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Studies of Radiative Decays of the W and Z

Marcel Demarteau

John Sculli

Kathleen Streets

1 Introduction

This note describes studies to determine the effect of radiative decays of the W and Z on the measurement of the W mass. The masses of the W and Z are obtained by fitting the experimental distributions (transverse mass for the W and the ee invariant mass for the Z) to a Monte Carlo simulation using a maximum likelihood method. The values presented below for the W and Z masses are not the DØ measurements of these quantities and should not be interpreted as such. Only the changes in the masses induced by the radiative processes are of interest.

2 Cases Considered

Radiative W and Z decays,

$$q\bar{q} \to \mathrm{Z} \to \ell\ell(\gamma)$$

and

$$q'\bar{q} \to \mathrm{W} \to \ell\nu(\gamma),$$

must be properly simulated in the Monte Carlo program to extract correct values for the W and Z masses. Because the $\ell\ell$ (Z) and $\ell\nu$ (W) invariant masses are smaller in these decays than the corresponding vector boson masses, the experimentally measured mass distributions are shifted toward lower values. The rates and distributions in lepton and photon momenta calculated to order α by Berends and Kleiss [1] were added to the Monte Carlo simulation. In the decay of the Z, either of the electrons (but not both) may radiate. In W decay, the electron or W may radiate the photon. Approximately 30% of W $\to e\nu$ decays and 60% of Z $\to e^+e^-$ decays have a photon of 50 MeV or more in the final state. The calculation does not include processes that in the limit of a zero width boson would be considered Wγ or Zγ production. Therefore, initial state (quark) radiation is not included in the calculation, nor is the production of a virtual high mass W which decays to an on-shell W and a photon. In Wγ and Zγ production, the W and Z masses are correctly obtained from the dilepton invariant masses ($e\nu$ or ee) and the γ direction is not strongly correlated with that of either lepton. Its presence produces a background not fundamentally different from that of a myriad of other processes.

The case of external bremsstrahlung in the beam pipe material and the central detector was calculated and determined not to affect the W or Z mass measurement. This is because the photon never gets far enough away from the electron to cause any mismeasurement in energy. The case when the electron radiates a photon which carries most of the electron energy was also considered. The resulting low energy electron would then multiple scatter away from the photon, but still produce a track in the road. This case was also determined to have a negligible effect on the W mass.

3 Experimental Considerations

In implementing radiative decays in the Monte Carlo simulation, three experimental scenarios were considered. We discuss $W^{\pm}$ decay below. Similar considerations apply to the decays of the Z.

When the γ is produced inside the electron (e) cone (cone size radius $R = .2$ in $\eta - \phi$ space), the γ is measured as part of the electron. The neutrino momentum, obtained from the missing P_T in the event, is calculated correctly. Therefore, the invariant mass of the $e\nu$ system is the W mass, and the transverse mass and transverse momentum of the W are properly calculated. If the γ is far from the electron (i.e. outside a cone of $R = .4$) the electron energy is measured correctly, and the γ P_T becomes part of the recoil against the W, $\vec{P}_T^W(meas) = \vec{P}_T^W(generated) - \vec{P}_T^\gamma$. The transverse mass of the $e\nu$ system is calculated correctly, but is shifted downward because the $e\nu$ invariant mass is smaller than the W mass. Therefore, M_W is mismeasured. In the third case, the γ is produced in the region between $R = .2$ and $R = .4$, altering the shape of the shower. Isolation and H-matrix cuts then result in inefficiencies that can affect the W mass if not properly simulated in the Monte Carlo.

In the Monte Carlo simulation, the fraction of the electron's energy in the region between R=.2 and R=.4 is generated according to the experimental distribution measured in W events. The photon energy is added to the electron's and the event discarded if it fails the isolation cut. If the event survives the isolation cut and the radial distance R between the γ and e is less than $R_{e\gamma}$(nominal value of $R_{e\gamma} \equiv 0.3$), the γ momentum is added to the electron's and the W mass is correctly calculated, as in the first case above. If the radial distance is greater than $R_{e\gamma}$, the photon energy is not added to the electron's and the reconstructed W mass and transverse momentum are shifted downward. The dependence of the W mass on $R_{e\gamma}$ for $.2 < R_{e\gamma} < .4$ is small and given in section 5.1 below.

4 Monte Carlo Simulation of W Production and Decay

After incorporating the Berends and Kleiss calculation into the Monte Carlo simulation program, twenty-one distributions in transverse mass were generated at equally spaced values of the W mass between 79.0 and 81.0 GeV . These curves, obtained from large Monte Carlo samples, were regarded as theory and compared to either the experimental data or smaller MC samples (mock data) in which one or more of the parameters related to the radiative process was altered.

The theoretical distributions were generated using two different detector simulations. In the first (detector smearing), the electron energy and W transverse momentum were smeared using simple resolution functions and the distribution in $\not{E}_T$ was taken from the distribution measured in W events. In the second ("toy" calorimeter smearing [2]), a more complete simulation was done using minimum bias events to model the event underlying the W and the pileup associated with multiple interactions, including the residual energy left over from previous crossings.

Several sets of theoretical distributions were generated and are described below. In each case, transverse mass distributions were generated for each of the 21 W mass values. When radiative decays are modelled in the theoretical distributions, the minimum photon energy, $E_\gamma(\min)$, is set to 50 MeV, which is taken as the nominal value.

Set 1 Using detector smearing, five million events were generated at each of the 21 values of the W mass separated by 0.1 GeV intervals between 79.0 and 81.0 GeV . Approximately 1.2 million events survived the cuts at each point and were used in the fits. The sample includes radiative decays.

Set 2 Two samples, one with radiation and one without, were generated using the toy calorimeter simulation. In both samples, twenty million events were generated in 100 MeV intervals for each of the twenty-one W mass points. Approximately 3.5(3.6) million events survived the cuts and were used in the fits when comparing to the theory with(without) radiation.

Set 3 Events were generated without radiation at 50 MeV intervals between 79.5 and 80.5 GeV , using the toy calorimeter simulation. Twenty million events were generated at each point, of which approximately 3.6 million survived the cuts and were used in the fits.

5 Results of Studies - W mass

We summarize the results of the studies in a series of tables. Either actual DØ data or data generated by the Monte Carlo program (MC data) were used to compare with the theoretical distributions to determine the sensitivity of the W mass to the radiative processes. The results presented below were obtained by fitting the spectrum in transverse mass between 60 and 90 GeV to the theoretical distributions. The analysis was done for events in the CC only. The errors listed are statistical only.

5.1 Monte Carlo Data

The Monte Carlo studies include: (i) a comparison of high statistics MC data that includes radiative decays to a theoretical sample that does not, (ii) a determination of the sensitivity to the cut on the photon-electron separation, $R_{e\gamma}$, (iii) a determination of the dependence of the W mass on the minimum photon energy, and (iv) a study of the dependence on the percentage of radiative decays. In all cases, when comparing MC data to theory generated under the same conditions, the fitted mass should come out to be 80.0 GeV within statistical errors.

STUDY 1

In study 1, MC data generated with and without radiative decays were fit to the theoretical distribution in Set 2 (described above) that did not include radiation. Ten million events were generated in each data sample. The results are listed in Table 1 and show that the mass obtained when there are radiative decays in the data but not in the theory is low by ≈ 60 MeV . The mass obtained when there were no radiative events in the MC data is 80.0 GeV within errors, as expected.

% Events with γ	M_W GeV	ΔM_W MeV
0.0	80.007 $\pm$ .006	
30.6	79.947 $\pm$ .008	-60 $\pm$ 10

Table 1: Masses obtained from fits to MC data generated with radiative decays (line 2) and without (line 1). The theory did not include radiative decays.

STUDY 2

In study 2, the effect on the W mass of varying the minimum energy of the radiated photon from 50 MeV (nominal value) to 5 GeV was determined. The percentage of events with a γ varied according to the calculation of Berends and Kleiss. There were 10 million MC events generated for each value of $E_\gamma(min)$ and these were divided into sets of 8000 events each and fit to the Set 3 theoretical distributions. The results listed in the first part of Table 2 show the average of the fitted W masses and the differences from the sample without radiative decays (line 1). The W mass is measured to drop by ≈ 40 MeV when going from no radiative decays in the sample to a sample with $E_\gamma(min) = 50$ MeV or more. This difference in mass is approximately constant as the minimum energy is raised. It (perhaps) becomes smaller at a minimum γ energy equal to 5 GeV. The shift in mass of the six independent MC data samples for different $E_\gamma(min)$ is about 2 sigma smaller than the shift expected from Study 1. For this study, large MC data samples were fit to a theory sample with relatively small statistics, causing the theory sample to be sensitive to statistical fluctuations. Since we are interested only in this study in the change in mass for deviations of $E_\gamma(min)$ from its nominal value, the slightly smaller observed overall shift should not impact this study.

The results show little dependence of the W mass on E_γ(min). To understand better this dependence and avoid generating even larger statistical samples, the difference between the $e\nu$ invariant mass and the generated W mass was calculated on an event by event basis as a function of E_γ(min). Cuts were applied as in the data on the resolution smeared variables: electron and neutrino $P_T > 25$ GeV/c, and W $P_T < 30$ GeV/c. The (resolution smeared) transverse mass was limited to the range used in the fits, $60.0 < M_T < 90.0$. The results are given in the second part of Table 2. The shift downward in mass actually grows slightly with E_γ(min) before falling to zero at large E_γ(min), as it must. There is very little dependence on this variable for values in the region of several hundred MeV or less.

STUDY 3

A number of MC data sets were generated in which the fraction of radiative decays was varied from 0 to 100%. The minimum photon energy in these data was 50 MeV. The percentage of events with a photon is fixed by calculation once the photon energy is set (30.6% with the nominal minimum photon energy). The data are fit to theory Set 1. The mass obtained when the percentage of events with radiative decays is the nominal value is 80 GeV , within errors, as expected, since the theory also included radiative decays. The results are listed in Table 3. A 100% increase in the fraction of decays changes the mass by 15-20 MeV . This fraction can be calculated directly from QED with small uncertainty, certainly less than 10%. The uncertainty in the W mass due to the variation of the fraction of events with a radiative decay is entirely negligible.

STUDY 4

The cut on the photon-electron separation, $R_{e\gamma}$, was varied in study 4 to determine the sensitivity of the W mass to this separation. The nominal value of the separation, which determines when the γ and electron coalesce, is 0.3. Three million MC events with $E_\gamma(\text{min}) = 50$ MeV were generated at each value of the separation cut and were fit in groups of 8000 to theory Set 2 with radiation. The average values of the fitted W mass are shown in the first part of Table 4. The differences in mass are much smaller than the errors.

The dependence of the $e\nu$ mass on the separation cut was also obtained directly from the Monte Carlo sample. The same cuts placed on the data were placed on the Monte Carlo sample. The difference between the effective $e\nu$ mass, where e includes the γ or not depending on whether the distance between the γ and e is less than or greater than $R_{e\gamma}$, and the generated W mass was calculated exactly, without resolution smearing, for the radiative decays. The results are shown in the second part of Table 4. The dependence of the $e\nu$ invariant mass on this cut is now clear, but varies by only 10 MeV when the cut is varied from $R_{e\gamma}= 0.2$ to $R_{e\gamma}= 0.4$. This calculation also reproduced the result of Study 1 for the shift in the W mass (here 63 MeV). The result suggests that the shift in the fitted W mass is due entirely to the reduced $e\nu$ invariant mass in the radiative sample.

5.2 DØ Data

The studies using actual DØ data include fitting different data sets to theoretical distributions with and without radiative decays. The data sets are

Part 1		
min γ energy (MeV)	M_W (GeV)	ΔM_W (MeV)
∞	80.004 $\pm$ .008	–
50	79.965 $\pm$ .008	-39 $\pm$ 11
100	79.965 $\pm$ .008	-39 $\pm$ 11
200	79.971 $\pm$ .008	-33 $\pm$ 11
400	79.963 $\pm$ .008	-41 $\pm$ 11
1000	79.968 $\pm$ .008	-36 $\pm$ 11
5000	79.983 $\pm$ .008	-21 $\pm$ 11

Part 2			
min γ energy (GeV)	# events with γ	ΔM_W events with γ (GeV)	ΔM_W all events (MeV)
0.05	7839	-.218	-57.5
0.075	7414	-.230	-58.2
0.1	7051	-.241	-58.9
0.2	6096	-.278	-60.7
0.5	4890	-.337	-61.7
1.0	3924	-.396	-60.3
2.0	3007	-.460	-55.6
3.0	2503	-.502	-51.6
5.0	1889	-.518	-41.2
10.0	1108	-.408	-19.7
15.0	739	-.350	-11.4
20.0	523	-.317	-7.41
25.0	372	-.219	-4.87
35.0	146	-.132	-.872
50.0	27	-.063	-.078
# events without γ: 21861			

Table 2: Part 1: Masses obtained from fits to MC data generated with varying values of $E_\gamma(\min)$. The theory did not include radiative decays. Part 2: The $e\nu$ invariant mass minus the generated W mass as a function of $E_\gamma(\min)$ for events selected using standard cuts.

% Events with γ	M_W GeV	ΔM_W MeV
0.0	80.071 $\pm$ .018	+74 $\pm$ 24
30.6	79.997 $\pm$ .016	
50.0	79.982 $\pm$ .016	-15 $\pm$ 24
70.0	79.975 $\pm$ .016	-22 $\pm$ 24
100.0	79.864 $\pm$ .021	-133 $\pm$ 28

Table 3: Masses obtained from fits to MC data generated with varying % of events containing a radiative decay. The minimum photon energy is fixed at 50 MeV (% events = 30.6). The data are fit to theory with radiative decays.

Part 1	
γe Separation	M_W (GeV)
.20	80.024 $\pm$ .015
.25	80.019 $\pm$ .015
.3	80.019 $\pm$ .015
.35	80.030 $\pm$ .015
.40	80.026 $\pm$ .015
Part 2	
γe Separation	ΔM_W (MeV)
.10	-97.7
.15	-79.0
.20	-67.9
.25	-65.0
.30	-62.6
.35	-59.7
.40	-57.5
.45	-52.7
.50	-47.1

Table 4: Part 1: Masses obtained from fits to MC data generated with varying values of the $R_{e\gamma}$ cut. The theory included radiative decays. Part 2: Direct determination of the difference in W invariant mass in MC data when varying the separation cut.

those on which the results presented at Tsukuba in October 1993 [2] are based, and newer data sets from the expressline and all-stream. All the data were processed using RECO 11.19 and were selected using the standard cuts.

STUDY 5

The Tsukuba data set, which had 4817 events in the fit in the CC, was fit to theory Set 2 with and without radiative decays to determine what effect adding radiation to the theory makes to real data. For the results shown in Tsukuba, the data was fit to theory without radiative decays. The results are listed in Table 5 and show that properly including radiative decay in the theory raises the W mass by 50 MeV.

theory	M_W (GeV)
with radiation	79.940 $\pm$ .16
without radiation	79.890 $\pm$ .16

Table 5: DØ Tsukuba data fit to theory with and without radiation.

STUDY 6

The RECO 11.19 expressline data set was fit to theory Set 2 with and without radiative decays. There were 5883 events in the fit. The results are listed in Table 6. In this sample, the mass is 49 MeV higher when radiative decays are included in the theory.

theory	M_W (GeV)
with radiation	80.070 $\pm$ .137
without radiation	80.021 $\pm$ .139

Table 6: DØ Expressline data fit to theory with and without radiation.

STUDY 7

The RECO 11.19 all-stream data set was fit to theory Set 2 with and without radiative decays. There were 5888 events in the fit and the results are listed in Table 7 and again show a mass difference of 50 MeV between fits to theory with radiation and theory without radiation.

STUDY 8

The RECO 11.19 all-stream data was processed with different H-matrix cuts (nominal cut: H-matrix $\chi^2 < 100$) and fit to theory Set 2 with radiative decays to determine the impact of this cut on the mass of the W. The results, given in Table 8, show no statistically significant dependence of the W mass on the H-matrix χ^2 cut.

6 Monte Carlo Simulation of Z

Radiative decays of the Z were also added to the Monte Carlo program which simulates Z decay. Theoretical distributions of the ee invariant mass were generated at twenty-one values of the Z mass centered approximately on the measured Z mass and separated by 200 MeV increments. Two million events with radiative decays and two million without were generated at each point and approximately 25% end up in the theoretical spectra. The data (Run 1a data and mock data generated by the MC program) were then compared with these theoretical distributions and a maximum likelihood method used to determine the best value for M_Z. The theoretical distributions were generated using the "toy" calorimeter, as in the W mass analysis.

7 Results of Studies - Z mass

7.1 Monte Carlo Data

STUDY 9

In this study the dependence of the Z mass on $E_\gamma(\text{min})$ was studied. The minimum photon energy was varied in the data from 50 MeV (the nominal value) to 400 MeV. The theoretical distribution did not include radiative decays. The results are listed in Table 9. The first line of the table is a fit to MC data in which radiative decays were turned off and gives back the generated 90 GeV for the mass. The 139 MeV difference between the masses obtained when radiative decays were and were not included in the data is insensitive to the minimum photon energy, certainly for thresholds below several hundred MeV.

STUDY 10

Monte Carlo data generated with radiative decays were fit to theoretical distributions with and without radiative decays. Comparisons to theory were made separately for the cases in which both electrons were in CC, one electron was in CC and one in ECS, and both electrons were in ECS. Fits of the data with radiative decays in the theory give back the generated mass within the statistical errors. The average and rms of the difference in mass obtained from fitting the data with radiative decays to the theory with and without radiative decays are listed in Table 10. For the case in which both electrons

theory	M_W (GeV)
with radiation	80.040 ± .139
without radiation	79.990 ± .141

Table 7: DØ All-Stream data fit to theory with and without radiation.

H-Matrix Cut	# Events Fit	M_W (GeV)
75	5719	80.077 ± .139
100	5888	80.065 ± .138
200	6137	80.065 ± .135
300	6214	80.084 ± .134
400	6250	80.073 ± .134
500	6269	80.065 ± .134
1000	6300	80.063 ± .134
no cut	6319	80.068 ± .134

Table 8: DØ data with varying H-Matrix cuts fit to theory with radiation.

min γ energy (MeV)	M_Z (GeV)	ΔM_Z (MeV)
∞	89.994 ± .015	–
50	89.855 ± .019	-139 ± 24
100	89.856 ± .015	-138 ± 21
400	89.886 ± .011	-108 ± 19

Table 9: Masses obtained from fits to MC data generated with different values of $E_\gamma(\mathrm{min})$. The theory did not include radiative decays.

are in the CC, the fitted Z mass is 161 MeV higher when radiative decays are included in the simulation. The corresponding shift for W events, in which just one electron is in CC, is 60 MeV (Studies 1, 2 and 4). It is the ratio of these two processes which is used in DØ to determine the W mass.

data type	# events in each sample	ΔM_Z (MeV)
CC-CC	500	161 $\pm$ 9
CC-ECS	200	193 $\pm$ 21
ECS-ECS	100	230 $\pm$ 53

Table 10: Differences in mass obtained from fits of many samples of MC data to theory generated with and without radiation.

STUDY 11

The cut on the photon-electron separation, $R_{e\gamma}$, was varied to determine the sensitivity of the Z mass to this cut. As in the corresponding study for the W's (Study 4), the nominal value for this cut is 0.3. There were 100 samples of Monte Carlo Z events, each with approximately 400 events, generated for each of 5 different $R_{e\gamma}$ values. These were fit to a 10 million event theoretical distribution that included radiative decays. The average and error of the difference in mass obtained from fitting the data with the different cuts is listed in Table 11. The difference in the mass of the Z from that at the nominal value is not substantially different than the error.

To get a better handle on this variation, we repeat the analysis done for the W's and study the difference between the ee invariant mass and the generated Z mass as a function of $R_{e\gamma}$. The radiated photon momentum is added to that of the nearest electron if its separation is less than $R_{e\gamma}$. The results are given in Part 2 of Table 11. The variation with $R_{e\gamma}$ is again small. The Z mass obtained from a fit to the ee invariant mass between 70 and 110 GeV shifts by 160 MeV (at $R_{e\gamma} = 0.3$) when radiative decays are included in the theory. This differs from the mean shift, which is 227 MeV . It should not be surprising that the shift in mass, calculated from an average of the di-electron invariant mass differs from the fitted mass. The average photon energy for events in the high mass tail of the Z resonance is about 25% higher than for events from the low mass tail of the resonance. This effect skews the distribution. A straight average of the invariant mass will therefore yield a mass lower than the one obtained from a mass fit.

γe Separation	ΔM_Z (MeV)
.20	-24 $\pm$ 23
.25	-45 $\pm$ 25
.30	–
.35	18 $\pm$ 22
.40	4 $\pm$ 23

Part 2

γe Separation	ΔM_Z (MeV)
.10	-325
.15	-277
.20	-243
.25	-234
.30	-227
.35	-222
.40	-218
.45	-206
.50	-196

Table 11: Part 1: Difference in mass obtained from fits to MC data for different values of $R_{e\gamma}$with respect to the nominal value of $R_{e\gamma}$= 0.3. The theory included radiative decays. Part 2: The ee invariant mass minus the generated Z mass for different values of $R_{e\gamma}$.

7.2 DØ Data

STUDY 12

The data were fit to the same theoretical spectra used in Study 11. The 3 different types of data samples were selected using the standard cuts. The mass obtained when fitting to the theory with radiation and also the difference in mass when fitting to theory without radiation are listed in Table 12. For events in CC, the fitted Z mass is 142 MeV higher when radiative decays are included in the theory. This agrees well with the 162 MeV expected from MC studies (Study 10). The observed shift in the mass for CC-ECS events is smaller than expected from the Monte Carlo, but is well within the statistical error.

data type	# events in sample	M_Z (GeV)	ΔM_Z (MeV)
CC-CC	373	87.719 ± .18	142
CC-ECS	150	87.604 ± .37	90
ECS-ECS	142	88.492 ± .76	238

Table 12: Masses obtained from fits of DØ data to theory with radiation and the difference in mass between fits to theory with and without radiation.

data type	ΔM_W (MeV)	ΔM_Z (MeV)	e	ΔM_W^{D0} (MeV)
MC	60 ± 10	161± 9	$(-10.4 \pm 1.6) \times 10^{-4}$	-83 ± 13
Data	50	142	-9.9×10^{-4}	-79

Table 13: Change in W and Z masses when adding radiative decays to the theory for MC and DØ data (columns 1 and 2). Change in final DØ W mass result when adding radiative decays to theory (column 4).

8 Conclusions

The Z mass determined from the Run 1a data increased by 142 MeV when radiative decays were properly included in the theoretical simulation. This increase was expected on the basis of MC studies to be 161 $\pm$ 10 MeV . The W mass increased by 50 MeV (studies 5,6,7). An increase of 60 $\pm$ 9 MeV was expected from MC studies.

In DØ we define the mass of the W as

$$M_W^{D0} = (M_W/M_Z) \times M_Z^{LEP}$$

where M_W and M_Z are the fitted values. Therefore

$$M_{Wrad}^{D0} \approx M_{Wnorad}^{D0} \times [1 + e]$$

where

$$e = \Delta M_W/M_W - \Delta M_Z/M_Z,$$

with ΔM the shift in mass when taking radiation into account.

We calculate the factor e using the differences in fitted values from the results of study 1 and 10 for the MC data and study 5 and 12 for the DØ data. The change in the W mass resulting from the inclusion of radiation is calculated as $e \times M_W$. Thus, the inclusion of radiative decays in the theoretical distribution used to fit the data lowers the W mass by approximately 83 $\pm$ 13(stat) MeV in the high statistics MC data samples, and by approximately 79 MeV in the data. These results are listed in Table 13.

The systematic uncertainty resulting from the dependence on the $R_{e\gamma}$ cut is small. A full MC simulation shows no dependence at the 10-15 MeV level. An exact calculation of the dependence of the $e\nu$ invariant mass on this cut gives a variation of the W mass of $\pm$ 5 MeV when $R_{e\gamma}$ is varied by ± 0.1 from its nominal value of $R_{e\gamma} = 0.3$. The corresponding variation for Z decays is $\pm$ 13 MeV (Table 11). The variation in the fitted mass should be somewhat less, and that in $(M_W/M_Z) \times M_Z^{LEP}$ is $\pm$ 7 MeV . The dependence on the calculated fraction of decays with $E_\gamma > 50$ MeV (the hard fraction of radiative decays in the language of ref. 1) was also small. The choice $E_\gamma(min) = 50$ MeV is sufficiently low, resulting in negligible dependence of the W (or Z) mass on this parameter. Although the isolation cut is properly taken into account in the Monte Carlo simulation, uncertainties can arise from inefficiencies caused by the H-matrix cut when the $R_{e\gamma}$ separation is between 0.2 and 0.4. Radiative decays generated by this MC program were put through the full plate level GEANT simulation and efficiencies extracted as a function of the photon energy and separation [3]. Incorporating these efficiencies into the MC program resulted in a change in the W mass of (9 ± 14) MeV . The plate level GEANT studies also confirm the choice $R_{e\gamma}$= 0.3, with an uncertainty less than the ± 0.1 variation considered above.

In conclusion, we assign an overall systematic error of 10 MeV to the ratio of the W to Z mass due to radiative effects. Finally it should be noted that we have considered only order α corrections to the lowest order diagrams and have not included processes in which two or more photons are radiated. We leave this to future studies.

References

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