DØ Note #2921

W Mass Analysis - Run 1A
Electron Direction Definition and Resolution

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1 Introduction

To calculate the angles of the electron direction in the W and Z mass analysis, we use the center of gravity (*cog*) of the CDC track (from the DTRK bank) and the hit in the calorimeter (taken at shower max - EM3). In the W data sample, the event vertex and angle of the electron are obtained by fitting the *cog* of the CD hit and the CAL hit to a line and projecting to the z-axis. In the Z data sample, a constrained fit is used with the 2 CD hits and 2 CAL hits to determine the vertex and angles.

This paper describes why these quantities were chosen to define the electron angles and how the resolutions of these quantities are extracted from the data. From the data, we extract the RECO vertex resolution, the CD hit resolution, the angular resolution, the CAL hit resolution and the vertex resolution when the electron direction is determined using various methods.

In the analyses described below, the standard W and Z requirements are placed on the data. Additional requirements on the number of RECO vertices in the event are made when specified.

2 Corrections to the Data

The W and Z data sample are from RECO V11.19 DSTs which have been corrected using CORRECTEM. The calorimeter hit position is further corrected in the data using CM3POS_PV$'$. The CAL hit correction uses the vertex (determined from the intersection of the electron with the z axis) and therefore the correction and vertex are iterated until they are stable.

For electrons in RECO V11.19, there exists a bias between the z position of the electron hit calculated using the *cog* from tracking, z_{CD}, and using the calorimeter z_{CAL}. Analysis of cosmic ray data [1] showed that applying a scale factor to the z position of the CD hit

$$z_{CD}^{\text{true}} \rightarrow 0.988 \times z_{CD}^{\text{measured}}$$

removes this bias. This scale factor is applied to the final W and Z data sample used in the W mass analysis.

3 RECO Vertex - Problem Distinguishing Vertices

The method currently used to calculate the electron direction angles is in contrast to the method previously used which was to use the vertex from RECO and the calorimeter (CAL) hit position. However from studies of the data, it was determined that in events with greater than two RECO vertices, there were problems distinguishing the vertices if they were closer than 10 cm apart. This is seen in Fig. 1 which shows the distribution of the difference in the z positions of the first two RECO vertices in W and Z events. In addition to the standard event selection requirements, the requirements also were made that there be more than 1 vertex and that multiple interaction flag (MIFLAG) be greater than 2. As seen in the figure, there is a hole in the distribution at ± 10 cm and therefore two vertices closer than 10 cm are not consistently distinguished. Therefore, using the RECO vertex in the determination of the electron direction would lead to many problems and another method was chosen. The following sections describe analyses to extract resolutions from the data and show how the method we use to define the electron angles was chosen.

4 Determination of RECO Vertex Resolution

To determine the RECO vertex resolution, we use Z events and calculate the electron direction using the z positions of the CD and CAL hit, z_{CD} and z_{CAL}. The intersection of each electron with the z axis, z_v, and the difference between these intersections S_1 is determined:

$$z_v(1) = z_{CD}(1) - \frac{R_1}{R_2}\left(z_{CAL}(1) - z_{CD}(1)\right)$$

$$z_v(2) = z_{CD}(2) - \frac{R_1}{R_2}\left(z_{CAL}(2) - z_{CD}(2)\right)$$

and

$$S_1 = z_v(1) - z_v(2).$$

Here R_1 is the radial distance from the z-axis to the center of gravity of the hit in the CD (typically 61.8 cm), and R_2 is the radial separation of the CD hit and the calorimeter hit (typically 31 cm). We can also calculate the deviation of the average of the projections of the two electron tracks in the Z event from the reconstructed vertex z_{vtx}^{RECO}:

$$S_2 = \frac{z_v(1) + z_v(2)}{2} - z_{vtx}^{RECO}$$

The errors in S_1 and S_2 are

$$(\delta S_1)^2 = 2(1 + \frac{R_1}{R_2})^2(\delta z_{CD})^2 + 2(\frac{R_1}{R_2})^2(\delta z_{CAL})^2$$

$$(\delta S_2)^2 = 0.5(1 + \frac{R_1}{R_2})^2(\delta z_{CD})^2 + 0.5(\frac{R_1}{R_2})^2(\delta z_{CAL})^2 + (\delta z_{vtx}^{RECO})^2$$

where we have ignored the correlation introduced by the fact that z_{vtx}^{RECO} uses the electron tracks. From these two equations we can calculate directly the resolution in z_{vtx}^{RECO}:

$$\delta z_{vtx}^{RECO} = \sqrt{(\delta S_2)^2 - \frac{(\delta S_1)^2}{4}}.$$

The statistical error in the resolution is

$$\Delta(\delta z_{vtx}^{RECO}) = \frac{1}{\delta z_{vtx}^{RECO}} \left[(\delta S_2)^2 (\Delta \delta S_2)^2 - \frac{(\delta S_1)^2 (\Delta \delta S_1)^2}{16} \right]^{\frac{1}{2}}$$

The distributions of S_1 and S_2 for all events and with the added requirement that there be only 1 RECO vertex in the event are shown in Figs. 2 and 3. Using the sigmas and obtained from Gaussian fits to these distributions for δS_1 and δS_1, we obtain the vertex resolution

$$\delta z_{vtx}^{RECO} = 1.491 \pm .1543, 1.781 \pm .1462 \text{ cm}$$

for events with only 1 vertex and for all events, respectively. The resolutions obtained are underestimates in that there are non-Gaussian tails in the distributions.

5 Determination of CD Hit and Angle Resolution

We can also estimate the CD hit and CD angle (from the DTRK bank) resolution from the data. The CD *cog* hit and the angle are determined by fitting up to eight measurements at r = 52.093, 55.913, 57.275, 61.095, 62.491, 66.311, 67.733, and 71.553 cm which are the r coordinates of the eight delay lines. If the track is observed in all eight delay lines, then

$$< r >= 61.808 \text{ cm}$$

$$\sqrt{< (r- < r >)^2 >} = 6.133 \text{ cm}$$

and the errors in the *cog* of the hits in the CD, z_{CD}, and the slope of the line fit to the hits $m = dz_{CD}/dr$ are

$$\delta z_{CD} = \frac{\delta z_{CD}^i}{\sqrt{N}} \text{ with } N = 8$$

$$\delta m = \frac{\delta z_{CD}}{6.133}$$

where z^i_{CD} is the individual CD hit resolution. The errors are uncorrelated because the *rms* value of r was calculated about the mean $< r >$. One can exhibit explictly the dependence of the slope on the radial extent, R_{CD}, of the measurements in the CD. The distance between the first and last hit is $R_{CD} = 19.46$ cm and therefore the resolution of the slope can be written as

$$\delta m = \frac{3.17\delta z_{CD}}{R_{CD}}.$$

The angular resolution is

$$\delta\theta_{CD} = sin^2\theta\delta m$$

These results change slightly if all eight hits in the CD are not present.

These resolutions can be obtained directly from Z events as follows. Project both electron tracks to the z axis as before, but this time use the angle measured in the CD and the position of the CD center of gravity,

$$z^*_v(1) = z_{CD}(1) - m_1 R_1$$

$$z^*_v(2) = z_{CD}(2) - m_2 R_1$$

and define the difference

$$S_3 = z^*_v(1) - z^*_v(2)$$

The error in this distribution is then

$$\delta S_3^2 = 2(\delta z_{CD})^2 + 2R_1^2(\delta m)^2$$

and therefore

$$\delta S_3 = \sqrt{2}(\delta z_{CD})\left[1 + \left(\frac{3.17R_1}{R_{CD}}\right)^2\right]^{\frac{1}{2}}$$

Substituting in for the R_1 and R_{CD} gives

$$\delta z_{CD} \approx \frac{\delta S_3}{14.3}.$$

The errors on the resolutions are

$$\Delta(\delta z_{CD}) = \frac{\Delta(\delta S_3)}{14.3},$$

$$\Delta(\delta m) = \frac{3.17}{14.3 R_{CD}} \Delta(\delta S_3),$$

and

$$\Delta(\delta z^i_{CD}) = \frac{\sqrt{N}}{14.3} \Delta(\delta S_3).$$

The error is dominated by the error in the slope. The distributions in the variable S_3 are shown in Fig. 4. From the width of the fit to a Gaussian, δz_{CD} is found to be

$$\delta z_{CD} = .272 \pm .0229, .296 \pm .0198 \text{ cm}$$

for events with only 1 vertex and for all events, respectively. The error in the slope, which is the CD angular resolution at $\theta = \pi/2$, is found to be

$$\delta m = \delta\theta_{CD} = 44 \pm 3.7, 48 \pm 3.2 \text{ mrad}$$

for events with only 1 vertex and for all events, respectively. The average resolution in the individual CD hits is $\sqrt{8}$ times larger than that in the center of gravity

$$\delta z^i_{CD} = .770 \pm .0648, .837 \pm .0560 \text{ cm}$$

for events with only 1 vertex and for all events, respectively.

6 Determination of the Calorimeter Hit Resolution

The resolution in z_{CD} may be put into the expression for δS_1 above to obtain the calorimeter hit resolution:

$$(\delta z_{CAL})^2 = \frac{(\delta S_1)^2 - 2(1+\frac{R_1}{R_2})^2(\delta z_{CD})^2}{2(\frac{R_1}{R_2})^2}$$

and the error in the resolution is

$$\Delta(\delta z_{CAL}) = \frac{1}{2(\frac{R_1}{R_2})^2(\delta z_{CAL})}\left[(\delta S_1)^2(\Delta\delta S_1)^2 + \frac{4(1+\frac{R_1}{R_2})^4}{(14.3)^4}(\delta S_3)^2(\Delta\delta S_3)^2\right]^{\frac{1}{2}}.$$

Using the sigma of a fit to a Gaussian of the S_1 distribution from Fig. 2 gives

$$\delta z_{CAL} = 0.8492 \pm .0774, 0.7433 \pm .0734 \text{ cm}$$

for events with only 1 vertex and for all events, respectively.

The calorimeter hit resolution can also be obtained from W events. The electron track determined from the RECO vertex and the CD hit is projected to the calorimeter radius, $R_3 = 91.68$ cm, and compared to the calorimeter hit:

$$S_4 = z_{CD} + \frac{R_2}{R_1}\left(z_{CD} - z_{vtx}^{RECO}\right) - z_{CAL}.$$

The error in S_4 is

$$(\delta S_4) = \left[(1+\frac{R_2}{R_1})^2(\delta z_{CD})^2 + (\frac{R_2}{R_1})^2(\delta z_{vtx}^{RECO})^2 + (\delta z_{CAL})^2\right]^{\frac{1}{2}}$$

and therefore

$$\delta z_{CAL} = \left[(\delta S_4)^2 - (1+\frac{R_2}{R_1})^2(\delta z_{CD})^2 - (\frac{R_2}{R_1})^2(\delta z_{vtx}^{RECO})^2\right]^{\frac{1}{2}}$$

The error in the resolution is then

$$\Delta(\delta z_{CAL}) = \frac{1}{\delta z_{CAL}} \left[(\delta S_4)^2 (\Delta\delta S_4)^2 + (1 + \frac{R_2}{R_1})^4 (\delta z_{CD})^2 (\Delta\delta z_{CD})^2 + (\frac{R_1}{R_2})^4 (\delta z_{vtx}^{RECO})^2 (\Delta\delta z_{vtx}^{RECO})^2 \right]^{\frac{1}{2}}.$$

Using the *rms* of the S_4 distribution shown in Fig. 5 and the previously determined values of δz_{CD} and δz_{vtx}^{RECO} gives

$$\delta z_{CAL} = .8836 \pm .0714, .7875 \pm .0883 \text{ cm}$$

for events with only 1 vertex and for all events, respectively.

7 Determination of the Angular Resolution using CD and CAL Hits

The angle defined using the CD and CAL hits is defined as

$$tan\theta = \frac{z_{CAL} - z_{CD}}{R_2}$$

and the error on the angle is

$$\delta\theta = \left(\frac{cos^2\theta}{R_2}\right) \left[\delta z_{CAL}^2 + \delta z_{CD}^2\right]^{\frac{1}{2}}$$

This resolution is calculated by substituting in the values of δz_{CD} and δz_{CAL} from the Z data studies. The error on the resolution is

$$\Delta\delta\theta = \frac{\delta\theta}{(\delta z_{CAL}^2 + \delta z_{CD}^2)} \left[\delta z_{CAL}^2 (\Delta\delta z_{CAL})^2 + \delta z_{CD}^2 (\Delta\delta z_{CD})^2\right]^{\frac{1}{2}}$$

Using the previously measured values, we obtain at $\theta = 0$

$$\delta\theta = 28.8 \pm 2.4, 25.8 \pm 2.2 \text{ mrad}$$

for events with only 1 vertex and for all events, respectively.

8 Determination of Vertex Resolutions

The z position of the RECO vertex is not used in the electron direction definition because of the problem of distinguishing vertices in events with multiple interactions (as discussed in section 3). We use Z data to determine the resolution in the z position of the vertex. Two cases are considered. In the first case, the electron track is obtained from the CD hit and CAL hit and the the resolution is extracted from the distribution in the variable S_1, defined above. In the second case, the electron track is determined from the CD hit and CD angle and the resolution is extracted from the distribution in the variable S_3, also defined above. Then

$$\delta S_1^2 = (\delta z_{v1})^2 + (\delta z_{v2})^2 = 2(\delta z_v)^2$$

$$\delta S_3^2 = (\delta z^*_{v1})^2 + (\delta z^*_{v2})^2 = 2(\delta z^*_v)^2$$

and the errors on the resolutions are then

$$\Delta(\delta z_v^{(*)}) = \frac{1}{\sqrt{2}}(\Delta\delta S_{1(3)}).$$

The vertex resolution obtained from a single electron track (as in W events) obtained from these distributions is:

$$\delta z_v = 1.884 \pm .1364, 1.732 \pm .1223 \text{ cm} \quad \text{(using CD and CAL hit)}$$

and

$$\delta z_v^* = 2.751 \pm .2314, 2.994 \pm .2004 \text{ cm} \quad \text{(using CD hit and angle)}$$

for events with only 1 vertex and for all events, respectively. To get the vertex resolution for two electrons (Z events), we divide the above values by $\sqrt{2}$ and obtain

$$\delta z_v = 1.332 \pm .0964, 1.225 \pm .0865 \text{ cm} \quad \text{using CD and CAL hit}$$

and

$$\delta z_v* = 1.945 \pm .1636, 2.117 \pm .1417 \text{ cm} \quad \text{using CD hit and angle}$$

for events with only 1 vertex and all for events, respectively.

9 Summary of Results

Listed in Tables 1 and 2 are the results of the above studies. We see from the tables that using the CAL and CD hits gives the best resolution in the measurement of the vertex and angle. In W events, the RECO vertex resolution is comparable to that obtained using the single electron track. It is considerably worse than the resolution obtained using the two tracks in Z events. Given the problems with the RECO vertex in a multi-hit environment, illustrated graphically in Figure 1, we have abandoned the use of the RECO vertex in the reconstruction of the electron momentum in the W mass analysis.

We examine the relative contributions of the energy resolution and angular resolution in the error in the W mass. Since $M_W \approx 2P_T$, we examine the contributions of the energy and angular resolution to $\frac{\delta P_T}{P_T}$ The energy resolution contribution is

$$\frac{\delta P_T}{P_T} \approx \frac{0.13}{\sqrt{P_T}} \oplus 0.02$$

and is ≈ 0.04 at $P_T = 40$ GeV. The angular resolution contribution is

$$\frac{\delta P_T}{P_T} \approx \frac{-(\delta\theta)^2}{2} + \delta\theta cot\theta.$$

If one uses the angle from the CD with $\delta\theta = 48$ mrad, this value is 0.012 at 75°, 0.027 at 60° and 0.048 at 45° and contributes more to the electron P_T resolution than the energy resolution of the calorimeter at small angles ($< 45°$)! Using the angle obtained from the CD and CAL hit with $\delta\theta = 26$ mrad the value is 0.0066 at 75°, 0.015 at 60°, 0.026 at 45° and contributes more than the energy resolution at angles smaller than $\approx 30°$.

Position and Angular Resolutions obtained from Z events

	1 vertex events	all events
RECO vtx	1.49 $\pm$ .154 cm	1.78 $\pm$ .146 cm
CD hit - *cog*	.272$\pm$ .023 cm	.296 $\pm$ .020 cm
CD hit - individual	.770 $\pm$ .065 cm	.837 $\pm$ .056 cm
CAL hit	.849$\pm$ .077 cm	.743 $\pm$ .073 cm
angle from DTRK bank	44$\pm$ 3.7 mrad	48$\pm$ 3.2 mrad
angle from CD and CAL hit	29 $\pm$ 2.4 mrad	26 $\pm$ 2.2 mrad

Position Resolutions obtained from W events

CAL hit	.884 $\pm$ .071 cm	.788 $\pm$ .088 cm

Table 1: Resolutions determined from data of quantities used to determine the electron direction.

References

[1] N. Graf and T. Yasuda.

quantities used	1 vertex events	all events
Position Resolution (cm) of vertex for 1 electron (W events)		
CAL and CD hit	1.88 ± .136	1.73 ± .122
CD hit and angle	2.75 ± .231	2.99 ± .200
RECO vertex	1.49 ± .154	1.78 ± .146
Position Resolution (cm) of vertex for 2 electrons (Z events)		
CAL and CD hit	1.33 ± .096	1.23 ± .087
CD hit and angle	1.94 ± .164	2.12 ± .142
RECO vertex	1.49 ± .154	1.78 ± .146

Table 2: Resolutions of vertex in W and Z events using different quantities to define the electron direction

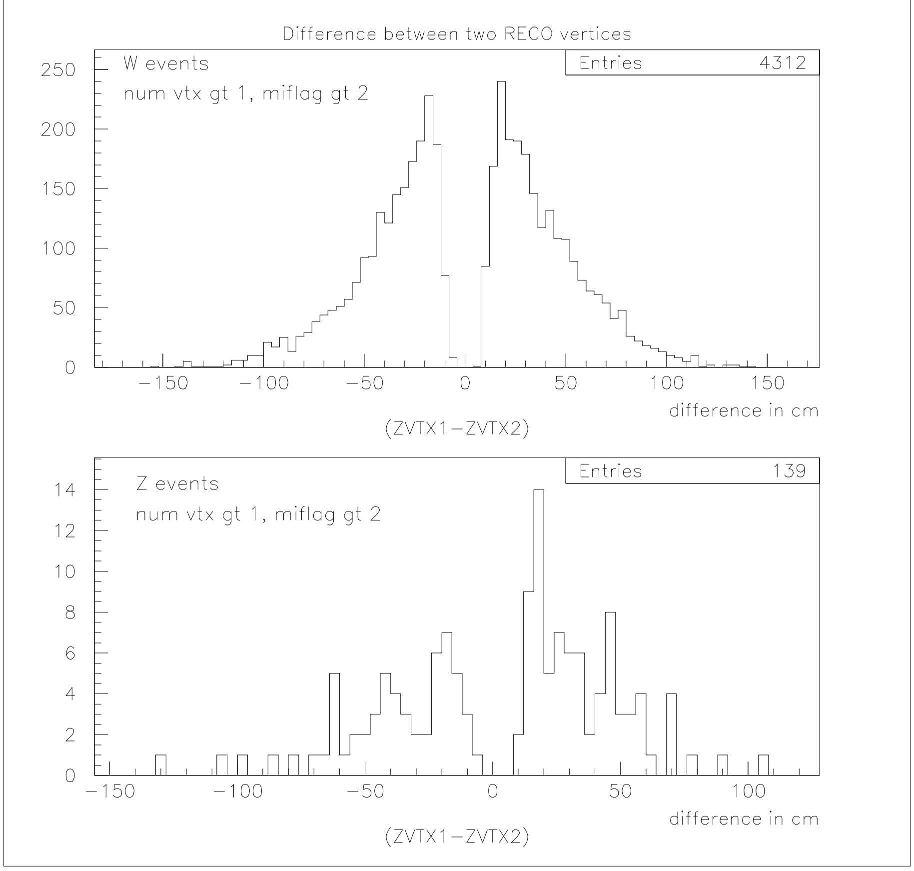

Figure 1: Distribution of the difference along the z axis between two RECO vertices in W and Z events with at least two RECO vertices and with the multiple-interaction flag greater than 2.

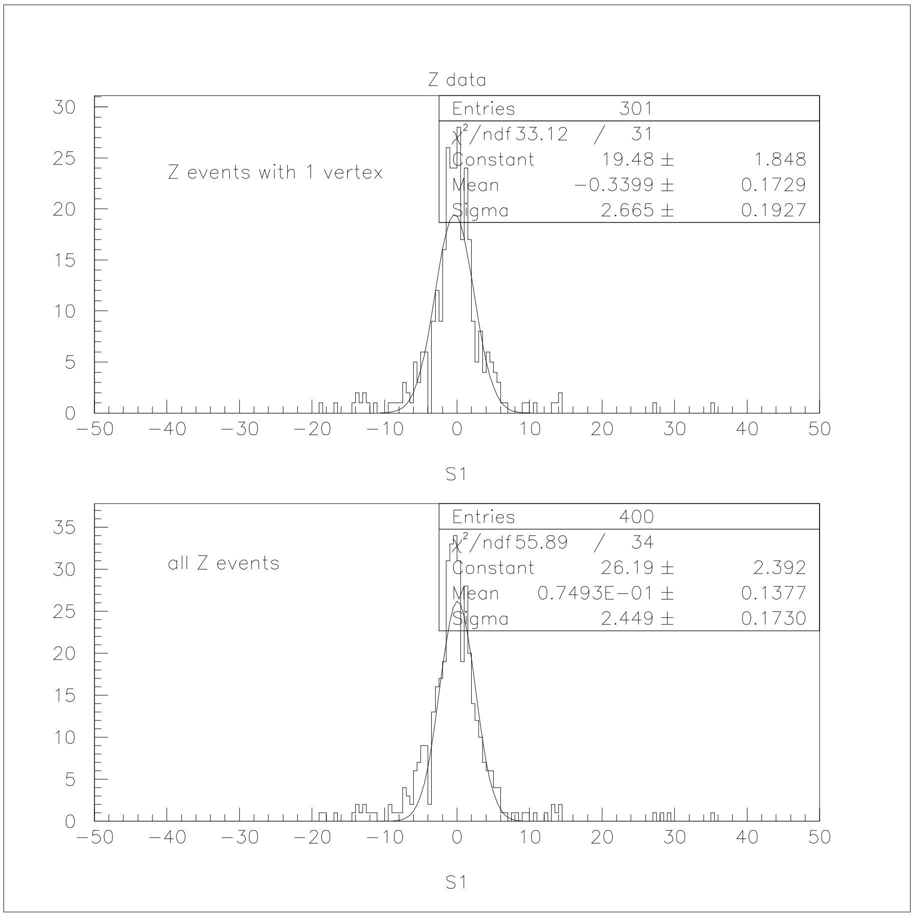

Figure 2: Distribution of quantity $S1$.

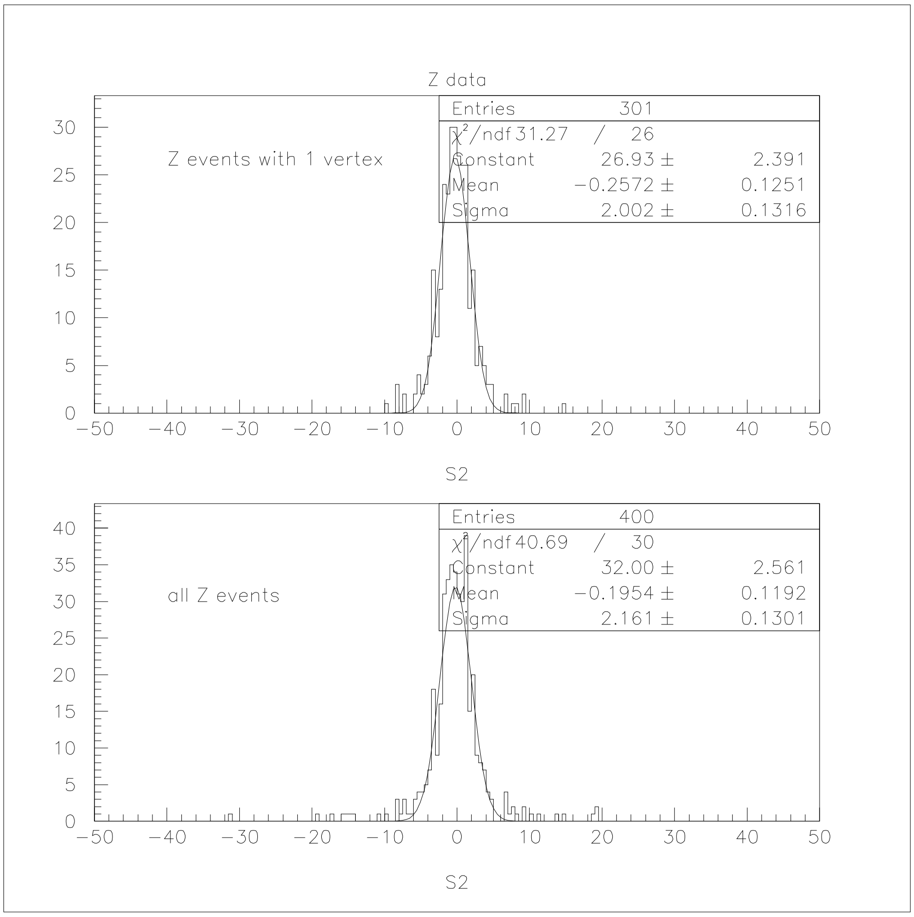

Figure 3: Distribution of quantity $S2$.

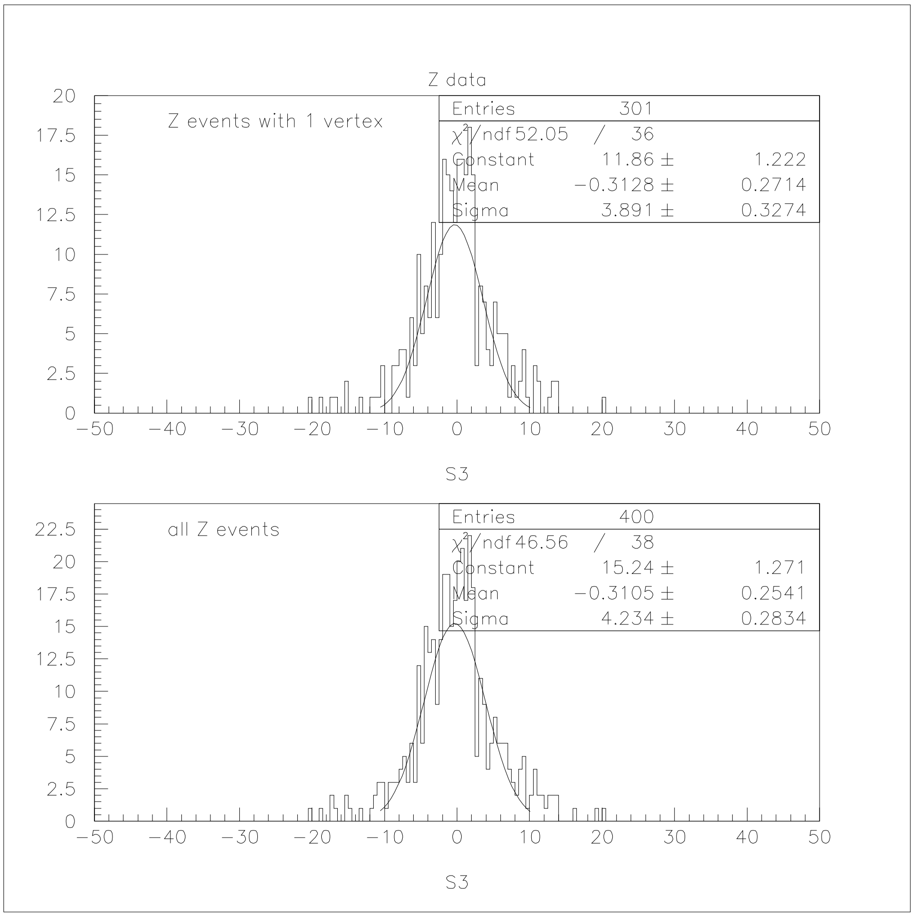

Figure 4: Distribution of quantity $S3$.

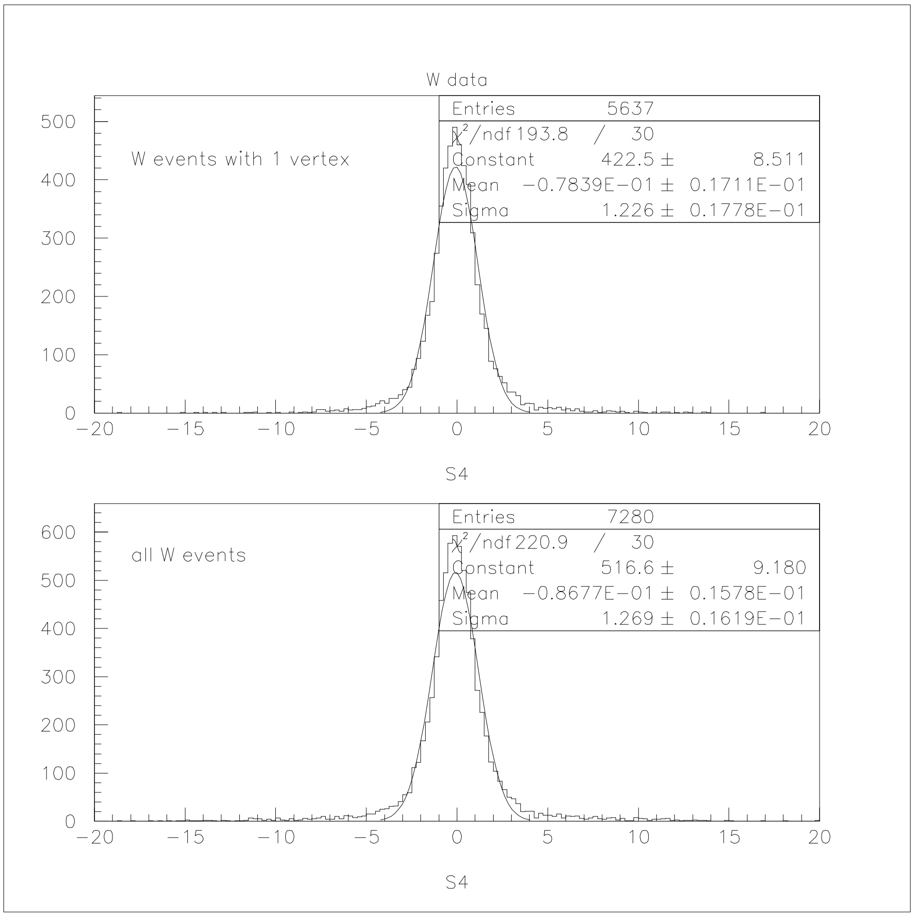

Figure 5: Distribution of quantity $S4$.