q}) \cong \exp(-R^2_{\text{side}} q^2_{\text{side}} - R^2_{\text{out}} q^2_{\text{out}} - R^2_{\text{long}} q^2_{\text{long}})$$

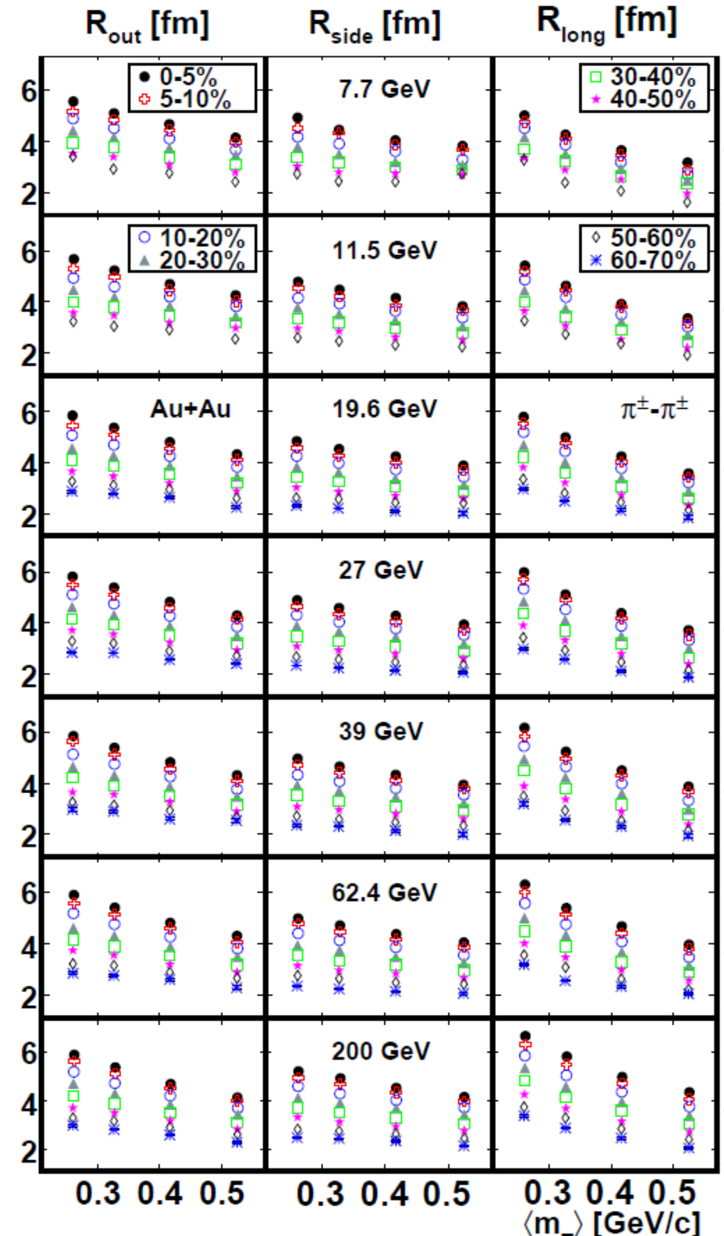
HBT Measurements

These data are used to search for non-monotonic patterns as a function of $\sqrt{s_{NN}}$

ALICE -PoS**WPCF2011**



STAR - 1403.4972



Exquisite data set for study of the HBT excitation function!

Scaling properties of HBT

Viscous Hydrodynamics – B. Schenke

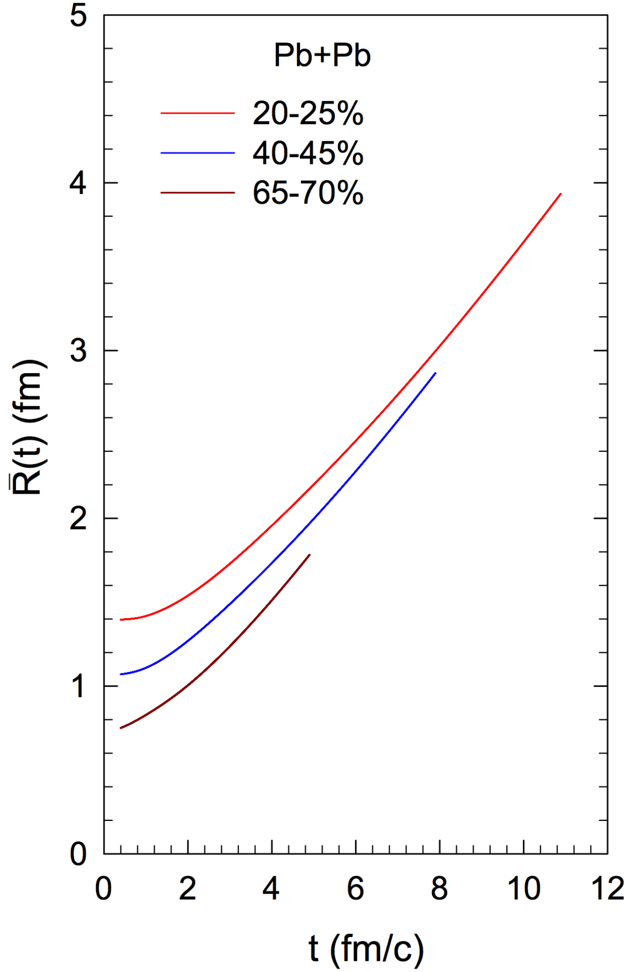
Freezeout size reflects;

- ➢ **Initial size**
- \+ **expanded size**

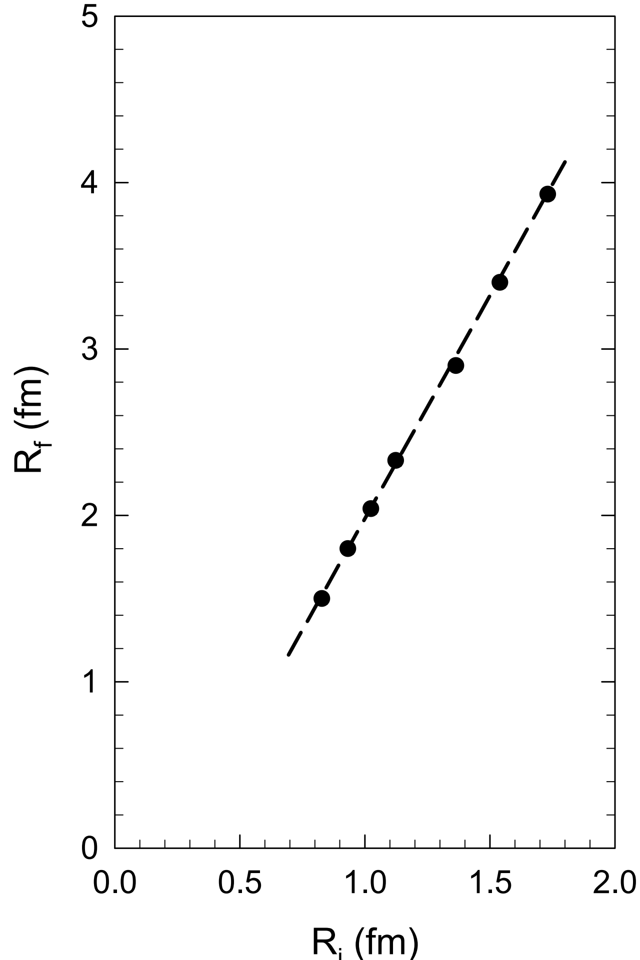
Characteristic acoustic Scaling validated in Viscous hydrodynamics

$t \propto \bar{R}$

E. Shuryak, I. Zahed
Phys.Rev. D89 (2014) 094001

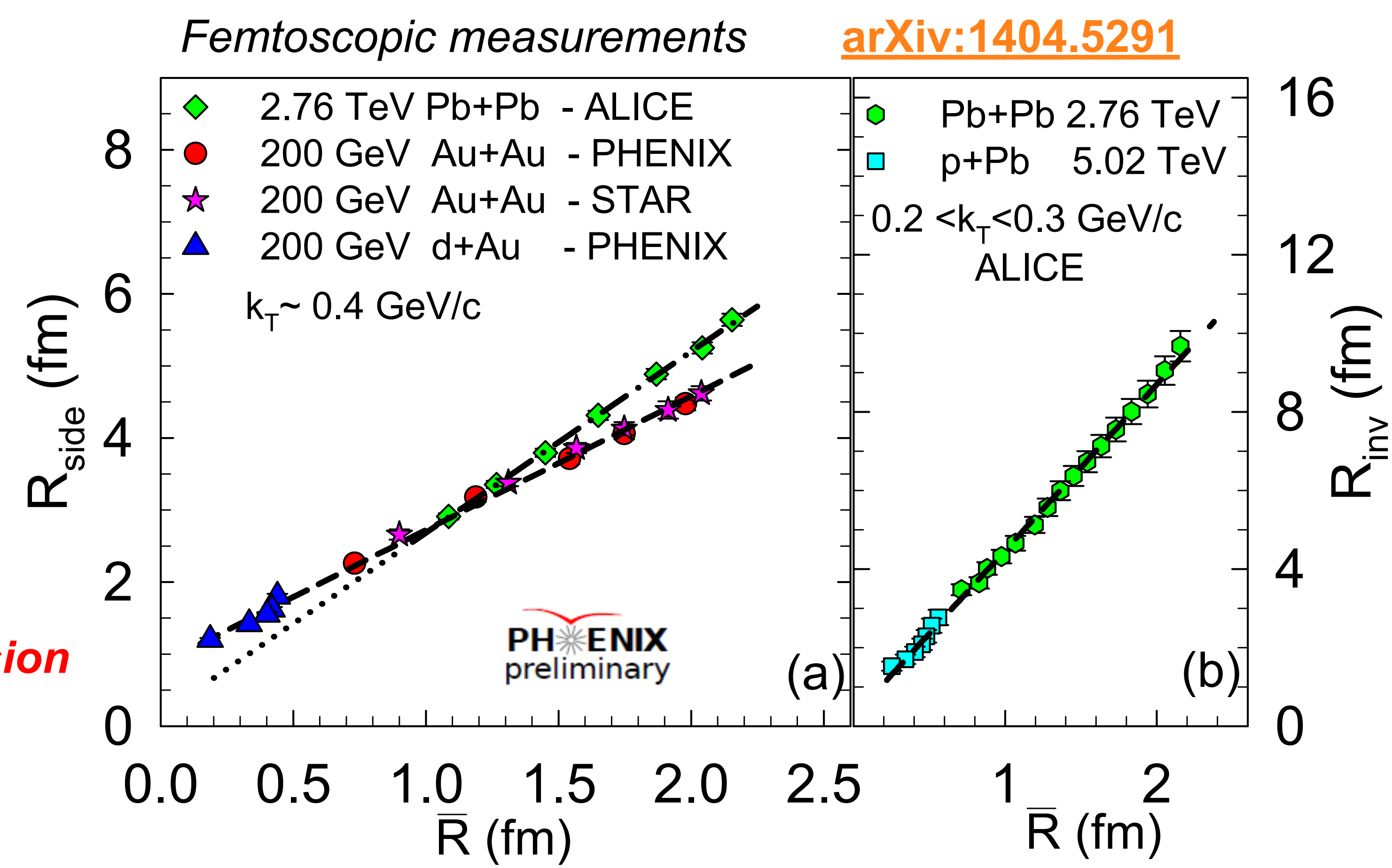


Freeze-out time varies with initial size



Freeze-out transverse size proportional to initial transverse size

Acoustic Scaling of HBT Radii



➢ ***Larger expansion rate at the LHC***

$t \propto \bar{R}$ ***exquisitely demonstrated via HBT measurement for several systems***